A dynamic model for corporate financing and investment with expansion under asymmetric information

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Abstract

In this paper we develop a dynamic model that takes into consideration the optimal stopping time of an investment, where the investment cost is financed from the equity market. We introduce two types of firms, a good one and a bad one, with similar investment projects but different expansion rates. We also examine information asymmetry; that is, both firms know the different expansion rates but equity investors are unaware of the differences between the two firms. We derive the equilibrium strategies for both firms in a competitive environment with asymmetric information. The main results indicate the following. First, when the expansion rate of a good firm is relatively large, the good firm can choose the optimal timing for its expansion independently. Second, even when the expansion rate of the good firm is relatively small, a good firm can still distinguish itself from a bad firm by investing earlier than the optimal timing under symmetric information. We evaluate different types of equilibrium through numerical

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experiments and examine the conditions of these equilibrium types. We also perform sensitivity analysis to examine the effect of different parameter values in the model.

Keywords: Asymmetric information; Signaling; Real options and game theory

JEL: G31,G13,G14,D82

1 Introduction

The existence of information asymmetry could cause serious problems in financial markets. A typical problem, known as adverse selection, involves investors underestimating the quality of goods or services owing to the information asymmetry. Akerlof (1970) first pointed out that a "lemon" exists in used car markets in the presence of asymmetric information. Adverse selection affects management decisions in corporate finance such as investment, financing, and equity issues. Hence, it is important to take this into consideration when examining financial problems.

We focus on a capital market where different types of firms attempt to obtain finance in the presence of asymmetric information between equity investors and the firms. We would expect that a relatively good investment firm would raise less finance than it needs as a result of the effect of adverse selection owing to the fact that investors cannot distinguish the good firm from the bad one. Myers and Majluf (1984) discussed the effect of adverse selection on firms' financing decisions and proposed a pecking order theory. In the theory, they claim that there is an order of priority with respect to financing; that is, a firm prefers a debt issue to financing with equity. On the one hand, a firm with full information prefers to issue equity when it is overestimated and its share price is higher than the fair market value. On the other hand, a rational investor with partial information takes the firm's strategy into account and invests less money in the equity. Consequently, the share price falls and the firm raises less finance than expected. Since the effect of information asymmetry on debt issues is relatively small, Myers and Majluf (1984) explain that firms tend to issue debt first.

Another important aspect of the investment decision to be considered is flexible managerial decisions under uncertainty. For better investment decisions, managers of a firm should take this flexibility into account, e.g., to expand or shutdown their investment opportunities, and avoid underestimating the value of the flexibility. Real option analysis has gained popularity in analyzing this aspect since it naturally considers flexibility under uncertainty as an option that should be exercised at the optimal time. As an early example, McDonald and Siegel (1986) examined a real option to defer investment, while Pindyck (1988) and Trigeorgis and Mason (1987) discussed real options to expand investment opportunities. For further examples of real options, the reader should consult the work of Schwartz and Trigeorgis (2001) and the references therein.

In this paper, we discuss investment timing and financing from a financial market in the presence of both uncertainty and asymmetric information. Myers and Majluf (1984) discussed a firm's financing and investment decisions under asymmetric information using a static model. In contrast, Morellec and Schürhoff (2011) extended this to a dynamic model to determine the optimal investment timing. They showed that a relatively profitable firm can send the correct signal to outside investors to prove that the firm is indeed more profitable than the other less profitable firm. Signaling is one of the approaches for solving the problem related to information asymmetry. Approaches have been proposed and discussed in several studies including Rothschild and Stiglitz (1976), Spence (1974), and Wilson (1977). The work of Morellec and Schürhoff (2011) indicates that the pecking order theory does not always hold when the firm sends a signal to outside investors by changing the timing of its investment.

Tirole (2005), however, showed that the adverse selection problem could occur when firms attempt to obtain finance to expand existing investment projects. This suggests that a firm with a high-valued project could miss an opportunity to expand it owing to not finding the necessary finance for the expansion. This is due to the static model used in the work of Tirole (2005) that does not consider the timing of investment and finance. In the model, it is shown that a firm decides not to expand an existing project unless the real option value is sufficiently large, owing to the presence of asymmetric information.

Motivated by the work of Tirole (2005), in this paper, we propose a dynamic investment model that derives an optimal investment and financing policy. Considering that the investment for expansion could exercise the growth option, we can use a real option approach to derive the optimal stopping time to expand the existing project under uncertainty and asymmetric information.

