Sensitivity of Demand Function Choice in a Strategic Real Options Context

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Abstract

This paper studies the effect of the three most commonly used demand functions, i.e. additive, multiplicative and iso-elastic demand, on the investment decisions of two competitive firms.

We show that the relative investment decision of the two firms can be very sensitive to the choice of a specific demand function. We find that the use of the multiplicative demand function results in a market where the leader has a bigger capacity than the follower. This is caused by the fixed price intercept of the multiplicative demand model, which implies that there is a fixed market size that has to be shared among the two firms in the market. Because the leader makes its investment decision first, it will be the firm with the largest capacity in the market. The opposite result occurs for the iso-elastic and additive demand model, because these models have no upper bound on demand. Then a follower will delay investment, in order to optimally invest in a large capacity amount. However, the introduction of convex costs violates the previous multiplicative result with linear costs. When the convex costs are sufficiently high, also for this demand function we find that the follower is the firm with the largest capacity. Furthermore, for the iso-elastic demand function, we show that only for a low elasticity parameter the monopoly profit of the leader is large enough that it is optimal to use the deterrence strategy.

1 Introduction

Most papers that take a real options approach to consider optimal investment decisions under uncertainty, concentrate on the question of optimal investment timing in the presence of irreversibility and uncertainty. In most of those papers, either the project value V(t) or the product price P(t) is chosen to be uncertain, where there is no need to more precisely specify the dependence of V(t) and P(t) on other parameters as for example quantity or a demand intercept (e.g. Dixit and Pindyck (1994), Nishide and Nomi (2009), Egami (2010), Nishihara and Shibata (2010), Armada, Kryzanowski and Pereira (2011) or Grenadier and Malenko (2011)).

Where this stream of literature discusses the timing of investment, it completely ignores a firm's capacity choice, which is an important part of the irreversible investment decision. When capacity or quantity optimization becomes an issue, one has to specify the demand function in more detail (e.g. Hagspiel (2011), Dangl (1999) and He and Pindyck (1992)). The choice of demand function is important and will be very determining for the outcomes in a model. Building a model, one should take into account how the demand function choice affects the optimal choice of capacity, timing and the resulting project value.

According to our knowledge the following two papers are the only ones in the literature that explicitly address the impact of the demand function choice on optimal investment decisions. Anupindi and Jiang (2008) categorize demand functions into two streams with different demand shocks, multiplicative and additive. In their paper these are respectively defined by $P(q, A) = A(a - q^n)^+$ and $P(q, A) = (A - q^n)^+$, for $n \geq 1$, a a constant, q the demand, and A the demand shock parameter that evolves according to a geometric Brownian motion. However, their results with respect to the two types of demand shocks focus on the choice between dedicated and flexible capacity and not the optimal capacity size. Even though they consider stochastic demand, their model does not allow them to optimize the timing of investment. Also they consider two symmetric firms and not a leader-follower setting. Ming-Gao Lin-Lin and Xiang-yang (2011) extend the paper of Fontes (2008), who takes a look at three types of flexibility (contract capacity - switch to a lower capacity level, expand capacity - switch to an upper capacity level, or both subtract and expand capacity), by comparing results for the multiplicative and additive demand function. They find that the capacity flexibility premium is significantly higher under the additive demand uncertainty than under the multiplicative demand shock. While those papers notice that the choice of a specific demand function can have an impact on their results, we did not come across any paper that elaborates on the explicit impact of this choice on timing and capacity of an optimal investment decision. That is where we want to make a contribution to the literature. This paper studies the effect of the three most commonly used demand functions, i.e. additive, multiplicative and iso-elastic, on the investment decisions of two competitive firms.

While Anupindi and Jiang (2008) discuss investment decisions in a competitive setting by comparing two types of demand functions, the majority of literature that take into account capacity or quantity optimization in a competitive setting, chooses one specific type of demand function. For example Aguerrevere (2003), and Holmberg and Willems (2011) are real options papers that optimize capacity under the assumption of a demand function with an **additive demand shock**. Grenadier (2000), Gotoa (2007), and Wang and Zhou (2004) are examples of real options papers that consider competition in a model using the **multiplicative demand shock**. They define their price function as the demand shock multiplied by a function that depends on demand: $P(t) = A(t) * D(Q(t))$, where A(t) is the demand shock. In Huisman and Kort (2012) demand $D(Q(t))$ is defined more specific by $D(Q(t)) = 1 - \eta Q(t)$. Furthermore, we want to categorize a third type of demand function, namely the **iso-elastic shock**. This model is used for example in Aguerrevere (2009), Novy-Marx (2007), Musshoff, Hirschauer and Balman (2007) and Schwartz and Torous (2003). The iso-elastic demand model is defined by $P(t) = A(t)Q(t)^{-1/\gamma}$, with $\gamma > 1$ being the elasticity of demand.

In this paper we discuss the impact of the three most commonly used demand functions in the literature, i.e. additive, multiplicative and iso-elastic demand, on the optimal investment decisions of firms in a competitive setting. We define those by $P_t = X_t - \eta Q_t$, $P_t = X_t(1 - \eta Q_t)$, and $P_t = X_t Q_t^{-\gamma}$, where demand uncertainty is captured in uncertainty parameter $\{X_t\}$, following a geometric Brownian motion. The models are not comparable in absolute terms, however, the competitive setting enables us to compare relative performances of the two firms for the three types of demand models. This paper follows the analysis used in Huisman and Kort (2012), that assume the multiplicative demand function, considering the additive and iso-elastic demand function.

We show that results can vary a lot between these three types of demand shocks. One of the main results of this paper is that the use of the additive or iso-elastic demand function will lead to a larger investment choice of the follower than the leader. However, the use of the multiplicative demand model for similar market conditions, will give the opposite result. The additive or iso-elastic demand model choices would, for example, be suitable in a situation where a foreign (but longer existing) firm enters a domestic market. Because the foreign firm was already active in other countries, and therefore it has already built up enough capital to take the risk of a big investment. The domestic market can already consist of a (smaller) market leader. Take for example the online shopping website Zalando GmbH, who was founded in 2008 in Germany. In 2009 Zalando began operations abroad by starting offering deliveries in Austria and in 2010 in Netherlands and France. Due to a huge and effective marketing compagne it became one year after entry already the market leader in online shoe sales in France in 2011. The multiplicative demand model would be more appropriate for discussing two firms that are both in the same build-up stage. One should make a deliberate choice for the type of demand function, in order to fit the right model to the presumed market situation. Another result that we find is that including convex costs in the model will abrogate the previous result with respect to the multiplicative demand function. When the convex cost are a sufficiently large part of the total investment costs, also for the multiplicative demand model choice the follower is the firm with the largest capacity in the market. Large convex costs lead to a low optimal capacity of both firms, however, the follower also enjoys indirectly from the low capacity of the leader, and therefore the follower will be the firm with the largest capacity in the market. Furthermore, we find that for the iso-elastic demand model choice the deterrence strategy is only preferred by the leader for low values of the elasticity parameter and a low discount factor. A low elasticity parameter gives the leader enough monopoly profit to make it optimal to deter the entrant, and a low discount factor causes that the leader assigns less attention to the impact of a future entry of the follower in its current investment decision.

