ESTIMATING THE APPROPRIATE RISK PROFILE FOR THE TAX SAVINGS: A CONTINGENT CLAIM APPROACH

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Abstract

Tax saving's valuation is crucial for discounted cash flow valuation and WACC estimation. There is an ongoing debate about the appropriate discount rate for tax savings under CAPM approach. On this paper we evaluate tax savings from a contingent claim approach in order to establish a framework which could be compatible with discounted cash flow valuation while being consistent with tax saving's nature. We present two approach based on contingent claims with continuous real and risk neutral probabilities, and propose a strategy based on sequential portfolios of European options for its valuation. We state that the value of tax savings is the present value of tax savings, and provide a valuation formula that can be integrated with a traditional discount cash flow model under WACC and APV.

Key words: contingent pricing, tax savings, WACC, asset pricing, rate of return, risk analysis, discount rate for tax shields, pricing kernel, risk neutral probabilities, stochastic discount factor.

JEL Clasification: C19, G11, G12, G13

1. Introduction

Considering that there has been enough theoretical and empirical evidence about the validity of discounted cash flow valuation for tax saving's valuation (Velez, 2010),(Tham & Wonder, 2002),(Miles & Ezzell, 1985) and (Cooper & Nyborg, 2006); our proposal for tax shield's valuation is based on this fundamental assumption and it's centered on determine the appropriate risk measure and methodology its valuation.

There are three major schools of thought regarding the nature of the risk associated with tax savings and therefore the appropriate discount rate applicable to this cash flow. The first one argues that the risk profile of TS is equal to the debt's (Modigliani & Miller, 1958)(Miles & Ezzell, 1985). The other argues that the appropriate risk profile is the unleveraged asset's (Velez, 2010)(Tham & Velez-Pareja, 2001). The third (Kolari, 2010) argues that the discount rate is the levered cost of equity.

All of the major discussion has been made within a framework based on continuous financial instruments; with cash flows which can be discounted using deterministic discount factors. The validity of this approach depends mainly on the underlying assumptions about the nature of the tax shield as a financial instrument, and its associated risk.

We propose to study tax shields from an unrestricted or generalized view based on stochastic discount factors; which could take into account the actual risk profile of that instrument and translate it into a comprehensive framework consistent with the basic tenet of finance. Under this unrestricted environment; tax shields should be interpreted as stochastic variables, with stochastic cash flows and discount factors, determined by the tax shield's calculation methodology. Such a framework should allow deducting a set of formulae compatible to those used discounted cash flow valuation.

Our proposed framework assumes that EBIT is a stochastic variable which follows a continuous probability distribution, which allows estimating the tax shield's payoff. The later is a discontinuous function of EBIT and risky debt with probability of credit default and cost of debt risk.

2. Tax shields

No matter the way tax shield is seen from the risk point of view, the fulfillment of tax savings depends on the existence of sufficient EBIT and other income which will allow the investor to get the full amount of possible tax benefit; assuming that the company is not on financial distress and payment of debt is guaranteed. This is due to the fact that TS are originated from an accounting record on the income statement and there is no direct relation with the amount of cash available to pay the debt.

This implies that there is no direct relation between EBIT and the default risk; but they are in effect linked. Velez-Pareja (Velez, 2010) proposes that there is a need to differentiate between risky TS and risky debt (in terms of default risk and cost of debt's risk); in a way that they depend on different financial statements, which are in fact very different in their nature and implications.