A distinct feature of the proposed model is that each firm in our model can choose the timing of its investment. In the model we consider two different types of firms. The "good" firm has a growth option with a higher expansion rate than the "bad" firm. Both firms determine the optimal timing of their investment through financing from equity investors. We assume that there is asymmetric information between the firms and the investors; in other words, the investors cannot distinguish the good firm from the bad one.

In this environment, we derive equilibrium strategies for the two firms. Two types of equilibrium are possible, namely, separate equilibrium and simultaneous equilibrium. Separate equilibrium is achieved when the two firms invest in the expansion project with different stopping times, whereas simultaneous equilibrium occurs when the two firms invest in the projects at the same time. Owing to the presence of asymmetric information, the bad investment firm attempts to mimic the good one to obtain financing from the equity market under better conditions. This could result in simultaneous equilibrium.

In this paper, we prove that simultaneous equilibrium never occurs. Intuitively, the good investment firm attempts to finance preemptively so that the investor can distinguish it from the bad one. Similarly, a good investment firm, can be recognized as such and obtain financing under better conditions. Furthermore, we derive different types of separate equilibrium. When the expansion rate of the good firm is sufficiently larger than that of the bad one, the former can determine its optimal timing exclusively. On the other hand, when the expansion rate for the good firm is relatively low, the good investment firm must preempt the investment to be recognized as a good firm, thereby decreasing the real option values. However, in both cases, the good investment firm can be distinguished from the bad one, thus contradicting the results obtained by the static model.

Comparing the proposed model with that of Morellec and Schürhoff (2011) highlights two differences. First, instead of a new project, we suppose that the firm has an existing project and consider the managerial flexibility to expand the project. Hence, the uncertainty affects both the existing project and the growth option. Second, Morellec and Schürhoff (2011) assume that the money for the new investment is financed by newly issued equity, resulting in the assumption that the new issued equities are financed by new investors. In our model, on the other hand, we assume that this is financed by selling shares in the secondary market.

The rest of the paper is organized as follows. Section 2 first introduces some assumptions and then develops the dynamic investment model. In Section 3, we discuss equilibrium under symmetric information while in Section 4, we derive equilibrium strategies under asymmetric information. We explain the optimization problems for both firms and derive possible equilibrium strategies. In Section 5, we demonstrate through numerical experiments the effect of the parameter values in the model on the choice of the equilibrium strategies. Section 6 presents some concluding remarks.

2 Model

Suppose that there are two competitor firms each of which has a similar existing project and an option to expand. One of the firms, referred to as the good firm, has a higher expansion rate than the other firm, called the bad firm. We assume that the firms and investors are risk neutral and that the risk-free rate r is constant for the duration of the project. Let X_t be the instantaneous cash flow at time t given by the following geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dZ_t,\ X_0 > 0,$$

where μ and σ are the drift and volatility, respectively, $\mu < r$, and $(Z_t)_{t\geq 0}$ denotes standard Brownian motion. The investment cost for the expansion is denoted by I and is irreversible. Let $k, k \in \{g, b\}$ denote the type of firm and $\theta_k, k \in \{g, b\}$ be the expansion rate for firm k, assuming $\theta_g > \theta_b > 1$. Hence, firm k obtains cash inflow of $\theta_k X_t$ at time t after the expansion.

Each firm knows the type of the other firm, whereas investors have less information regarding the types of the firms. The investors believe that a firm is a good one with probability p and a bad one with probability $1 - p$.

We assume that each firm holds all its stock at time 0, and that they will sell part of their own stock in the secondary market to raise finance. Without loss of generality we assume that the number of shares in each firm is equal to one. Let $\alpha(X; \theta_k)$ denote the number of shares sold when $X_t = X$ by the firm recognized as being of type k by the investors. In other words, the firm of type k collects money to finance its investment cost I by selling $0 < \alpha(X; \theta_k) < 1$ $(k \in \{g, b\})$ shares when X_t reaches X.