The paper is structured as follows. In the next section we present the general model. In Section 3 we analyze the capacity and timing decision of the two firms in a model where both firms have linear investment costs. Section 3.1 assumes the additive linear demand function, and Section 3.2 the multiplicative linear demand function. We discuss the results, considering the relative differences between the two firms for the two models, in Section 4. In Section 5 the iso-elastic demand function will be compared to the two linear demand function, under the assumption of convex costs. Section 6 concludes.

2 Model

We consider a duopoly game in the real options context, where two firms produce a single homogenous good. The firms are assumed to be risk neutral and value maximizing, where future profits will be discounted with constant discount rate r. Each firm is able to invest once in an irreversible investment. This setting will be analyzed for three different types of inverse demand functions, depending on the nature of the demand shock. The inverse demand functions at time t are defined by:

$$P_{t,mult} = X_t(1 - \eta Q_t) \tag{1}$$

for multiplicative demand,

$$P_{t,add} = X_t - \eta Q_t \tag{2}$$

for additive demand, and

$$P_{t,iso} = X_t Q_t^{-\gamma} \tag{3}$$

for iso-elastic demand. Equations (1), (2), and (3) are also known as the net-price functions. The gross-price functions are respectively denoted by $p_{t,mult}$, $p_{t,add}$, and $p_{t,iso}$, and variable production costs by parameter c. By subtracting the variable costs from the gross-price function, one obtains the net-price functions. For example $P_{t,mult} = p_{t,mult} - c$ is the net price function for the multiplicative demand. We impose the following assumption:

Assumption 1 *A firm can produce a positive quantity of a product that corresponds to a negative (net-)price.*

Among others, Mackintosh (2003) or Holweg and Pil (2004) argue that in practice it is very common to sell a product below unit cost. In the automotive industry, shutting production lines down is more costly than producing against a negative price. Also in the electricity market, during the nights, demand supply can be bigger than then demand and then the electricity price will be negative. This is caused for instance by the so-called must-run character of combined-cycle facilities, that are installed for the generation of heat (steam), and generate electricity as co-product. High shutdown costs are involved by reducing the must-run output, and therefore it is better to keep producing (Sewalt and de Jong (2003)). Since we include variable production costs in the net-price functions, prices can become negative if the variable costs exceed the gross price.

The total market output is given by Q_t, $\eta > 0$ is a constant, and demand uncertainty is modeled by $\{X_t\}$ following the following geometric Brownian motion

$$dX_t = \mu X_t dt + \sigma X_t d\omega_t \tag{4}$$

In this expression μ is the trend parameter, σ the volatility parameter and $d\omega_t$ the increment of a Wiener process. We impose the market clearance assumption, which states that a firm will use its full capacity for production. The investment costs are modeled similar to Aguerrevere (2003). A firm that enters the market with capacity amount Q_t has investment cost $\delta_1 Q_t + \delta_2 Q_t^2$, where δ_1 and δ_2 are positive constants. For analytical convenience, in Section 3 it is assumed that $\delta_2 = 0$, leaving us with the linear cost model. The investment decision, in a duopoly setting with exogenous firm roles, will be analytically compared for a model with multiplicative and additive demand. In Section 5 the general case is considered, with $\delta_2 > 0$. The use of convex investment costs enables us to compare the linear demand models numerically with the iso-elastic demand model. The use of linear investment costs in an iso-elastic demand model results in a model that is linear in capacity Q_t. Therefore, a finite solution for an optimal Q_t cannot be found.

3 Linear investment costs

This section considers a duopoly, consisting of a leader and a follower[1]. Denote the optimal capacity of the leading and following firm with Q_L and Q_F, respectively. Once both investors have invested, the total market output is equal to $Q = Q_L + Q_F$. In this section we consider linear investment costs, equal to $(\delta_1 Q_t)$.

[1]For the monopoly case, the monopolist's investment decision is equal to the follower's investment decision, where the optimal capacity of the leader is equal to zero. Huisman and Kort (2012) give for the multiplicative demand function also an analysis for the monopolist scenario.

The firm roles are assumed to be exogenous. In other words, the leading firm knows that it will be the first investor in the market and cannot be preempted by the other firm. The other firm is the follower, which knows that it will have to wait with its investment until the leader has invested. This section is divided into two subsections, where we will discuss the impact of the two types of demand functions, respectively.

3.1 Additive demand

This section assumes an additive inverse demand function, defined by (2). The steps taken that lead to the solutions in this paper, are analogous to Huisman and Kort (2012) who consider a multiplicative inverse demand function. Denote with X(0), the value of demand shock X at time zero. Assume that the initial level of demand shock is low enough to fall below any of the investment triggers derived in this section. We solve this game backwards, which implies that we start with deriving the optimal investment decision of the follower. The following firm invests when the leader is already in the market, and therefore it is assumed that the optimal decision of the leading firm is known to the follower. Consequently, the optimal investment timing of the follower ($X_F(Q_L)$) and its optimal capacity ($Q_F(Q_L)$) are a function of the optimal leader's capacity Q_L. The optimal investment thresholds of the follower are the content of Proposition 1.

