The Optimal Timing of the Announcement in Merger and Acquisition Activities

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1 Introduction

From the point of view of corporate finance, the gain of merger and acquisition (M&A) activity is measured by the difference between the acquirer's increased firm value after the merger and the cost of purchasing the target firm. Thus, to analyze the effect of M&A, the acquirer needs to estimate the increased cash flows after the merger, that is, to predict the differences between the cash flows consequent on taking over the target and those consequent on maintaining the present situation, and then needs to evaluate their present value, referred to here as the increased firm value. To predict the cash flows after a merger, the acquirer needs to analyze the synergies of the merger, for example, economies of scale, market share, effects of technologies, improvement in business risks. On the other hand, the purchasing cost is based on the stock price of the target. If the target is a publicly traded company, the stock price can be observed continuously. In contrast, the increased firm value varies under the influence of movements in economic situations, political environments, and the two firms' individual circumstances. Because of the complexity of estimating the increased firm value, reassessment may be difficult to carry out continuously, or the costs of such operations of reassessment may not be justified. In this paper, we consider the situation that the events which cause a reassessment of the increased firm value occur discretely, therefore the acquirer's increased firm value changes discretely. We assume that the increased firm value changes follow a Poisson process, and the size of the changes are also uncertain, its logarithm has an exponential distribution, and the purchasing cost changes following a geometric Brownian motion. Under these two stochastic variables, we derive the optimal timing of an announcement using a real option approach.

2 The Model

Assume that an acquirer plans to take over a target firm. The acquirer estimates the differences between the future cash flows involved in acting or not acting, and evaluates their present value. We call the present value of the differences between the future cash flows the increased firm value of the acquirer. Let $X_1(t)$ denote the increased firm value at time t. The estimated differences in the future cash flows shift with the macroeconomic or individual factors of the firms. In many situations, it may be difficult to continuously reassess these shifts, or it may be too costly. Therefore, we assume that reassessments will be carried out when the economic situation changes dramatically, or when there occurs an event that has a serious impact on the two firms involved. So we assume that $X_1(t)$ is a discrete stochastic variable. The events that cause upward jumps in the increased firm value occur following a Poisson process with parameter κ. When an upward jump occurs, $X_1(t)$ becomes $YX_1(t)$. The events that cause downward jumps in the increased firm value occur following a Poisson process with parameter λ. When a downward jump occurs, $X_1(t)$ becomes $X_1(t)/Z$. The multiples $Y > 1$ and $Z > 1$ are also stochastic variables: assume that $y = \log Y$ and $z = \log Z$ have exponential distributions with parameters ζ (> 1) and η (> 0).[1] Assume that Y, Z, and the two Poisson processes are all mutually independent.

The purchasing price is set to add a constant premium of C percent to the stock price of the target at the time of the M&A announcement. When the stock price of the target is $\hat{X}_2(t)$, and the number

[1]Kou and Wang[3] use the same jump process added to a geometric Brownian motion to model stock prices. The expected value of Y goes to infinity as $\zeta \leq 1$, here, we set $\zeta < 1$.

of issued stocks is N, the purchasing cost $X_2(t)$ becomes

$$X_2(t) = \hat{X}_2(t)(1 + C)N.$$

Assume that the stock price of the target firm $\hat{X}_2(t)$ follows a geometric Brownian motion with parameters μ and σ. Then $X_2(t)$ also follows

$$dX_2(t) = \mu X_2(t)dt + \sigma X_2(t)dW(t).$$

$X_1(t)$ and $X_2(t)$ are assumed mutually independent.

Let r denote the discount rate with consideration of M&A risks, and let τ denote the optimal timing of the announcement. Then the expected present value of the gain of the M&A is

$$V(x_1,\ x_2) = \mathrm{E}[e^{-r\tau}(X_1(\tau) - X_2(\tau)) \mid \{X_1(0),\ X_2(0)\} = \{x_1,\ x_2\}]$$

hereinafter referred to as the value of the M&A. The purpose is to derive the optimal timing of the announcement of the M&A that maximizes the value of the M&A.