In real terms, TS can be calculated using the following rule (Velez, 2010):

$$TS = \begin{cases} EBIT + OI < 0 & ; \quad 0 \\ 0 < EBIT + OI < FE; & (EBIT + OI)\tau \\ EBIT + OI \geq FE & ; \quad (FE)\tau \end{cases} \tag{2.1}$$

Assuming a stochastic variable

$$\zeta_i = \left\{ EBIT + OI \middle| (EBIT + OI) \in \Omega \right\} \tag{2.2}$$

Which follows a continuous distribution; which will be considered undefined by the moment; and using the rule stated in (2.1) to define 3 distinct events on a space (Ω, F):

$$A = \{\zeta_i \geq FE\} \tag{2.3}$$

$$B = \{0 < \zeta_i < FE\} \tag{2.4}$$

$$C = \{\zeta_i \leq 0\} \tag{2.5}$$

Where A, B and C are independent and mutually exclusive events; we can estimate the effect of certain probability restrictions over the risk of the TS.

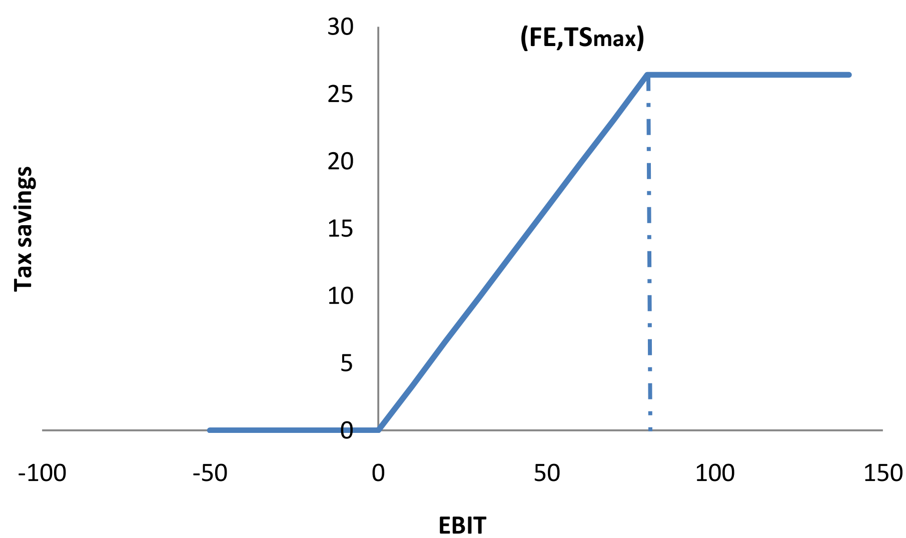

Figure 1. Tax saving's Payoff

First we will consider a scenario in which the event A is certain; or in other words, that $P(A)=1$. Under this assumption we have that all possible outcomes of ζ_i are greater than the financial expenditures (from now on FE) as shown on the next figure.