Proposition 1 *Suppose the additive inverse demand function $P(t) = X(t) - \eta Q(t)$. Given the current level of stochastic demand shock X, and the leader's capacity level Q_L, the optimal capacity level of the follower $Q_F(X, Q_L)$ is equal to:*

$$Q_F^*(X, Q_L) = \frac{Xr - (r-\mu)(r\delta + Q_L\eta)}{2\eta(r-\mu)}. \tag{5}$$

The value function of the follower $V^*(X, Q_L)$ is given by

$$V_F^*(X, Q_L) = \begin{cases} A_F(Q_L)X^\beta & \text{if } X < X_F^*(Q_L), \\ \frac{(Xr-(r-\mu)(r\delta+Q_L\eta))^2}{4r\eta(r-\mu)^2} & \text{if } X > X_F^*(Q_L), \end{cases}$$

where

$$A_F^*(Q_L) = \left(\frac{r(\beta-2)}{\beta(r\delta + Q_L\eta)(r-\mu)}\right)^\beta \left(\frac{(r\delta + Q_L\eta)^2}{(\beta-2)^2 r\eta}\right), \tag{6}$$

$$X_F^*(Q_L) = \frac{\beta(r\delta + Q_L\eta)(r-\mu)}{r(\beta-2)}, \tag{7}$$

and β the positive root of the quadratic polynomial

$$\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0, \tag{8}$$

so that

$$Q_F^*(Q_L) = Q_F^*(X_F^*(Q_L), Q_L) = \frac{r\delta + Q_L\eta}{r(\beta-2)}. \tag{9}$$

Notice that to ensure convergence, we have to assume that $\beta > 2$ (Hagspiel (2011)).

The next step is to determine the investment decision of the leader. The leader has two possible strategies. Entry deterrence corresponds to sequential investments, and gives the leading firm a monopoly profit for some amount of time right after its investment, till the moment where the follower enters. Entry accommodation leads to simultaneous investments, where the follower invests at the same time as the leader. The leader will use its optimal capacity Q_L as a tool to force either one of those strategies. Here it is important to realize that X_F is increasing in Q_L, i.e. the follower's investment threshold is increasing in the leader's capacity size. Besides delaying investment of the follower, another incentive for the leader to invest in a large capacity is that Q_F decreases in Q_L.

First we give a brief explanation of the leader's strategies in both cases, which will explain the triggers and boundaries used in the coming propositions. Figure 1 gives an illustrative support of this explanation, and shows the regions where a strategy is feasible. Afterwards, each strategy will be discussed comprehensively, and results will be stated in the following propositions.

Entry deterrence

Recall that entry deterrence implies that the entrant invests later than the leading firm. Given the current level of X, a large enough investment of the leading firm leads to a follower's investment trigger $X_F^*(Q_L)$ larger than X. Therefore we are looking for a *lower* bound $\underline{Q_L}$, for which $Q_L > \underline{Q_L}$ implies the entry deterrence strategy. Translating this in terms of X, entry deterrence can only occur when the value of demand shock X is below an *upper* bound $\overline{X^{det}}$, i.e. $X < \overline{X^{det}}$. Otherwise, the market is large enough, that it is optimal for the follower to enter immediately once the leader has invested. The fact that the leader's capacity is nonnegative, corresponds to a *lower* bound on X, denoted by $\underline{X^{det}}$, where for $X > \underline{X^{det}}$ entry deterrence is a feasible strategy. In that case the demand level is too low for an investment to be profitable.

Entry accommodation

Alternatively, a firm chooses for the accommodation strategy, where the follower invests simultaneous with the leader. The accommodation strategy can only occur for low capacity investments of the leader, i.e. $Q_L \leq \underline{Q_L}$. Entry accommodation only occurs for high levels of demand shock X, i.e. there is a *lower* bound $\underline{X^{acc}}$ for which $X > \underline{X^{acc}}$ leads to entry accommodation.

Figure 1 shows that there are three possible regions for the additive demand function and four regions for the multiplicative demand function (see proof Proposition 2). For low values of X there is no investment possible. Somewhat higher values of X enable the leader to choose for the deterrence strategy. In the third

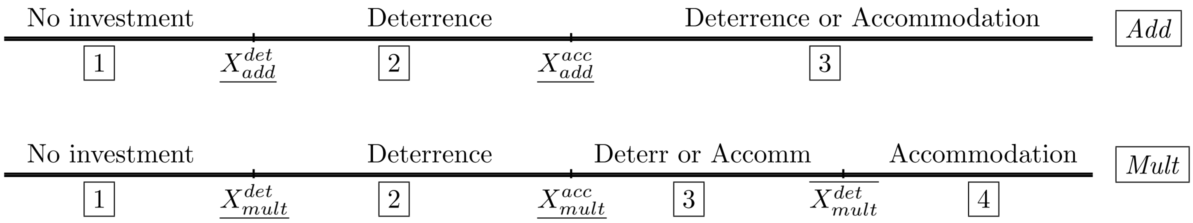

Figure 1: Location of accommodation and deterrence boundaries for the additive (upper figure) and multiplicative (bottom figure) demand functions.

region both strategies could be chosen. In this region, the leader will choose the strategy that results in the highest value. For some threshold $\hat{X}$, the leader is indifferent between the deterrence and the accommodation strategy, i.e.

$$\hat{X} = min\{X \in (\underline{X^{acc}}, \overline{X^{det}}) | V_L^{det}(X) = V_L^{acc}(X)\}. \tag{10}$$

In the last region (region 4), for i.e. very high levels of X, the leader is forced to apply the accommodation strategy. For the additive demand function, we find that there is no upper bound $\overline{X^{det}}$ on X for the deterrence strategy (see proof Proposition 2). For all X, it holds that the optimal capacity value $Q_L^{det}(X) > \underline{Q_L(X)}$. This is intuitive since the additive inverse demand function, given by equation (2), is also not restricted by a fixed price intercept. The leader's capacity has the ability to grow infinitely when the uncertainty parameter X grows. For high values of X, deterrence will always be possible, because the leader can place a correspondingly large capacity in the market to deter the entrant. Referring to Figure 1, this implies that for the additive demand function, there is no region for X where only the accommodation value is possible.

Let us discuss the two leader's strategies more comprehensively. We start with the investment decision of the leader that chooses for the deterrence strategy. The value function of a leader with an entry deterrence policy is given by

$$V_L^{det}(X, Q_L) = \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta Q_L - (\frac{\eta Q_L Q_F}{r})(\frac{X}{X_F^*(Q_L)})^\beta. \tag{11}$$

Since the leading firm uses the entry deterrence policy, it incurs monopoly profits for a certain amount of time, reflected by the first two terms of the value function. The third term are the investment costs necessary to install capacity at amount Q_L. However, at some point in time, the entrant will also invest in the market, which negatively influences the value of the leader. This impact is shown by the negative fourth term of the leader's value function, which is the respective difference between the leader's monopoly and duopoly profit, that is discounted from $X_F^*(Q_L)$ to X by discount factor $(\frac{X}{X_F^*(Q_L)})^\beta$.

Given that the leading firm will use the deterrence strategy, it will maximize (11) with respect to timing (X_L^{det}) and capacity (Q_L^{det}). Proposition 2 summarizes the optimal investment decision of the leader when it uses the entry deterrence policy.