The infinitesimal generator of $\{X_1(t),\ X_2(t)\}$ is

$$\begin{aligned}(\mathcal{L}V)(x_1,\ x_2) &= \lim_{t\to 0_+} \frac{\mathrm{E}[V(X_1(t),\ X_2(t)) \mid \{X_1(0),\ X_2(0)\} = \{x_1,\ x_2\}] - V(x_1,\ x_2)}{t} \\ &= \frac{1}{2}\sigma^2 x_2^2 V_{22}(x_1,\ x_2) + \mu x_2 V_2(x_1,\ x_2) \\ &\quad + \kappa\{\mathrm{E}[V(Yx_1,\ x_2)] - V(x_1,\ x_2)\} + \lambda\{\mathrm{E}[V(x_1/Z,\ x_2)] - V(x_1,\ x_2)\} \qquad (1)\end{aligned}$$

Here, $V_2(x_1,\ x_2) = \partial V(x_1,\ x_2)/\partial x_2$, $V_{22}(x_1,\ x_2) = \partial^2 V(x_1,\ x_2)/\partial x_2^2$. Let $\mathcal{A}$ denote the M&A announcement region of $\{X_1(t),\ X_2(t)\}$, then the timing of the announcement is $\tau = \inf\{t \mid \{X_1(t),\ X_2(t)\} \in \mathcal{A}\}$. Between time 0 and s ($< \tau$), that is, before $\{X_1(t), X_2(t)\}$ reaches region $\mathcal{A}$,

$$V(x_1,\ x_2) = e^{-rs}\mathrm{E}[V(X_1(s),\ X_2(s)) \mid \{X_1(0),\ X_2(0)\} = \{x_1,\ x_2\}]$$

is satisfied. So we obtain

$$(\mathcal{L}V)(x_1,\ x_2) = rV(x_1\ x_2). \qquad (2)$$

The value of the M&A goes to zero as the increased firm value goes to zero, and thus the boundary conditions are given by

$$V(0,\ x_2) = 0$$

$$V(x_1,\ x_2) = x_1 - x_2, \qquad \{x_1,\ x_2\} \in \mathcal{A}.$$

Under the above boundary conditions, we derive the solution of equation (2), and then derive the boundary of the region $\mathcal{A}$.

As $\{X_1(t), X_2(t)\}$ is a 2-dimensional Lévy Process, the Lévy exponent $\varphi(s,\ t)$ satisfies

$$\mathrm{E}[X_1(\tau)^s X_2(\tau)^t \mid \{X_1(0),\ X_2(0)\} = \{x_1,\ x_2\}] = x_1^s x_2^t \exp[\varphi(s,\ t)\tau]. \qquad (3)$$

Here, the Levy exponent is given by

$$\varphi(s,\ t) = \frac{1}{2}\sigma^2 t(t-1) + \mu t + \frac{\kappa s}{\zeta - s} - \frac{\lambda s}{\eta + s}. \qquad (4)$$

Define the increase rate of expected value of $X_i(\tau)$, $(i = 1, 2)$ as m_i, $(i = 1, 2)$:

$$m_i = \frac{d\mathrm{E}[X_i(\tau)]}{d\tau}/\mathrm{E}[X_i(\tau)] \qquad i = 1,\ 2,$$

so that

$$m_1 = \varphi(1,\ 0) = \frac{\kappa}{\zeta - 1} - \frac{\lambda}{\eta + 1}, \qquad m_2 = \varphi(0,\ 1) = \mu.$$

We assume that $r > m_i$ $(i = 1,\ 2)$ below.

3 The Optimal Solution

As $V(x_1,\ x_2)$ is a homogeneous function of degree one of $\{x_1,\ x_2\}$, we have $V(x_1,\ x_2) = x_2 V(x_1/x_2,\ 1)$. Let $u = x_1/x_2,\ \ V(u,\ 1) = W(u)$, when equation (2) becomes[2]

$$\frac{1}{2}\sigma^2 u^2 W''(u) - \mu u W'(u) - (r-\mu)W(u) + \kappa\{\mathrm{E}[W(Yu)] - W(u)\} + \lambda\{\mathrm{E}[W(u/Z)] - W(u)\} = 0. \quad (5)$$

The boundary conditions become

$$W(0) = 0; \qquad W(u) = u - 1, \quad u \geq u^*.$$

The results are the following.[3] The M&A announcement region of $\{X_1(t),\ X_2(t)\}$ is

$$\mathcal{A} = \{x_1,\ x_2 \mid x_1/x_2 > u^*\}, \quad (6)$$

here

$$u^* = \frac{\zeta - 1}{\zeta}\frac{\alpha_1}{\alpha_1 - 1}\frac{\alpha_2}{\alpha_2 - 1}. \quad (7)$$