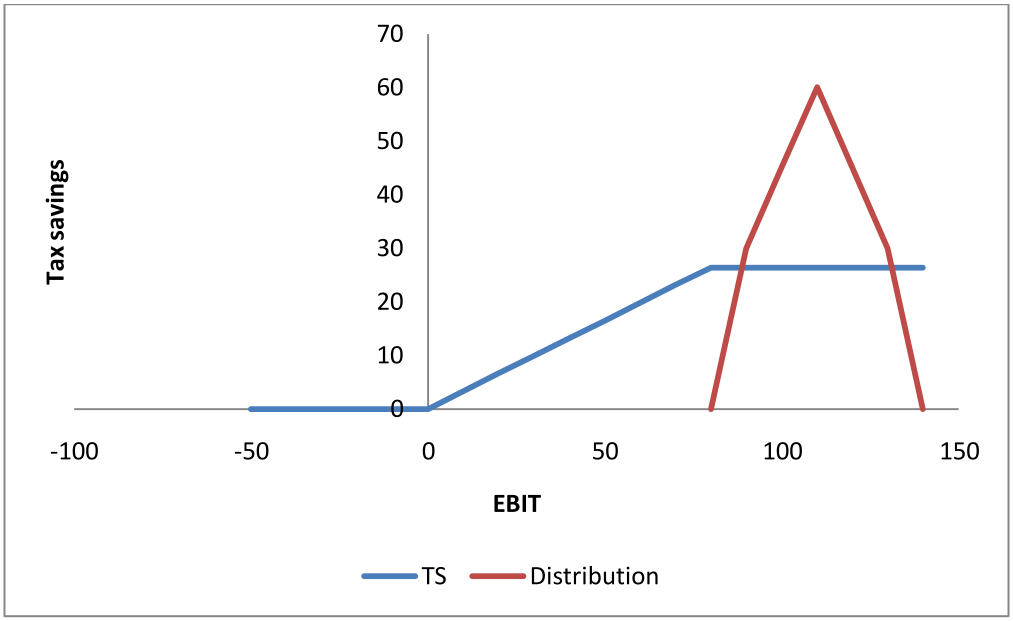

Figure 2. Scenario where P(A)=1

This will cause that no matter the value of EBIT+OI; the TS will be always equal to the FE. If we assume that the TS come solely from the payment of interests, TS_{t+1} will depend on the value of FE_t. Under this condition, TS will be certain (and by definition) risk free and its appropriate discount rate will be K_D.

Taking into account the reality of the operation for any business, the assumption that ζ_i will be always greater than FE is unrealistic. For this reason, it's not advisable to use K_D as the discount rate for TS.

On the other hand, if we assume that $P(A)<1$, there is a possibility that ζ_i will be less that FE; leading to the loss of some portion of the maximum amount of TS available for the company. Under this condition the possible outcomes for EBIT+OI may fall outside of the "risk free area" of the TS (see Figure 3).

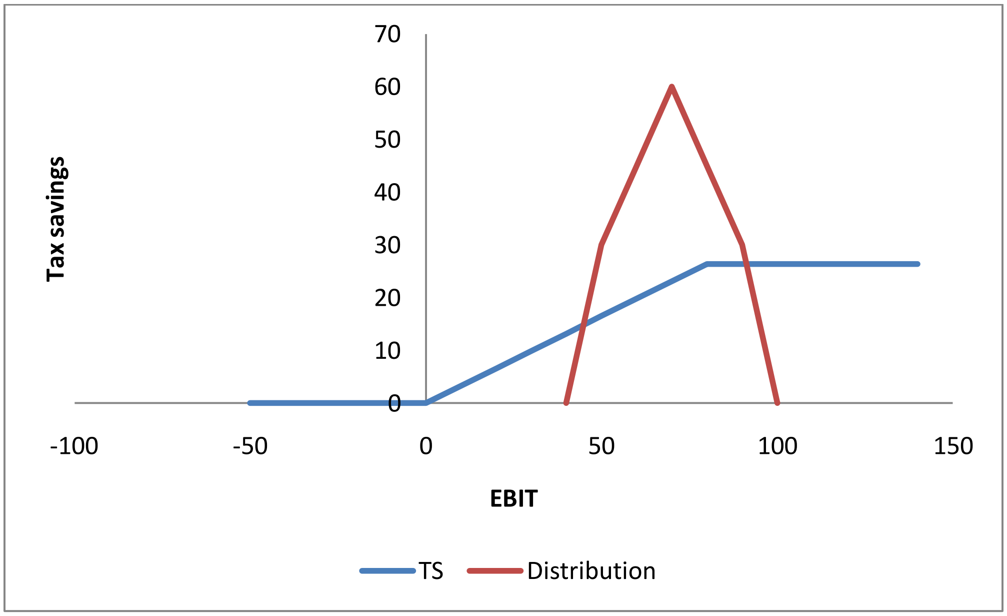

Figure 3. Scenario where P(A)<1

When the TS case is seen from this unrestricted view, it is very intuitive to state that K_D is no longer the appropriate discount rate for TS. We will take this statement further and we will not make any assumption regarding the validity of using K_U or K_E as the appropriate discount rate for TS.

These facts raise the obvious question about the value and methodology for deducting the right non-risk free discount rate for Tax savings. On the following sections we will discuss and propose a framework based on contingent claims in order to estimate the appropriate discount rate for TS, so it can be used for DCF valuation.