Proposition 2 *Consider the additive inverse demand function $P(t) = X(t) - \eta Q(t)$. The deterrence strategy occurs whenever the leader chooses a capacity level $Q_L > \underline{Q_L}$ such that $X_F^*(Q_L) > X$. $\underline{Q_L(X)}$ is defined by:*

$$\underline{Q_L(X)} = \frac{1}{\eta}\left(\frac{Xr(\beta-2)}{\beta(r-\mu)} - \delta r\right). \tag{12}$$

In terms of the demand shock parameter X, the leader will consider the entry deterrence strategy whenever the current level of X lies within the interval $(\underline{X^{det}}, \infty)$, where $\underline{X^{det}}$ is implicitly determined by

$$\frac{\underline{X^{det}}}{r-\mu} - \delta - \frac{\delta}{\beta-2}\left(\frac{\underline{X^{det}}(\beta-2)}{\delta\beta(r-\mu)}\right)^{\beta} = 0. \tag{13}$$

The value function for the leader's entry deterrence strategy, when the leader invests at X, $V_L^{det}(X)$, equals

$$V_L^{det}(X) = \frac{XQ_L^{det}(X)}{r-\mu} - \frac{\eta(Q_L^{det}(X))^2}{r} - \delta Q_L^{det}(X) - \left(\frac{Q_L^{det}(X)(\delta r + Q_L^{det}(X)\eta)}{r(\beta-2)}\right)\left(\frac{Xr(\beta-2)}{\beta(\delta r + Q_L^{det}(X)\eta)(r-\mu)}\right)^{\beta}. \tag{14}$$

The optimal investment threshold X_L^{det} and the corresponding Q_L^{det} is given by

$$X_L^{det} = \frac{\delta\beta(r-\mu)}{\beta-2}, \tag{15}$$

$$Q_L^{det} = \frac{\delta r}{(\beta-2)\eta}. \tag{16}$$

The alternative for the leader is to use the entry accommodation policy, where it allows the entrant to immediately invest once it has invested itself. The value function of the leader, using the accommodation policy, accordingly V_L^{acc} is given by

$$V_L^{acc}(X, Q_L) = \frac{XQ_L}{r-\mu} - \frac{\eta((Q_L)^2 + Q_L Q_F)}{r} - \delta Q_L. \tag{17}$$

The first two terms represent the leader's expected discounted duopoly profit. Notice that the leader will not obtain any monopoly profit, when choosing for the accommodation strategy. The third term in this expression are the investment costs resulting from a capacity amount of Q_L. Proposition 3 gives the investment decision of a leading firm that uses the entry accommodation policy.

Proposition 3 *Suppose additive inverse demand function $P(t) = X(t) - \eta Q(t)$. The leader will consider the entry accommodation strategy whenever the current level of X is larger than or equal to $\underline{X^{acc}}$, where*

$$\underline{X^{acc}} = \frac{\delta\beta(r-\mu)}{\beta-4}. \tag{18}$$

The leader's value of the entry accommodation strategy, when investment takes place at X, is equal to:

$$V_L^{acc}(X) = \frac{r(X - \delta(r-\mu))^2}{8\eta(r-\mu)^2}. \tag{19}$$

The optimal investment threshold and corresponding capacity level for the entry accommodation strategy are given by

$$X_L^{acc} = \frac{\delta\beta(r-\mu)}{\beta-2}, \tag{20}$$

$$Q_L^{acc} = \frac{\delta r}{(\beta - 2)\eta}. \quad (21)$$

Notice that for $\beta < 4$, the accommodation boundary $\underline{X_{det}^{acc}}$ will be negative and hereby dispensable. We elaborate on this in Section 4.

Endogenous firm roles

The first part of this section analyzed the investment decisions of the two firms under the assumption of exogenous firm roles. However, in reality, firms do not know their market position beforehand. For the analysis of endogenous firm roles we can use the knowledge of the previous part of this section. It is profitable to be the leader in a competitive market, because this comes together with a period of monopoly profits, therefore both firms will try to become the market leader. Once it is known which of the two firms grabbed the market first, the other firm becomes the market follower. After investment of the leader, the follower acts as if the market positions are exogenously determined, because there are no strategic aspects related to this investment decision. Therefore, for the investment decision of the follower, under the assumption of endogenous firm roles, we can refer to Proposition 1.

Both firms want the leader's position, and try to preempt each other, therefore, investment will take place at preemption trigger X_P, the moment where a firm is indifferent between waiting for the followers position and investing in the leaders position. Among others, Huisman (2001) shows that the preemption trigger can be obtained by solving the following equation for X_P:

$$V_L^*(X_P, Q_L^*(X_P)) = V_F^*(X_P, Q_L^*(X_P)). \quad (22)$$

A firm does not want to invest for $X < X_P$ because then it is more profitable to wait for the followers position. The firms are assumed to be symmetric, so none of the firms will invest. For $X > X_P$, it is more profitable for a firm to invest and become the leader, than wait with investment. However, this is the case for both firms. Assume that firm 1 wants to invest at level X, then firm 2 will preempt this firm and invest at $X - \epsilon$. The reaction of firm 1 is to invest even before firm 2, at $X - 2\epsilon$. This preemption mechanism proceeds until $X - n\epsilon = X_P$, where one of the firms invests. Because the firms are symmetric, both have equal probabilities to become the market leader at the preemption trigger. The preemption trigger X_P has to be implicitly determined by equation(22), and one cannot derive analytical expressions for X_P, the corresponding optimal capacity of the leader $Q_L(X_P)$, and the investment trigger and optimal capacity of the follower $X_F(Q_L(X_P))$ and $Q_F(Q_L(X_P))$.

3.2 Multiplicative demand

This section assumes the multiplicative inverse demand function, defined by (1). This case has been elaborately investigated by Huisman and Kort (2012). Therefore, we refer to their paper for the analysis that lead to multiplicative results. Notice that, in contrast to the use of the additive demand model, a multiplicative demand function results in an upper bound on X for the deterrence strategy. Investment above threshold $\overline{X^{det}}$ makes entry deterrence impossible. Then demand is so high that it is optimal for the follower to invest immediately once the leader has invested. Therefore a model using the multiplicative demand has a smaller region where the entry deterrence strategy can be exercised, compared to an additive demand model. This is due to the restricted property of the multiplicative demand: there is a fixed market share that has to be divided by the two firms.