α_1 and α_2 are the two positive roots of the equation

$$F(x) = \frac{1}{2}\sigma^2 x(x-1) - \mu x - (r-\mu) + \frac{\kappa x}{\zeta - x} - \frac{\lambda x}{\eta + x} = 0. \quad (8)$$

The value of the M&A is

$$V(x_1,\ x_2) = x_2 W(u).$$

Here,

$$W(u) = \begin{cases} \displaystyle\sum_{j=1}^{2} A_j u^{\alpha_j} & u < u^* \\ u - 1 & u \geq u^* \end{cases} \quad (9)$$

and

$$A_1 = \frac{\zeta - \alpha_1}{\zeta}\frac{1}{\alpha_1 - 1}\frac{\alpha_2}{\alpha_2 - \alpha_1}\frac{1}{u^{*\alpha_1}}, \quad (10)$$

$$A_2 = \frac{\zeta - \alpha_2}{\zeta}\frac{1}{\alpha_2 - 1}\frac{\alpha_1}{\alpha_1 - \alpha_2}\frac{1}{u^{*\alpha_2}}. \quad (11)$$

The expected value of the first passage time $T(u)$, that is, the length of time which x_1/x_2, starting from an arbitrary value $u < u^*$, takes to reach or exceed u^*, is given by

$$\mathrm{E}[T(u)] = \begin{cases} \dfrac{1}{\bar{\mu}}\left[\log\left(\dfrac{u^*}{u}\right) + \dfrac{\beta - \zeta}{\beta\zeta}\left(1 - \left(\dfrac{u^*}{u}\right)^{-\beta}\right)\right] & \bar{\mu} > 0 \\ \infty & \bar{\mu} \leq 0. \end{cases} \quad (12)$$

Here,

$$\bar{\mu} = \frac{\kappa}{\zeta} - \frac{\lambda}{\eta} - \mu + \frac{\sigma^2}{2} \quad (13)$$

and β is the positive root of equation

$$G(x) = \frac{1}{2}\sigma^2 x(x-1) + (\sigma^2 - \mu)x + \frac{\kappa x}{\zeta - x} - \frac{\lambda x}{\eta + x} = 0. \quad (14)$$

Letting $V(x_1,\ x_2) = \log(x_1/x_2)$, from equation (1) we obtain

$$(\mathcal{L}V)(x_1,\ x_2) = \frac{\sigma^2}{2} - \mu + \frac{\kappa}{\zeta} - \frac{\lambda}{\eta} = \bar{\mu}.$$

Thus, $\bar{\mu}$ is the drift rate of $\log[X_1(t)/X_2(t)]$.

[2] The transformation of a differential equation related with 2-dimensional geometric Brownian motion is demonstrated in McDonald and Siegel[1], Dixit and Pindyck[2].

[3] See the Appendix for details.

4 Comparative Statics Analysis

In this section, we analyze the influences on the threshold u^* of changing the values of the parameters. As u^* is a decreasing function of α_1 and α_2, the effect of a parameter is clear if α_1 and α_2 increase or decrease simultaneously when the parameter's value changes, except for ζ. Moreover it is clear that if equation (8) shifts upward, then α_1 and α_2 decrease simultaneously, and vice versa, when a parameter's value changes. Thus, the effects of the parameters are precisely as shown in Table 1 except for κ and ζ. The effects of parameter κ and ζ are confirmed by several numerical examples. As ζ and η are the parameters of exponential distributions, it is hard to understand the substantive meaning of their effects. The effects of changes by multiples of the upward shifts $\mathrm{E}[Y]$ and downward shifts $\mathrm{E}[1/Z]$ are added to the bottom of Table 1.

Table 1: Changes in the threshold u* as a parameter's value increases

variables	parameters	changes in u^*
$X_2(t)$: cost of purchasing	μ: drift rate	decreasing
	σ: volatility	increasing
$X_1(t)$: increased firm value	κ: expected frequency of upward jumps	increasing
	λ: expected frequency of downward jumps	decreasing
	ζ: parameter of upward jump size	decreasing
	η: parameter of upward jump size	increasing
	r: discount rate	decreasing
	$\mathrm{E}[Y]$: expected multiple of upward jumps	increasing
	$\mathrm{E}[1/Z]$: expected multiple of downward jumps	increasing

An increase in u^* delays the timing of the announcement of the M&A, and a decrease in u^* advances the timing. The factors that delay or advance the timing of the announcement can be summarized as follows. The effects of changing a parameter's value on $\bar{\mu}$ have the same directions as on u^*. Furthermore, the effects of changing a parameter's value on $m_1 - m_2$ have the same direction as on u^*. Although these two results suggest that an increase in expected firm value or a decrease in the expected purchasing cost relate to delaying the timing of the announcement, some numerical examples show that u^* increases due to changes in a parameters' value while keeping $\bar{\mu}$ or $m_1 - m_2$ unchanged. This means that it is not enough to explain the changes in u^* only using $\bar{\mu}$ or $m_1 - m_2$.