3. Tax savings payoff and its risk

When considering Figure 1, it's obvious that the characteristic payoff for the tax savings as a function of the EBIT could be interpreted as the result of a contingent claim's cash flow. In other words, TS cash flow depends on the fulfillment of some conditions; which in turn will determine the actual amount of TS which will be awarded to the company.

If we assume that tax savings come from debt payment and are effective on the same year that they are recorded (in other words, if the taxes are paid on the same year they are recorded); we can visualize the tax saving cash flow as a succession events trough time In which we can appreciate the risk sources for the tax shield and how they interact for tax shield configuration.

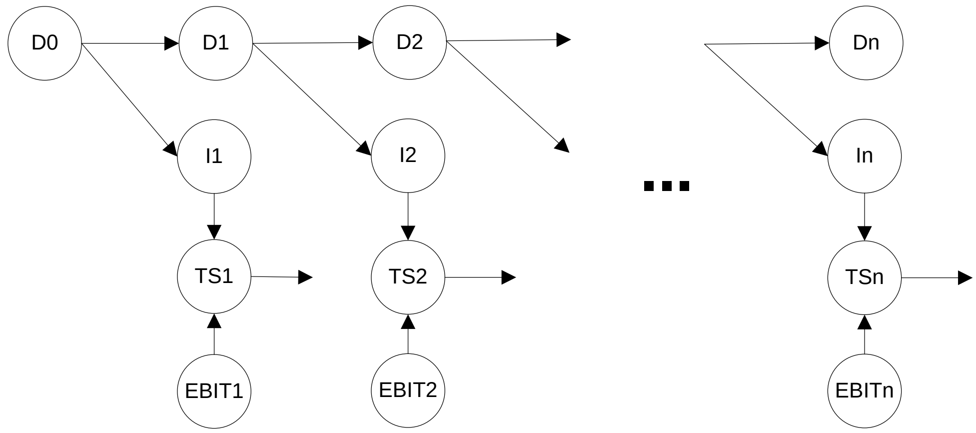

Figure 4. Tax savings and risk flow

From Figure 4 we can visualize that no matter that there is a clear relation between EBIT and FE with tax savings; the latter is a completely different financial instrument. In fact, TS can be viewed in very simple terms as a derivative.

Taking it further, it can be said that TS can be viewed as a portfolio of two call options as well; one long call with strike price k=0 and a short call with strike price k=FE and multiple exercise dates. This complete the approach proposed by Tham and Wonder (Tham & Wonder, 2002) in terms that it captures the complete reality of tax shields; which allows having not only a full tax shield or nothing states, but also an in-between state of partially achieved tax savings.

It is to be held in mind that any valuation method used for tax shield's valuation needs to be based on the nature of the financial statements by which TS is estimated and TS itself. If we take into consideration that TS are calculated from the income statement, and it is a periodic yearly statement; it can be said without hesitation that TS can only be exercised once the accounting period is closed and its strike price varies from period to period.

This means that no matter that the debt discount rate is K_D and EBIT discount rate is K_U; there is no reason to assume that the appropriate discount rate has any direct relation with any of these values. In simple terms, K_D is simply the appropriate discount rate for the instrument which establishes the strike price for one of the options, and K_U is the appropriate discount rate of the underlying asset of the two options constituting the TS payoff. For practical valuation purposes, it is required to estimate the appropriate risk measure for the financial instrument constituted by the tax savings, which will provide the appropriate risk premium for such cash flow.

On the following section, tax savings will be evaluated as a stochastic cash flow on a probability space (Ω, F) and with a real probability measure P; where F is a Borel field containing events A and B; so that $\Omega \in F$. In other words, we will present a framework for kernel valuation of tax savings, followed by its risk neutral equivalent and finalizing with an option approach.