Table 1 summarizes the optimal investment decision and boundaries of the leader and follower for additive and multiplicative inverse demand functions, when exogenous firm roles are assumed. For the endogenous firm role situation we refer to Huisman and Kort (2012).

Formula	Additive	Multiplicative
	Entry deterrence	
$\underline{Q_L(X)}$	$\frac{1}{\eta}\left(\frac{Xr(\beta-2)}{\beta(r-\mu)}-\delta r\right)$	$\frac{1}{\eta}\left(1-\frac{(\beta+1)\delta(r-\mu)}{(\beta-1)X}\right)$
$\underline{X^{det}}$	$\frac{\underline{X^{det}}}{r-\mu}-\delta-\frac{\delta}{\beta-2}\left(\frac{\underline{X^{det}}(\beta-2)}{\delta\beta(r-\mu)}\right)^{\beta}=0$	$\frac{\underline{X^{det}}}{r-\mu}-\delta-\frac{\delta}{\beta-1}\left(\frac{\underline{X^{det}}(\beta-1)}{\delta(\beta+1)(r-\mu)}\right)^{\beta}=0$
$\overline{X^{det}}$	∞	$\frac{2(\beta+1)\delta(r-\mu)}{\beta-1}$
X_L^{det}	$\frac{\delta\beta(r-\mu)}{\beta-2}$	$\frac{(\beta+1)\delta(r-\mu)}{\beta-1}$
Q_L^{det}	$\frac{\delta r}{(\beta-2)\eta}$	$\frac{1}{(\beta+1)\eta}$
$X_F^{det}(Q_L^{det})$	$\frac{\delta(\beta-1)\beta(r-\mu)}{(\beta-2)^2}$	$\frac{(\beta+1)^2\delta(r-\mu)}{\beta(\beta-1)}$
$Q_F^{det}(Q_L^{det})$	$\frac{\delta r(\beta-1)}{(\beta-2)^2\eta}$	$\frac{\beta}{(\beta+1)^2\eta}$
	Entry accommodation	
$\underline{X^{acc}}$	$\frac{\delta\beta(r-\mu)}{\beta-4}$	$\frac{(\beta+3)\delta(r-\mu)}{(\beta-1)}$
$Q_L(\underline{X^{acc}})$	$\frac{2\delta r}{(\beta-4)\eta}$	$\frac{2}{(\beta+3)\eta}$
$X_F^{acc}(Q_L^{acc}(\underline{X^{acc}}))$	$\frac{\delta\beta(r-\mu)}{\beta-4}$	$\frac{(\beta+3)\delta(r-\mu)}{(\beta-1)}$
$Q_F^{acc}(Q_L^{acc}(\underline{X^{acc}}))$	$\frac{\delta r}{(\beta-4)\eta}$	$\frac{1}{(\beta+3)\eta}$
X_L^{acc}	$\frac{\delta\beta(r-\mu)}{\beta-2}$	$\frac{(\beta+1)\delta(r-\mu)}{\beta-1}$
Q_L^{acc}	$\frac{\delta r}{(\beta-2)\eta}$	$\frac{1}{(\beta+1)\eta}$
$X_F^{acc}(Q_L^{acc}(X_L^{acc}))$	$\frac{\delta(\beta-1)\beta(r-\mu)}{(\beta-2)^2}$	
$Q_F^{acc}(Q_L^{acc}(X_L^{acc}))$	$\frac{\delta r}{2(\beta-2)\eta}$	

Table 1: Boundaries, optimal investment triggers and optimal capacities corresponding to the additive and multiplicative inverse demand function, respectively.

4 Results

4.1 Entry deterrence

The upper part of Table 1 compares, for the entry deterrence strategy, the boundaries on entry deterrence, and the investment decision for the two discussed types of inverse demand functions.

Notice that the resulting optimal investment triggers should be within the boundaries. In other words, the investment trigger of the leader, in case of entry deterrence should be in between $[\underline{X^{det}}, \overline{X^{det}}]$. For entry accommodation, the leader's investment trigger should be larger than $\underline{X^{acc}}$.

In Proposition 4 we show that in a model using the additive inverse demand, the follower will always invest in a larger capacity compared to the leader, given that firm roles are exogenously determined. However, in a similar model, but using the multiplicative demand, it is the leader that will always invest in a larger capacity than the follower.

Proposition 4 *Suppose additive inverse demand function $P(t) = X(t) - \eta Q(t)$.*

Given that firm roles are exogenously determined, the follower will always invest in a larger capacity compared to the leader:

$$Q^{det}_{L,add} = \frac{\delta r}{(\beta-2)\eta} = \frac{\delta r(\beta-2)}{(\beta-2)^2\eta} < \frac{\delta r(\beta-1)}{(\beta-2)^2\eta} = Q^{det}_{F,add}. \tag{23}$$

Suppose multiplicative inverse demand function $P(t) = X(t)(1-\eta Q(t))$. Given that firm roles are exogenously determined, the leader will always invest in a larger capacity compared to the follower:

$$Q^{det}_{L,mult} = \frac{1}{(\beta+1)\eta} = \frac{\beta+1}{(\beta+1)^2\eta} > \frac{\beta}{(\beta+1)^2\eta} = Q^{det}_{F,mult}. \tag{24}$$

The difference in magnitudes between the follower and the leader optimal capacity are caused by the shape of the inverse demand functions. Using additive inverse demand, the follower gains by waiting long with investment until a large threshold for X is hit. This gives it the opportunity to invest in a larger capacity amount, compared to investing at a smaller X. Obviously, the larger the price intercept (X(t)) is, the larger the market size at that moment, and therefore the follower will invest optimally in a larger optimal capacity. A follower invests after the investment of the leader, that has taken its share of the market already. However, the additive demand function enables the follower to wait for high X so that the market is so large that it is optimal for the follower to invest in with a large capacity amount, which will make the follower a strong competitor for the leader.

Multiplicative inverse demand restricts the capacity choice of a firm. Again, the leader invests first, and takes its share of the market. However, now the follower cannot justify a large capacity investment by waiting for a high X, because the multiplicative inverse demand has a fixed upper bound independent of the stochastic process X. Obviously, this is a disadvantage for the follower. It will invest in a relatively small capacity.

Using the additive demand function, Proposition 4 showed that the follower will invest in a larger capacity than the leader. However, it also invests late, in order to be able to justify a larger capacity investment. This explains the result of Proposition 5, which shows that the use of the additive inverse demand function gives the leader a longer period of monopoly profits, compared to the case of multiplicative demand.