3 Equilibrium strategy under symmetric information

In this section, we first derive the equilibrium strategies for both firms under symmetric information for comparative purposes. Suppose that investors can distinguish a good firm from a bad one. Let V_k denote the project values for a type k firm before the expansion, and $\Pi(x)$ be the present value of perpetual cash flow of X_t with initial value $X_0 = x$, given by

$$\Pi(x) = \mathbb{E}\left[\int_0^\infty e^{-rt} X_t dt | X_0 = x\right].$$

Note that Π can be considered as the share price of the firm without the option. Using the standard argument of real option analysis, we can derive the following ordinal differential equation:

$$rV_k = \mu X_t \frac{\partial V_k}{\partial X_t} + \frac{1}{2}\sigma^2 {X_t}^2 \frac{\partial^2 V_k}{\partial {X_t}^2} + X_t. \tag{1}$$

A general solution of Eq. (1) can be expressed as

$$V_k(X) = AX^{\xi} + BX^{\nu} + \frac{X}{r-\mu}, \tag{2}$$

where ξ and ν denote, respectively, the positive and negative solution of the characteristic function

$$\frac{1}{2}\sigma^2 y(y-1) + \mu y - r = 0.$$

To specify the coefficients of A and B in Eq. (2), we derive boundary conditions by applying the value matching condition, smooth pasting condition, and boundary condition of V_k at $X = 0$. In other words, suppose that the type k firm decides to expand the project at the stopping time when X reaches $\bar{X}_k$. Then, the conditions are given by the following three equations.

$$V_k(X)|_{X=\bar{X}_k} = (1-\alpha(\bar{X}_k;\theta_k))\theta_k\Pi(\bar{X}_k) - I, \tag{3}$$

$$\partial V_k/\partial X|_{X=\bar{X}_k} = (1-\alpha(X;\theta_k))\theta_k\Pi(X)/\partial X|_{X=\bar{X}_k}, \tag{4}$$

$$\lim_{X\to 0} V_k(X) = 0. \tag{5}$$

The coefficients are easily obtained from Eqs. (5) and (3), respectively, as

$$B = 0, A = \left[\left(1-\alpha\left(\bar{X}_k;\theta_k\right)\right)\theta_k\Pi\left(\bar{X}_k\right) - I - \frac{\bar{X}_k}{r-\mu}\right]\left(\bar{X}_k\right)^{-\xi}.$$

In addition, the optimal investment threshold $\bar{X}_k$ is obtained from Eqs. (3) and (4) as

$$\bar{X}_k = \frac{\xi}{\xi-1}\frac{I(r-\mu)}{(1-\alpha(\bar{X}_k;\theta_k))\theta_k - 1}. \tag{6}$$

where $\xi = (\sigma^2/2-\mu)/\sigma^2 + \sqrt{[\sigma^2/2-\mu)/\sigma^2]^2 + 2r/\sigma^2}$. Note that ξ is greater than one. Consequently, in the absence of asymmetric information, the project value of each firm can be written as

$$V_k(X) = \left[\left(1 - \alpha\left(\bar{X}_k; \theta_k\right)\right)\theta_k \Pi\left(\bar{X}_k\right) - I - \frac{\bar{X}_k}{r-\mu}\right]\left(\frac{X}{\bar{X}_k}\right)^{\xi} + \frac{X}{r-\mu}.$$

Now let $\bar{X}_g$ and $\bar{X}_b$ denote, respectively, the optimal threshold for a good firm and that for a bad one. The following lemma shows that given that the initial cash flow level x is sufficiently small, the good firm exercises the option to expand with a relatively lower threshold.

Lemma 1. *$\bar{X}_b$ is greater than $\bar{X}_g$.*

Proof. Owing to the budget constraint, it is easily shown that

$$\alpha(\bar{X}_k; \theta_k) = \frac{I}{\theta_k \Pi(\bar{X}_k)}, k \in \{g, b\}. \tag{7}$$

Equation (7) indicates that the investment cost I is financed by selling α shares. Substituting Eq. (7) in Eq. (6) leads to

$$\bar{X}_k = \frac{\xi}{\xi - 1}\frac{I(r-\mu)}{(1 - \frac{\xi-1}{\xi})\theta_k + \frac{\xi-1}{\xi} - 1}.$$

Since by definition $\theta_g > \theta_b > 1$, we have shown that $\bar{X}_g < \bar{X}_b$. □

In other words, the good firm tends to expand earlier than the bad one under symmetric information. This lemma is used to show the signaling effect of the good firm under asymmetric information.