4. Stochastic payoffs and discount factors

If we assume a stochastic payoff TS, which follows a probabilistic model (Ω, F, P); and a stochastic discount factor or pricing kernel X with a probability measure P, so that $0 \leq X \leq 1$; the pricing of any instrument is (Smith & Wickens, 2002)

$$P_t = E_t^P\left[TS_{t+s}X_{t+s}\right] \tag{4.1}$$

Or in terms of the absolute return $R_{t+s} = \dfrac{TS_{t+s}}{P_t} = 1 + r_{t+s}$

$$1 = E_t^P\left[X_{t+s}\frac{TS_{t+s}}{P_t}\right] = E_t^P\left[X_{t+s}R_{t+s}\right] \tag{4.2}$$

If we assume that the discount factor and the absolute return of TS are two correlated stochastic variables; it is possible to rewrite (4.2) as

$$E_t^P\left[X_{t+s}R_{t+s}\right] = E_t^P\left[X_{t+s}\right]E_t^P\left[R_{t+s}\right] + \operatorname{cov}\left(X_{t+s}, R_{t+s}\right) = 1 \tag{4.3}$$

If tax savings were risk free, TS payoff and its absolute return is known with certainty for t+s. From(4.2), the expected return for the stochastic discount factor would be

$$1 = E_t^P\left[\left(1 + r_t^f\right)X_{t+s}\right] = \left(1 + r_t^f\right)E_t^P\left[X_{t+s}\right] \tag{4.4}$$

$$E_t^P\left[X_{t+s}\right] = \frac{1}{\left(1 + r_t^f\right)} \tag{4.5}$$

And the stochastic discount factor will be a random variable

$$X_{t+s} = \left\{\frac{1}{\left(1 + r_t^f\right)} + \varepsilon_{t+s} \,\middle|\, E_t^P\left[\varepsilon_{t+s}\right] = 0\right\} \tag{4.6}$$

But as we stated before, tax saving's payoff doesn't fall on the "risk free area". This means that the appropriate discount rate has a premium over the risk free rate given by (Smith & Wickens, 2002)

$$E_t^P\left[r_{t+s}\right] - r_t^f = -\left(1 + r_t^f\right)\operatorname{cov}\left(X_{t+s}, R_{t+s}\right) \tag{4.7}$$

This leaves open the key task of defining the appropriate measure for risk and its relation with the stochastic discount factor. The analysis has to consider that the underlying asset $\zeta = EBIT + OI$ is non-traded, and cannot be assumed to follow a log-normal distribution.

It is the usual practice when valuating contingent claims; that the measurement of probability is taken to a risk neutral environment; So that the risk free rate could be used to discount the cash flows. On the next section we will introduce a framework for discounting TS under a risk neutral probability measure Q.

5. Risk neutral approach

When dealing with derivatives, such as options; there is a possibility to discount the cash flows with the risk free rate using a different probability measure Q. This probability measure is called a risk neutral probability.

If we assume that $P \wedge Q$ are probability measures on a space (Ω, F), and P is absolutely continuous with respect to Q, and vice versa; meaning that they are equivalent; there is a non-negative random variable called the Radon-Nikodym derivative

$$Z \big| Q(A) = \int_A Z dP; \forall A \in F \tag{5.1}$$

So that

$$E_t^Q \left[L_{t+s} \right] = E_t^P \left[L_{t+s} Z_{t+s} \right] \forall L \tag{5.2}$$

Or, if $M = LZ$

$$E_t^P \left[M_{t+s} \right] = E_t^Q \left[\frac{M_{t+s}}{Z_{t+s}} \right]; \forall M \tag{5.3}$$

Radon-Nikodym derivative allows us to transform TS from P to Q, and vice versa; so it's possible to use the risk free rate as the appropriate rate of return for a risk neutral investor under Q. For valuating purposes, it can be use to guarantee the consistency between risk neutral valuation, and pricing kernel valuation.