5 Numerical Examples

Set the drift rate of the target firm's stock price to 0.02, the volatility to 0.3, and the risk-adjusted discount rate to 0.2. To show how the variation pattern of the acquirer's increased firm value influences the optimal timing of the announcement and the expected value of the M&A, we consider the variation patterns according to the following seven cases. In Case 1, the increased firm value is assumed constant. In Case 2, there are 10% upward and downward jumps on average, expected to occur once in 5 years. In Case 3, there are bigger changes (20% upward and downward jumps in average) than case 2, but with lower frequency (once every 10 years on average). Case 4 and Case 5 consider only events which reduce the increased firm value, and Case 6 and 7 consider only events which augment the increased firm value. Table 2 shows parameters' values, u^*, $W(u)$, and the expectation of the first passage time $\mathrm{E}[T(u)]$ when $u = x_1/x_2 = 1.4$ at time 0 in each case.[4]

[4]When $\kappa = 0$, as equation (8) has only one positive root, the terms that include α_2 and ζ vanish in equations (7) and (10), and $A_2 = 0$ in equations (9) and (11). The expectation of the first passage time becomes

$$\mathrm{E}[T(u)] = \begin{cases} \dfrac{1}{\bar{\mu}} \log\left(\dfrac{u^*}{u}\right) & \bar{\mu} > 0 \\ \infty & \bar{\mu} \leq 0. \end{cases}$$

Here, $\bar{\mu}$ is as defined in equation (13) with $\kappa = 0$.

Table 2: Variation patterns of the increased firm value and the results

Case	1	2	3	4	5	6	7
κ	0	0.1	0.05	0	0	0.1	0.05
λ	0	0.1	0.05	0.1	0.05	0	0
$\mathrm{E}[Y]$	1.0	1.1	1.2	1.0	1.0	1.1	1.2
$\mathrm{E}[1/Z]$	1.0	0.9	0.8	0.9	0.8	1.0	1.0
u^*	1.5409	1.5542	1.5635	1.5064	1.5110	1.5942	1.5987
$W(u)$	0.4116	0.4136	0.4151	0.4073	0.4078	0.4195	0.4205
$\mathrm{E}[T(u)]$	3.837	4.593	5.409	5.274	6.104	3.844	4.056

$(\mu = 0.02,\ \sigma = 0.3,\ r = 0.2,\ u = 1.4)$

In Case 1, as the increased firm value is constant, the acquirer need only observe the variation of target stock price and wait for the timing of the announcement. If the increased firm value $X_1(t)$ is estimated to be 3.36 billion Yen, then the timing of the announcement is that the purchasing cost $X_2(t)$ should go below 2.18 billion Yen for the first time. That is, if the number of the issued stocks of target N is 100 thousand, and the premium C is 20%, then the timing of the announcement is when the stock price goes below 18,171 Yen for the first time. If the stock price now is 20,000 Yen, then u is 1.4, and the expected value of the M&A is $V(x_1,\ x_2) = x_2 W(u) = 0.98785$ billion Yen. In addition, the expectation of the first passage time of u from 1.4 to u^* is 3.837 years.

As we assumed in this paper that the increased firm value of M&A changes discretely, until an event which necessitated a reassessment of the increased firm value occurs, $X_1(t)$ remains unchanged. Thus, the timing of the announcement depends only on the stock price of the target, as in Case 1.

Table 3 shows the result of sorting the u^* of the 7 cases in Table 2 into ascending order. From equation (3), we see that the magnitude of the expectation of $X_1(t)$ depends on the magnitude of $\varphi(1,0)$, and the magnitude of the variance of $X_1(t)$ depends on $\varphi(2,0)$ when $\varphi(1,0)$ is the same. As shown in Table 3, u^* increases as the expectation and variance increase in the examples of Table 2.