4 Equilibrium strategy under asymmetric information

In this section we derive equilibrium strategies for both firms under asymmetric information. In the case where investors cannot distinguish the type of firm, the bad firm has an incentive to mimic the good one so that it can sell stock at a higher price. This action could result in a simultaneous equilibrium where the good firm is not recognized as being good, and stock is sold at a lower price than under symmetric information. The good firm, therefore, has an incentive to send a signal to the investors so that it can be correctly recognized. If the good firm succeeds in sending an appropriate signal, this results in a separate equilibrium, with the investors correctly recognizing the type of firm. Thus, the type of equilibrium depends on the success of the signaling.

4.1 Separate equilibrium

In this subsection, we discuss the existence of different types of equilibrium. The bad firm, on the one hand, determines whether it should invest early to mimic the good one or choose the optimal timing for a bad firm to invest. The good firm, on the other hand, determines whether it should send a signal by investing earlier than the bad firm to be recognized as a good type of firm or choose the same timing for investment as the bad firm. To derive the equilibrium strategy we need to consider the incentive compatibility constraints for both firms. Here, the incentive urges both firms to invest earlier in their expansion projects.

Let us first examine the incentive compatibility constraints for a bad type of firm. Let $V_b(X)$ denote the project value for the bad firm when it chooses the optimal timing for a bad firm, assuming $X \leqslant \bar{X}_b$. $V_b(X)$ is given by

$$V_b\left(X\right) = \left[\left(1-\alpha\left(\bar{X}_b;\theta_b\right)\right)\theta_b\Pi\left(X\right) - I - \frac{\bar{X}_b}{r-\mu}\right]\left(\frac{X}{\bar{X}_b}\right)^{\xi} + \frac{X}{r-\mu}.$$

The bad firm has an incentive to invest in the expansion project earlier than the good one if

$$V_b\left(X\right) \leqslant \left(1-\alpha\left(X;\theta_g\right)\right)\theta_b\Pi\left(X\right) - I. \tag{8}$$

The right-hand side of Eq. (8) represents the project value for the bad firm mimicking the good one. In this case, the bad firm can finance the investment cost to the same extent as the good one, which is mis-recognized by the investors. Let $\bar{X}_b^{\min}$ denote the value of X

satisfying the equality in Eq. (8).

In response to the strategy for the bad firm, the incentive compatibility constraint for the good firm can be considered as follows. The good firm has an incentive to invest earlier if the following condition holds:

$$(1-\alpha(X;\theta_g))\,\theta_g\Pi(X)-I \geqslant \left[\left(1-\alpha\left(\bar{X}_b;\theta_b\right)\right)\theta_g\Pi\left(\bar{X}_b\right)\right]\left(\frac{X}{\bar{X}_b}\right)^{\xi}. \tag{9}$$

The left-hand side of Eq. (9) represents the project value for the good firm when it succeeds in separating its investment timing from that of the bad firm. In this case, the good firm can sell $\alpha(X;\theta_g)$ shares of stock to finance investment cost I. The right-hand side of the equation represents the project value for the good firm when it is mis-recognized as a bad firm by the investors. Let $\bar{X}_g^{\min}$ be the value of X satisfying the equality in Eq. (9). Then, the good firm can anticipate the investment timing until X reaches $\bar{X}_g^{\min}$. The good firm knows that the bad firm will not invest when X is smaller than $\bar{X}_b^{\min}$. Consequently, the good firm can separate its investment timing if it invests when X is smaller than $\bar{X}_b^{\min}$. In addition, the following lemma can easily be shown.

Lemma 2. *$\bar{X}_b^{\min}$ is greater than $\bar{X}_g^{\min}$.*

The argument for the incentive compatibility conditions can be summarized in the following lemma.

Lemma 3. *It is better for the good firm to send a signal to separate its investment timing from that of the bad firm when*

$$\bar{X}_g^{\min} \leqslant X \leqslant \bar{X}_b^{\min}.$$

Lemmas 1 and 2 suggest that there are two possible cases that should be considered with respect to the order of $\bar{X}_g$ and $\bar{X}_b^{\min}$. To describe a set of strategies for both firms, let (X_1, X_2) denote the strategy whereby the good firm invests in the expansion project when process X_t reaches X_1, and the bad firm invests when X_t reaches X_2. Note that these timings denote stopping times. The first type of equilibrium strategy is obtained by the following proposition.