Proposition 5 *Suppose that the additive inverse demand function (add) is given by $P(t) = X(t) - \eta Q(t)$, and multiplicative demand function (mult) is given by $P(t) = X(t)(1 - \eta Q(t))$. Monopoly period is measured by ratio $\frac{X^x_{L,det}}{X^x_{F,det}}$, for both types of inverse demand $x = \{add, mult\}$. Consideration of an additive inverse demand function gives the leader a longer period of monopoly profits:*

$$\frac{X^{det}_{L,add}}{X^{add}_{F,det}} = \frac{\beta^2 - \beta - 2}{(\beta - 1)(\beta + 1)} \qquad < \qquad \frac{\beta^2 - \beta}{(\beta - 1)(\beta + 1)} = \frac{X^{mult}_{L,det}}{X^{det}_{F,mult}}. \tag{25}$$

Notice that we can only discuss relative differences between the two types of inverse demand functions. The two demand functions are incomparable in absolute terms. For this reason we cannot compare the deterrence value functions for the two types of inverse demand. However, it is interesting to know what a market leader prefers. The combination of a large period of monopoly profits, but facing a strong competitor in the future, or dominating the market with the largest capacity, but enjoying a shorter period of monopoly profits? Therefore we look at the effect that a follower has on the value function of the leader. The expression $\frac{V_{L,x}(X^{det}_{L,x})}{V_{M,x}(X_M)}$, with $x \in \{add, mult\}$, explains the relative downfall in profit for the leader when it knows there is a potential follower. L stands for the leader value in a duopoly, and M for the monopoly value. The smaller this expression, the more impact the entry of the follower has on the value of the leader. Huisman (2001, p.170) explains that the monopoly investment trigger is equal to the leader's investment trigger in a duopoly. This means, the determination of the threshold (X^{det}_L) has no effect on the optimal reply of the follower. Assume that the initial value X is sufficiently low, so that no firm has yet entered the market ($X < X_M = X^{det}_L$). Proposition 6 derives for the expression $\frac{V_{L,x}(X^{det}_{L,x})}{V_{M,x}(X_M)}$, for the two types of inverse demand functions $x \in \{add, mult\}$.

Proposition 6 *Suppose additive inverse demand function $P(t) = X(t) - \eta Q(t)$. Given that firm roles are exogenously determined, the relative downfall in profit at trigger $X_M = X^{det}_{L,add} = \frac{\beta\delta(r-\mu)}{\beta-2}$, when there will be a potential entrant in the market, is:*

$$\frac{V_{L,add}(X^{det}_{L,add})}{V_{M,add}(X_M)} = 1 - (\frac{\beta - 2}{\beta - 1})^{\beta - 1} \tag{26}$$

Suppose multiplicative inverse demand function $P(t) = X(t)(1 - \eta Q(t))$. Given that firm roles are exogenously determined, the relative downfall in profit at trigger $X_M = X^{det}_{L,mult} = \frac{(\beta+1)\delta(r-\mu)}{\beta-1}$, when there will be a *potential entrant in the market, is:*

$$\frac{V_{L,mult}(X^{det}_{L,mult})}{V^{mult}_M(X_M)} = 1 - (\frac{\beta}{\beta+1})^{\beta}. \tag{27}$$

It holds that the relative downfall in profit when there is a potential entrant in the market, is larger in case of multiplicative demand:

$$\frac{V_{L,add}(X^{det}_{L,add})}{V^{add}_M(X_M)} > \frac{V_{L,mult}(X^{det}_{L,mult})}{V^{mult}_M(X_M)}. \tag{28}$$

The results of Proposition 6 show that a firm would rather face a follower in the additive demand model, than in the multiplicative demand model. Apparently, enjoying a long period of monopoly profits dominates the effect of the entry of the follower.

4.2 Entry accommodation

When the leader invests relatively late in the market, the market is big enough for the follower to immediately enter once the leader has done so. The bottom part of Table 1 gives the accommodation strategies of the two firms for both types of inverse demand functions. Obviously, the leader and the follower invest simultaneously in this market, i.e. $X_L = X_F$. Notice that, for the multiplicative demand function, the optimal accommodation trigger of the leader will not lead to the entry accommodation strategy, since $X^{acc}_L < \underline{X^{acc}}$. Therefore, the leader has to invest at the minimal boundary $\underline{X^{acc}}$ that enables the accommodation strategy (see Huisman and Kort (2012)). The corresponding optimal leader's capacity at the boundary trigger is equal to $Q_L(\underline{X^{acc}})$, as denoted in Table 1. The follower invests at trigger $X^{acc}_F(Q^{acc}_L(\underline{X^{acc}}))$, in corresponding optimal capacity $Q^{acc}_F(Q^{acc}_L(\underline{X^{acc}}))$. In case of the additive demand, this depends on the value of β. For $\beta > 4$, the same result as in the multiplicative demand case occurs, and the leader invests at the boundary $\underline{X^{acc}_{add}}$. But for $4 > \beta > 2$, the accommodation boundary $\underline{X^{acc}_{add}}$ will be negative, and therefore dispensable. (Remember, to ensure convergence we need the assumption that $\beta > 2$, i.e $r > 2\mu + \sigma^2$.) The two firms will invest at the accommodation trigger $X^{acc}_{L,add} = X^{acc}_{F,add}$, with corresponding capacities $Q^{acc}_{L,mult}(X^{acc}_{L,mult})$ and $Q^{acc}_F(Q^{acc}_L(X^{acc}_{L,mult}))$.

Table 1 shows that even though the firms invest simultaneously, the optimal capacity choices of the two firms differ. Proposition 7 shows that the follower will always invest in a lower capacity level than the leader. This is because it is a Stackelberg equilibrium where the leader decides first about its capacity decision. It can be concluded that in the additive model using its accommodation strategy, in contrary to using its deterrence strategy, the leader is able to make the follower the smaller firm in the market. When the leader uses its accommodation strategy, the follower invests by definition at the same moment as the leader. Therefore, the follower invests in a lower capacity than the leader, when the leader applies the entry accommodation strategy. Still, the follower prefers to invest simultaneously with the leader, because the large accommodation trigger corresponds to a large optimal capacity investment for the follower, which causes that it does not want to delay investment to the future. In a model with multiplicative demand, the leader always invests in a larger capacity than the follower, no matter what strategy it uses.