For risk neutral tax saving's valuation we will have that; for a TS with probability measure Q

$$V_t^{TS} = E_t^Q \left[TS_{t+s} \left(1 + r_t^f\right)^{-s} \right] = E_t^P \left[TS_{t+s} \left(1 + r_t^f\right)^{-s} Z_{t+s} \right] \tag{5.4}$$

The same applies to real and risk neutral discount factors. From (4.7) we have that for a risk neutral investor, the appropriate rate of return is

$$E_t^Q \left[r_{t+s} + \left(1 + r_t^f\right) \mathrm{cov}_t \left(X_{t+s} R_{t+s} \right) \right] = r_t^f \tag{5.5}$$

Under risk neutral approach, it remains the challenge to define the appropriate model for the risk premium and the Radon-Nikodym derivative in such a way that it could be used on traditional discounted cash flow valuation models using WACC or an APV valuation approach.

Using (5.2) and(5.3), we have that

$$P_t = E_t^P\left[TS_{t+s}\left(1+\varphi\right)^{-(t+s)}\right] \tag{5.6}$$

$$P_t = E_t^P\left[TS_{t+s}\frac{\left(1+\varphi\right)^{-(t+s)}}{Z}\right] \tag{5.7}$$

$$P_t = E_t^P\left[TS_{t+s}\left(1+r_f\right)^{-(t+s)}\right] \tag{5.8}$$

$$P_t = E_t^P\left[TS_{t+s}\left(1+r_f\right)^{-(t+s)}Z\right] \tag{5.9}$$

From (5.6) and (5.9); or (5.7) and (5.8) we can deduct

$$Z = \frac{dQ}{dP} = \frac{\left(1+r_f\right)}{\left(1+\varphi\right)} \tag{5.10}$$

Where Z is the Radon-Nikodym derivative for transforming the probability measures between P and Q. (5.10) provides a tool for the estimation of the pricing kernel for TS given the risk free rate, and the probability measures for real and risk neutral spaces.

6. Option's portfolio approach

From Figure 4 and Figure 1, it is possible to build an option based strategy to mimic tax saving's payoff through time. We have said that the single period tax saving's payoff can be viewed as a portfolio of two call options (one long with k=0 and one long with k=FE); but this characterization has to be enriched taking into account the interaction of those payoff with its underlying and the financial statements by which its parameters are determined.

It is absolutely clear that contrary to limited claim's underlying assets (stocks, commodities, bonds, etc.) which follow log-normal distributions; EBIT can take negative values, so we assume that it follows at least a normal distribution. This requires that all formulation regarding option pricing for continuous and discrete models must be re-calibrated; so that traditional Black-Scholes and binomial tree pricing formulae cannot be used as they are. This is due to the fact that they are defined based on a log-normal underlying price process.

The series of long call options has no interaction with any financial statement in terms of strike price; which is constant and equal to cero. Its only interaction is with the underlying asset, which is calculated from the income statement and defines its payoff.

The later reasoning does not apply to the short call series. For these options, we have that their strike price depends on the value of debt one period before, and it is known with complete certainty; causing that strike prices varies as time passes. Under this assumption, tax shield cash flow is a succession of portfolios of options which are exercised every year as a function of the current financial expenditures.

Using the general equation for asset valuation(4.1), it is possible to apply a backward formulation for finite TS valuation. Using $C_t(k)$ as the proper notation for a call option value given its strike price, TS value for T-1 is given by

$$V_{T-1}^{TS} = C_{T-1}(0) - C_{T-1}(FE_T) + TS_{T-1} \tag{6.1}$$

Where T is the last year of analysis.

For T-2 we use a similar approach

$$V_{T-2}^{TS} = C_{T-2}(0) - C_{T-2}(FE_{T-1}) + \frac{V_{T-1}^{TS}}{1+r_{T-1}^{f}} + TS_{T-2} \tag{6.2}$$

For any given time, we can use the generalized formulae

$$V_{T-j}^{TS} = C_{T-j}(0) - C_{T-j}(FE_{T-j+1}) + \frac{V_{T-j+1}^{TS}}{1+r_{T-j+1}^{f}} + TS_{T-j}; \forall\ j = 1, \ldots, T \tag{6.3}$$

Under this formulation, TS can in effect be discounted using the risk free rate. Equation (6.3) is a general formula for TS valuation and allows the application of pricing kernel and risk neutral valuation alternatives showed before.