Proposition 4. *In the case where $\bar{X}_g < \bar{X}_b^{\min}$, a separate equilibrium is established, given as $\left(\bar{X}_g, \bar{X}_b\right)$.*

Proof. For the good firm choosing the investment boundary, $\bar{X}_g$ is the most favorable strategy under symmetric information. In this case, since $\bar{X}_g$ is smaller than $\bar{X}_b^{\min}$, the bad firm never mimics the good one owing to the incentive compatibility constraint. Therefore, the bad firm chooses the investment boundary $\bar{X}_b$. Consequently, the equilibrium strategy is established as $\left(\bar{X}_g, \bar{X}_b\right)$. $\square$

Proposition 4 implies that when the difference in the expansion rates of the good and bad firms is sufficiently large, the presence of asymmetric information does not affect the equilibrium strategy for either firm.

Proposition 5. *In the case where $\bar{X}_g > \bar{X}_b^{\min}$, a separate equilibrium is established, given as $\left(\bar{X}_b^{\min}, \bar{X}_b\right)$.*

Proof. In the case where $\bar{X}_g$ is greater than $\bar{X}_b^{\min}$, the bad firm mimics the good one and attempts to invest earlier than the good firm if the latter chooses boundary $\bar{X}_g$. In response, the good firm needs to choose a smaller threshold for the investment. We show that the good firm chooses $\bar{X}_b^{\min}$ as its investment boundary.

Suppose that the good firm chooses $\bar{X}_b^{\min}$ as its investment boundary. The bad firm must determine whether to mimic the good firm or act as a bad one. If the bad firm mimics the good one and chooses $\bar{X}_b^{\min}$ as the investment boundary, this results in simultaneous investment as the investors cannot distinguish the types of the firms. Hence, the investors buy $\alpha\left(\bar{X}_b^{\min}; \theta_{pool}\right)$ shares from each firm where θ_{pool} is given by $\theta_{pool} = p\theta_g + (1-p)\theta_b$. In other words, the bad firm fails to mimic the good one. Because inequality $\theta_b < \theta_{pool} < \theta_g$ holds, this simultaneous investment breaks the incentive compatibility condition for the bad firm. Consequently, it decides to behave as a bad firm and chooses $\bar{X}_b$ as the investment boundary.

Next, we show that the choice of $\bar{X}_b^{\min}$ is optimal for a good firm. If the good firm invests before process X_t reaches $\bar{X}_b^{\min}$, the project value of the good firm decreases. If, on the other hand, the good firm invests after X_t reaches $\bar{X}_b^{\min}$, the bad firm invests at $\bar{X}_b^{\min}$ to mimic the good one, thereby decreasing the project value of the good firm. Thus, the good firm chooses $\bar{X}_b^{\min}$ as its investment boundary. Consequently, the separate equilibrium $\left(\bar{X}_b^{\min}, \bar{X}_b\right)$ is established. $\square$

Contrary to the results given by Tirole (2005), the proposed model shows that a good firm can avoid simultaneous equilibrium by differentiating the timing of its investment from that of the bad firm. However, the equilibrium strategy for the good firm is different from that under symmetric information, which decreases the project value of the good firm.

Proposition 5 suggests that in the case where $\bar{X}_g > \bar{X}_b^{\min}$, which implies that the difference in the expansion rates is relatively small, the good firm needs to invest in the expansion project earlier than the optimal timing under symmetric information, so as to send a signal to the investors to prove that it is indeed a good firm.

5 Numerical examples

In this section we present several numerical examples illustrating the different equilibrium strategies. Figure 1 shows investment thresholds for both firms plotted against the good firm ’ s expansion rate. Parameter values used in the figure are $r = 0.05, \mu = 0.01, \sigma = 0.25, I = 10$, and $\theta_b = 1.1$. In the figure, “a”, “b”, and “c” represent $\bar{X}_b^{\min}$, $\bar{X}_g^{\min}$, and $\bar{X}_g$, respectively. In the figure, let θ_g^* be the expansion rate where $\bar{X}_g = \bar{X}_b^{\min}$. Figure 1 indicates that the separate equilibrium given by Proposition 4 is established if θ_g is greater than θ_g^*; otherwise the separate equilibrium given by Proposition 5 occurs. Figure 2 illustrates similar results with different expansion rates for the bad firm. We consider three cases, i.e., $\theta_b = 1.05, 1.15, 1.2$.