Table 3: The order of u^* and the expectation and variance of $X_1(t)$

order	case	u^*	$\varphi(1.0)$	$\varphi(2,0)$
1	Case 4	1.506	-0.01	-0.0182
2	Case 5	1.511	-0.01	-0.0167
3	Case 1	1.541	0.00	0.0000
4	Case 2	1.554	0.00	0.0040
5	Case 3	1.563	0.00	0.0083
6	Case 6	1.594	0.01	0.0222
7	Case 7	1.599	0.01	0.0250

In Table 2, we showed the values of $W(u)$ and $\mathrm{E}[T(u)]$ for u equal to 1.4. Figures 1 and 2 show how these values vary when the value of u changes in Cases 1, 2, 4, and 6. The same figures could be constructed for Cases 3, 5, and 7, but we have omitted these cases to avoid complexity. In these numerical examples, we see that the higher the value of u^*, the higher the expected present value of the M&A, $W(u)$.

6 Conclusion

This paper treated the determination of the optimal timing of the announcement of M&A activity using the real option approach. The problem was modeled by using a 2-dimensional stochastic process containing double exponential jump processes and a geometric Brownian motion. A closed form solution for the optimal timing, expected value of the M&A, and the expectation of the first passage time were obtained. The results of a comparative statics analysis suggest that an increase in the increased

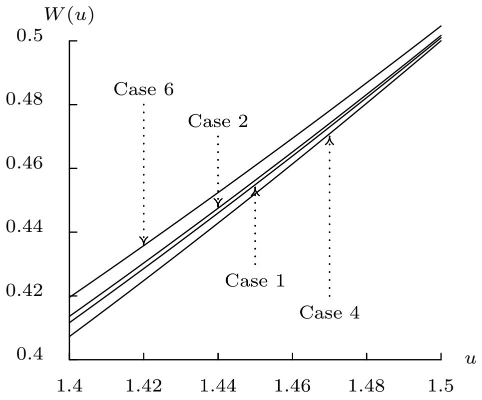

Figure 1: The expected present value of M&A

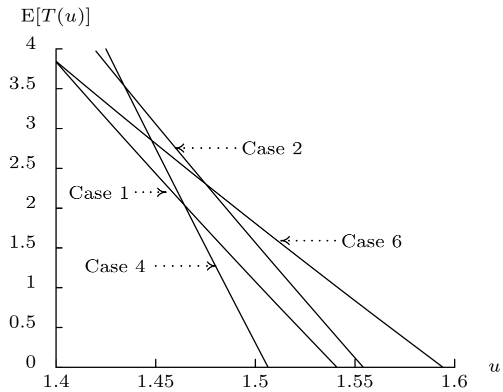

Figure 2: The expectation of first passage time

firm value or a decrease in the purchase cost relates to delaying the timing of the announcement. Numerical examples show that a higher threshold of the M&A leads to a higher expected value of the M&A.

Appendix A: Derivation of the Optimal Solution

Take the boundary conditions of equation (5) into consideration and assume the solution to be as follows. (We discuss the range of the index j later.)

$$W(u) = \begin{cases} \sum_{j=1}^{2} A_j u^{\alpha_j} & u < u^* \\ u - 1 & u \geq u^* \end{cases} \tag{A1}$$

Then, equation (5) becomes

$$\sum_{j=1}^{2} A_j u^{\alpha_j} \left[\frac{1}{2}\sigma^2 \alpha_j(\alpha_j - 1) - \mu\alpha_j - (r - \mu) + \frac{\kappa\alpha_j}{\zeta - \alpha_j} - \frac{\lambda\alpha_j}{\eta + \alpha_j} \right]$$

$$-\kappa\left(\frac{u}{u^*}\right)^{\zeta}\left[\sum_{j=1}^{2}A_j u^{*\alpha_j}\frac{\zeta}{\zeta-\alpha_j}-u^*\frac{\zeta}{\zeta-1}+1\right]=0. \tag{A2}$$

Thus, the α_j must be the roots of the following equation to insure that equation (A2) is satisfied for arbitrary value of u.

$$F(x)=\frac{1}{2}\sigma^2 x(x-1)-\mu x-(r-\mu)+\frac{\kappa x}{\zeta-x}-\frac{\lambda x}{\eta+x}=0. \tag{A3}$$

The function $F(x)=0$ has two positive and two negative roots, and the two positive roots are bigger than 1 if m_1 and m_2 are smaller than r. Only the two positive roots α_j satisfy the boundary condition $W(0)=0$, we denote them by α_1 and α_2.