Proposition 7 *Suppose the additive inverse demand function $P(t) = X(t) - \eta Q(t)$, or multiplicative inverse demand function $P(t) = X(t)(1 - \eta Q(t))$. Given that firm roles are exogenously determined, the leader will invest in a larger capacity compared to the follower, when it uses the accommodation strategy, i.e.*

$$\begin{array}{lll} Q^{acc}_{L,add}(\underline{X^{acc}}) = \frac{2\delta r}{(\beta-4)\eta} & > & \frac{\delta r}{(\beta-4)\eta} = Q^{acc}_{F,add}(Q^{acc}_{L,add}(\underline{X^{acc}})) \quad \text{if } \beta > 4, \\ Q^{acc}_{L,add}(X^{acc}_{L}) = \frac{\delta r}{(\beta-2)\eta} & > & \frac{\delta r}{2(\beta-2)\eta} = Q^{acc}_{F,add}(Q^{acc}_{L,add}(X^{acc}_{L})) \quad \text{if } 4 \geq \beta > 2, \\ Q^{acc}_{L,mult}(\underline{X^{acc}}) = \frac{2}{(\beta+3)\eta} & > & \frac{1}{(\beta+3)\eta} = Q^{acc}_{F,mult}(Q^{acc}_{L,mult}(\underline{X^{acc}})) \end{array}$$

5 Convex investment costs

In this section, we relax the assumption of linear investment costs and assume that investment costs are equal to $\delta_1 Q_t + \delta_2 Q_t^2$. This means that the price of investment is an increasing function of the quantity invested. Using convex investment costs allows us to take into account the iso-elastic demand model and compare it with the linear models. The analysis of the iso-elastic demand is similar to the analysis of the additive demand case in Section 3. However, in this situation, the solutions cannot analytically be given, and results will be given in figures. Equation (3) shows that the iso-elastic inverse demand model has an even more unrestricted character than the additive demand function, since the optimal capacity is not restricted by any price intercept. This section will show the differences in results that are driven by a specific choice of demand function considering a quadratic cost function. It also serves as a robustness check for the results from Section 4.

5.1 Entry deterrence

Recall that entry deterrence implies that the follower invests later than the leading firm, which gives the leader a period of monopoly profit. In this section we consider that parameter $\delta_2 > 0$, i.e. investment costs are convex. As a consequence we cannot solve the investment decisions of the firms analytically anymore. Two parameters are especially of interest, namely δ_1 and δ_2. Figure 2 and 3 show how sensitive the investment decisions of the leader and follower are with respect to the two cost parameters.

By analyzing the iso-elastic demand function we show that there is a restricted parameter region where the deterrence strategy is optimal for the leader. In contrast to the linear demand functions, the iso-elastic demand function contains elasticity parameter γ. A high γ gives the leader such a low monopoly profit that it can dominate the total leader's value. Only for a low elasticity parameter γ in combination with low β (note that it should still hold that $\beta > 2$), a positive leader value function can be found. High values of β, either results from a low interest rate or a high uncertainty level. In that situation the entry of the follower will have such a big negative impact on the value of the leader, that it makes the leader's deterrence value function negative. In other words, the deterrence strategy is too expensive for the leader for high values of β. Therefore, it will use the accommodation strategy. In this subsection we want to compare the deterrence strategy for the three types of demand functions. We assume that the parameters are chosen such that it is also optimal for a leader in a model with iso-elastic demand to choose the deterrence strategy.

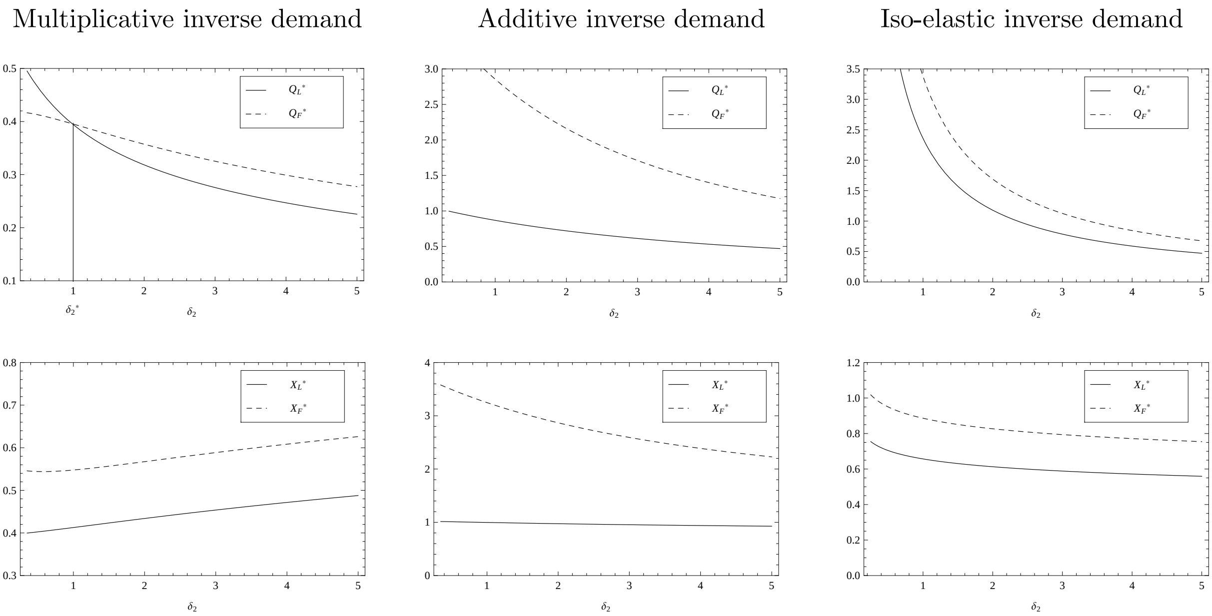

Figure 2: The optimal investment timing and capacities for the leader and the follower, for a change investment parameter δ_2. Take r =0.1, μ =0.02, σ =0.1, η =0.5, γ =0.1 and δ_1 =2.