In a similar way, it is possible to solve the problem using a binomial tree approach under risk neutral probability q. From (6.3) it is possible to define a formulation for the long call with strike price k=0

$$C_{T-j}^{0} = \frac{q\left(TS_{T-j+k}\tau + C_{T-j+1}^{0}\right) + (1-q)C_{T-j+1}^{0}}{1+r_{T-j+1}^{f}}; \forall j = 1, \ldots, T \tag{6.4}$$

The short call with strike price k=FE

$$C_{T-j}^{0} = \frac{q\left(TS_{T-j+k}\tau + C_{T-j+1}^{0}\right) + (1-q)C_{T-j+1}^{0}}{1+r_{T-j+1}^{f}}; \forall j = 1, \ldots, T \tag{6.5}$$

Finally, the TS portfolio can be formulated as

$$V_{T-j}^{TS} = C_{T-j}^{0} - C_{T-j}^{FE} + TS_{T-j} \tag{6.6}$$

For (6.5) and(6.4), C_T^k is the terminal tax shield. Now, the task standing points to the estimation of q under a non log-normal underlying process.

7. Conclusions

Tax shield have to be valued using methods which take into account its unique nature and risk. It should be clear now that tax saving's pricing needs to be focused on contingent claim valuation. On an unrestricted scenario, tax savings risk cannot be interpreted as those of traditional financial instruments; due to the fact that the conditionality of its cash flows imposes different probability measures, and the underlying instrument is non-traded and accountant based.

We have proved that the use of cost of debt as the appropriate discount rate can only be done under the assumption that EBIT plus other income always exceeds financial expenditures; in other words, only if profit after tax are greater than zero. This assumption is in fact unrealistic and non-consistent with the natural development of most businesses.

In a similar way we have conclude that no matter EBIT risk is in fact linked to the unleveraged cost of capital; tax saving's risk is not. Unlike EBIT, tax shield is a contingent claim cash flow with a non-negative value. This characteristic nature makes that its risk follows a completely different stochastic function related to its payoff, its probability function and contingent cash flows. For this reason, it is not possible to state that the unlevered cost of capital is the appropriate discount rate for tax savings.

For the cost of levered equity approach, it is clear that equity is a completely different financial instrument than tax shield; even though their cash flows are related. There is no reason to assert that tax savings should be discounted at the levered cost of equity just because TS is used for its calculation. Equity is in fact a portfolio of debt, tax savings and the value of the unlevered assets, and as such its opportunity cost is the weighted average cost of capital of the financial instruments that formed it.

In other words, we conclude that under a generalized contingent claim framework, it cannot be argued that using cost of debt, cost of the unlevered assets or the cost of the levered equity as the appropriate discount rate for tax shield is the right way to value tax savings. Tax shields valuation requires the estimation of either a pricing kernel or its equivalent risk neutral valuation model; as the cash flows in which they are based are in fact a derivative.

We have presented two discounted cash flow continuous approaches for tax saving's valuation, which are consistent with (Modifliani & Miller, 1963),(Tham & Wonder, 2002),(Miles & Ezzell, 1985) regarding the fact that the value of tax savings is the present value of the tax savings.

Further work needs to be done on the pricing kernel estimation method for non-traded instruments under nontraditional distributions.

Finally, we have presented a methodology for the valuation of tax shields as a succession of nested portfolios of European options with maturity of one year. This methodology allows the calculation of the value of tax shavings for every year and its integration with traditional discounted cash flow valuation using the generalized WACC approach and the risk free rate as the appropriate discount rate for the risk neutral tax shield valuation.

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