Combining the high-contact condition and the condition that the value in the square bracket of the second item in equation (A2) must be zero, the optimal solution satisfies the following simultaneous equations.

$$\begin{cases} A_1u^{*\alpha_1}+A_2u^{*\alpha_2}-u^*+1=0\\ \alpha_1A_1u^{*\alpha_1}+\alpha_2A_2u^{*\alpha_2}-u^*=0\\ A_1u^{*\alpha_1}\dfrac{\zeta}{\zeta-\alpha_1}+A_2u^{*\alpha_2}\dfrac{\zeta}{\zeta-\alpha_2}-u^*\dfrac{\zeta}{\zeta-1}+1=0\end{cases} \tag{A4}$$

Solving for u^* and $A_1,\ A_2$ from the above simultaneous equations, we obtain

$$u^*=\frac{\zeta-1}{\zeta}\frac{\alpha_1}{\alpha_1-1}\frac{\alpha_2}{\alpha_2-1},$$

$$A_1=\frac{\zeta-\alpha_1}{\zeta}\frac{1}{\alpha_1-1}\frac{\alpha_2}{\alpha_2-\alpha_1}\frac{1}{u^{*\alpha_1}},$$

$$A_2=\frac{\zeta-\alpha_2}{\zeta}\frac{1}{\alpha_2-1}\frac{\alpha_1}{\alpha_1-\alpha_2}\frac{1}{u^{*\alpha_2}}.$$

Appendix B: The Expectation of the First Passage Time

Define $T(u)$ to be the first passage time of $U(t)=X_1(t)/X_2(t)$ starting from $U(0)=u$ to u^*, and define

$$V(u)=\mathrm{E}[e^{-rT(u)}]. \tag{B1}$$

Then $V(u)$ satisfies the following equation.

$$\frac{1}{2}\sigma^2u^2V''(u)+(\sigma^2-\mu)uV'(u)+\kappa\mathrm{E}[V(uY)-V(u)]+\lambda\mathrm{E}[V(u/Z)-V(u)]=rV(u) \tag{B2}$$

with boundary conditions

$$V(0)=0;\qquad V(u)=1,;\ u\geq u^*.$$

Assume that the solution of $V(u)$ is

$$V(u)=\begin{cases}\displaystyle\sum_{j=1}^{2}A_ju^{\alpha_j} & u<u^*\\ 1 & u\geq u^*\end{cases}$$

so that equation (B2) becomes

$$\sum_{j=1}^{2}A_ju^{\alpha_j}\left[\frac{1}{2}\sigma^2\alpha_j(\alpha_j-1)+(\sigma^2-\mu)\alpha_j-r+\frac{\kappa\alpha_j}{\zeta-\alpha_j}-\frac{\lambda\alpha_j}{\eta+\alpha_j}\right]$$

$$-\kappa\left(\frac{u}{u^*}\right)^{\zeta}\left[\sum_{j=1}^{2}A_ju^{*\alpha_j}\frac{\zeta}{\zeta-\alpha_j}-1\right]=0. \tag{B3}$$

This means that $\alpha_1,\ \alpha_2$ are the two positive roots of equation

$$F(x) = \frac{1}{2}\sigma^2 x(x-1) - (\sigma^2 - \mu)x - r + \frac{\kappa x}{\zeta - x} - \frac{\lambda x}{\eta + x} = 0,$$

and $A_1,\ A_2$ satisfy

$$\begin{cases} A_1 u^{*\alpha_1} + A_2 u^{*\alpha_2} = 1 \\ A_1 u^{*\alpha_1}\dfrac{\zeta}{\zeta - \alpha_1} + A_2 u^{*\alpha_2}\dfrac{\zeta}{\zeta - \alpha_2} = 1, \end{cases}$$

that is,

$$A_1 = \frac{\zeta - \alpha_1}{\zeta}\frac{\alpha_2}{\alpha_2 - \alpha_1}\frac{1}{u^{*\alpha_1}}$$

$$A_2 = \frac{\zeta - \alpha_2}{\zeta}\frac{\alpha_1}{\alpha_1 - \alpha_2}\frac{1}{u^{*\alpha_2}}.$$

As the expectation of the first passage time is given by

$$\mathrm{E}[T(u)] = -\lim_{r \to 0}\frac{\partial}{\partial r}V(u),$$

we obtain the result as equation (12).

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