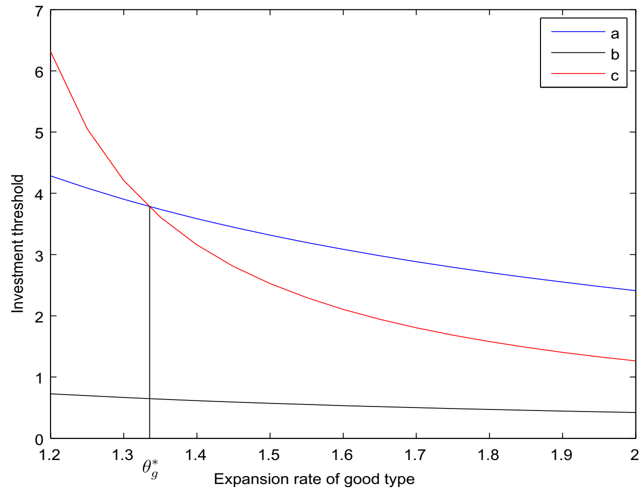


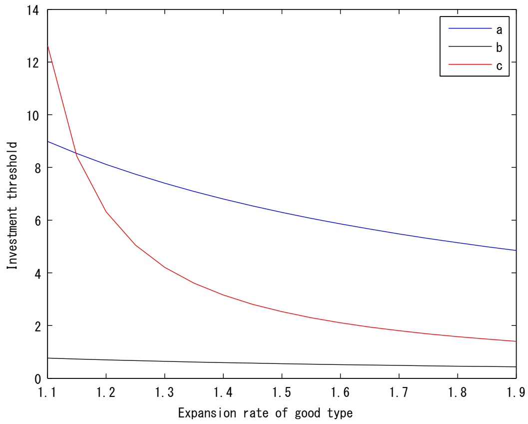
Figure 1: Investment threshold versus the expansion rate of the good firm
In the figure, “a”, “b”, and “c” represent $\bar{X}_b^{\min}$, $\bar{X}_g^{\min}$ and $\bar{X}_g$, respectively.

From the figures we can confirm that the larger the expansion rate for the bad firm is, the larger the value of θ_g^* tends to become. In other words, the good firm ’s expansion tends to occur earlier as the expansion rate for the bad firm increases.

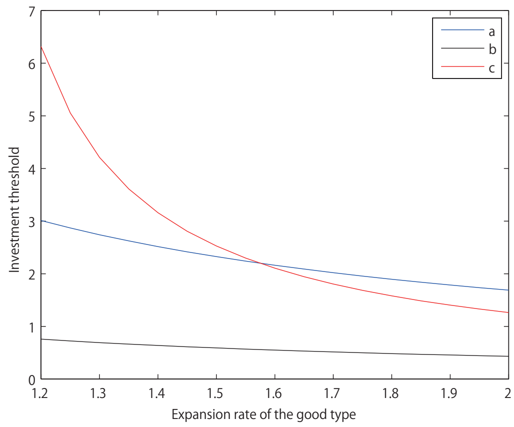
5.1 Sensitivity of the equilibrium strategy

It is important to examine the effect of certain parameter values on the equilibrium strategy for both firms. First, we consider the sensitivity with respect to the investment cost I. Figure 3 shows the investment threshold for different investment costs. We consider three cases, i.e., $I = 5?AI = 20?AI = 30$. Other parameter values used in the figure are $r = 0.05, \mu = 0.01, \sigma = 0.25$, and $\theta_b = 1.1$. Figure 3 indicates that the critical expansion rate θ_g^* is independent of the investment cost. This is because our model assumes a symmetric investment cost.

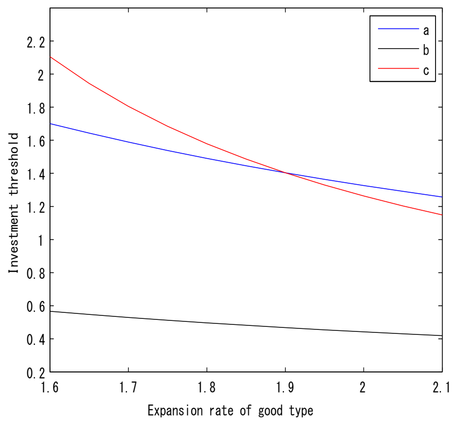
Next, we consider the sensitivity with respect to volatility σ. Figure 4 shows the investment threshold for different volatilities. We consider three cases, i.e., $\sigma = 0.1, 0.3, 0.5$. Other parameter values used in the figure are $r = 0.05, \mu = 0.01, I = 10$, and $\theta_b = 1.1$. Figure 4 indicates that the critical expansion rate θ_g^* tends to increase as the volatility increases.



(a) $\theta_b = 1.05$

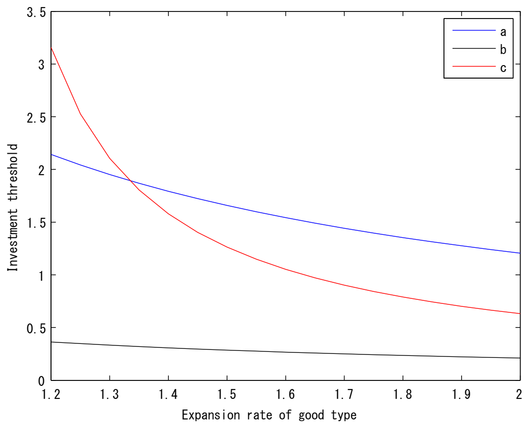


(b) $\theta_b = 1.15$

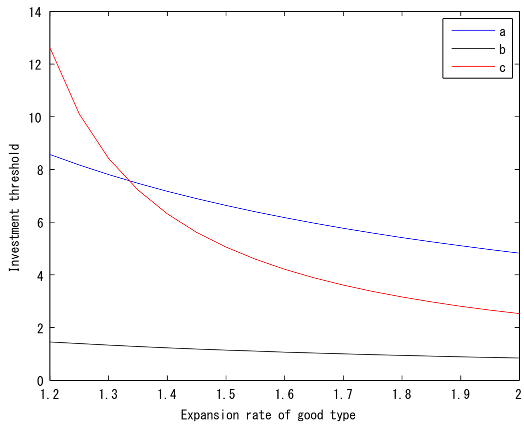


(c) $\theta_b = 1.2$

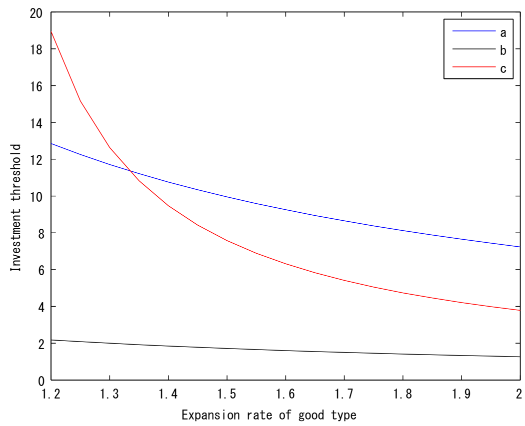
Figure 2: Investment threshold versus different expansion rates for the bad firm
In the figure, "a", "b", and "c" represent $\bar{X}_b^{\min}$, $\bar{X}_g^{\min}$, and $\bar{X}_g$, respectively.



(a) $I = 5$



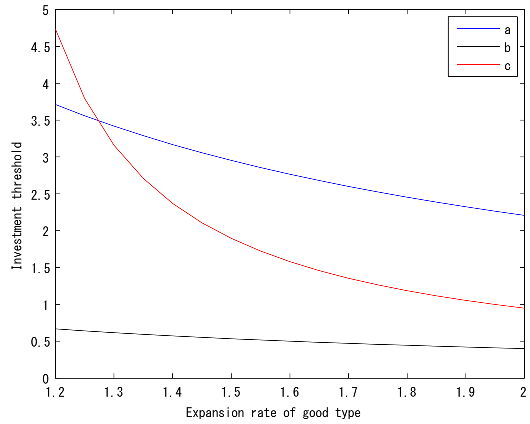
(b) $I = 20$



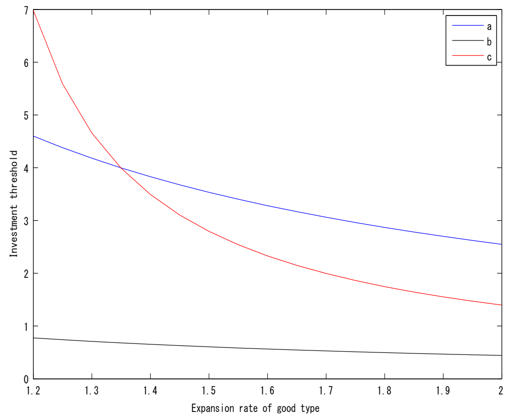
(c) $I = 30$

Figure 3: Investment threshold versus investment cost

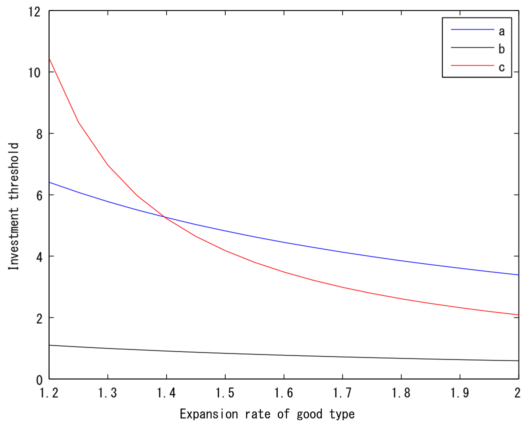
In the figure, “a”, “b”, and “c” represent $\bar{X}_b^{\min}$, $\bar{X}_g^{\min}$ and $\bar{X}_g$, respectively.



(a) $\sigma = 0.1$



(b) $\sigma = 0.3$



(c) $\sigma = 0.5$

Figure 4: Investment threshold for different volatilities

In the figure, “a”, “b”, and “c” represent $\bar{X}_b^{\min}$, $\bar{X}_g^{\min}$ and $\bar{X}_g$, respectively.

In other words, the good firm is put in a more difficult position to distinguish itself as the volatility increases. For further analysis, Figure 5 shows the investment threshold for differ-

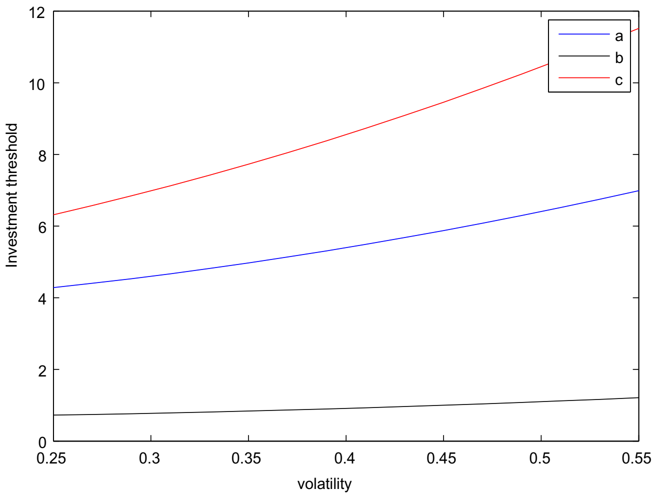


Figure 5: Investment threshold versus volatility
In the figure, "a", "b", and "c" represent $\bar{X}_b^{\min}$, $\bar{X}_g^{\min}$ and $\bar{X}_g$, respectively.

ent volatilities. Parameter values used in the figure are $r = 0.05, \mu = 0.01, I = 10, \theta_g = 1.2$, and $\theta_b = 1.1$. Figure 5 implies that $\bar{X}_b^{\min}$, $\bar{X}_g^{\min}$, and $\bar{X}_g$ increase as the volatility increases. It is well known that the value of an option increases as the volatility increases. Intuitively, this implies that the real option value for the bad firm becomes relatively larger compared to the value obtained by mimicking the good firm as the volatility increases. We also notice that the sensitivity of the three thresholds are all different in this numerical example, that is, $\bar{X}_g^{\min} < \bar{X}_b^{\min} < \bar{X}_g$. Consequently, as shown in Figure 4, the value of θ_g^* increases as the volatility increases.

6 Concluding remarks

In this paper we developed a dynamic investment model for expanding an existing project under asymmetric information. The model revealed that a good firm with a larger expansion rate can distinguish itself by sending a signal to the investors. More concretely, the good firm invests earlier in the project than the bad firm so that outside investors can distinguish it as being good.

In contrast to the work of Tirole (2005), the proposed model shows that: (i) when the expansion rate of a good firm is relatively large, the good firm can choose the optimal timing for its expansion independently, and (ii) even when the expansion rate of the good firm is relatively small, a good firm can still distinguish itself from a bad firm by investing earlier than the optimal timing under symmetric information. In other words, a separate equilibrium is always established in the proposed model.

Finally, we discuss two future works. First, we need to consider debt issues in addition to equity funding and second, taking risk averse investors into consideration is also an important topic.

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