Figure 2 shows that investment decisions in a model with multiplicative demand responds differently to a change in cost parameter δ_2 compared to the additive and iso-elastic demand model. Notice that $\delta_2 = 0$ represents the model from Section 3. For the model with multiplicative demand there is a direct effect and an indirect effect of δ_2 on the optimal capacity of the follower. When the quadratic part of the cost function becomes larger (δ_2 increases), this directly decreases the optimal capacity of the leader and follower. However, the lower level of optimal leader's capacity gives the follower an indirect strategic advantage, which would result in an upward shift of the optimal follower's capacity. As can be seen in Figure 2, the direct effect dominates the optimal capacity decision of the follower. However, due to the indirect strategic effect, the optimal capacity of the follower decreases less fast than the leader's optimal capacity in the cost parameter δ_2. Therefore, when the quadratic investment cost part dominate the total costs, the follower will have a larger capacity than the leader, i.e. for $\delta_2 \geq \delta_2^*$ such that $\delta_2^*(\delta_1) = \min\{\delta_2 | Q^{mult}_{L,det} = Q^{mult}_{F,det}, \delta_1\}$. We can conclude that the result from Proposition 4 in Section 3, where the leader invests more than the follower, is sensitive to the inclusion of quadratic costs. The just described pattern is not observed in the model with additive demand and the model with iso-elastic demand. Since demand in these models is not restricted by a fixed upper bound on market quantity, an increase in the optimal capacity of the leader has less impact on the optimal capacity of the follower. In these model the follower is able to wait until the market is large enough for a large investment. In the model with multiplicative demand, an increasing capacity investment of the leader will occupy a bigger part of the fixed market share.

The effect of δ_2 on the timing of investment, in a model with multiplicative demand, can in a similar fashion be explained with a direct and indirect effect. An increase in δ_2 directly delays investment of both firms. However, it also decreases the optimal capacity of the leader, which speeds up the investment for the follower. Again the direct effect dominates, but the follower will not delay its investment as much as the leader when cost parameter δ_2 increases. In the model with additive demand or iso-elastic demand, the direct effect dominates. The decreasing capacities of both firms cause that the firms want to invest sooner. In these models, timing is more sensitive for a change in the optimal capacities, because it means that the total market can be smaller. The scope of the total market size can in this model be controlled with timing parameter X.

In Proposition 5 in Section 3 we show that the additive demand model has a longer period of monopoly profits. Figure 4 supports that the inclusion of quadratic costs does not change this result. Figure 4 illustrates for a chosen set of parameters that $\frac{X^{det}_{L,add}}{X^{det}_{F,add}} < \frac{X^{det}_{L,mult}}{X^{det}_{F,mult}}$, implying that the the additive demand model leads to a longer period of monopoly profits. Figure 4 also shows that the relative monopoly position of the iso-elastic demand is not affected by the quadratic part of the investment costs.

Figure 3 shows that the investment behavior, when a multiplicative demand function is assumed, behaves differently in for a change in δ_1, than for the assumption of an additive or iso-elastic demand function. The multiplicative demand function illustrates that for a low value of cost parameter δ_1, the follower invests in a higher capacity than the leader. This is due to the presence of quadratic costs $\delta_2 = 1$, that dominate the total costs. We explained in Figure 2 that quadratic costs have as indirect strategic effect that it leads to a low leader's capacity, which in turn leads to a higher follower's capacity. However, when the linear cost parameter δ_1 increases, this directly *increases* the optimal capacity of the leader and follower. But the increase of the capacity of the leader indirectly *decreases* the follower's capacity. Therefore for high values of cost parameter δ_1, the leader has a higher level of capacity than the follower i.e. for $\delta_1 \geq \delta_1^*$ such that $\delta_1^*(\delta_2) = \min\{\delta_1 | Q^{mult}_{L,det} = Q^{mult}_{F,det}, \delta_2\}$. Notice that, in case of a multiplicative demand function, depending on the dominating part of the cost function, the corresponding *strategic effect* will determine which firm will have the larger capacity in the market.

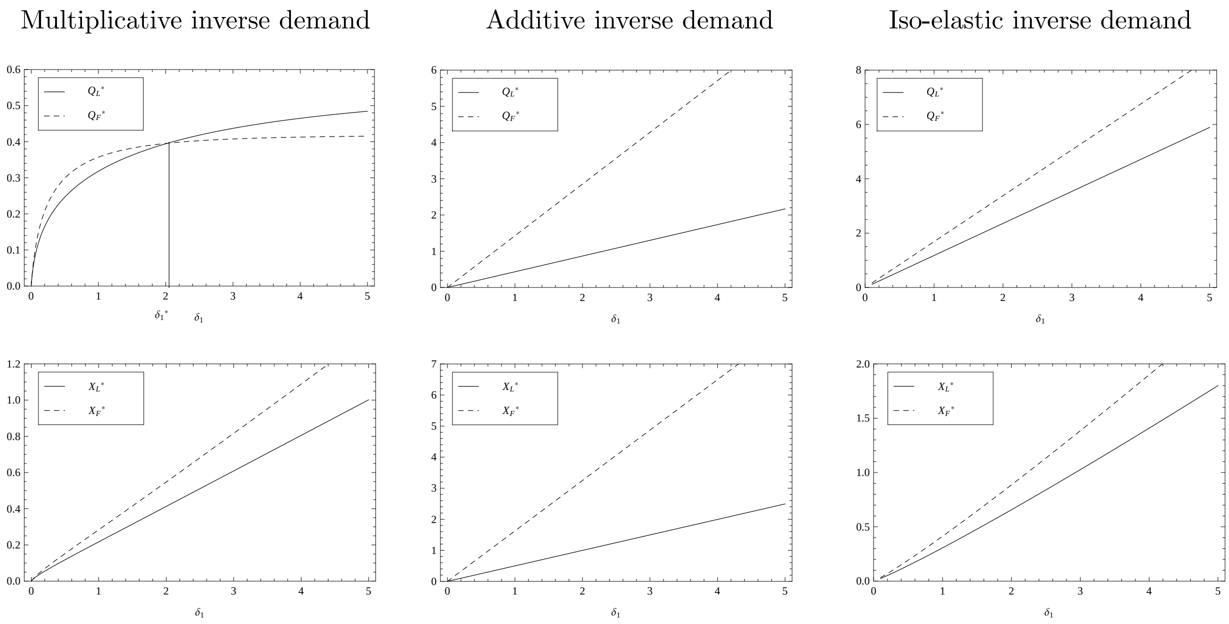

Figure 3: The optimal investment timing and capacities for the leader and the follower, for a change investment parameter δ_1. Take r =0.1, μ =0.02, σ =0.1, η =0.5, γ =0.1 and δ_2 =1.

Conjecture 1 *For the multiplicative demand function, the strategic effect of cost parameters δ_1 and δ_2 determines which firm will have the larger capacity in the market. When the linear costs dominate the total cost function ($\delta_1 > \delta_1^*(\delta_2)$), the corresponding high optimal leader's capacity, makes the follower invest in small capacity. The leader will be the larger firm in the market. When the quadratic costs dominate the total cost function ($\delta_2 > \delta_2^*(\delta_1)$), the corresponding low optimal leader's capacity, makes the follower invest in a large capacity. The follower will be the larger firm in the market. Where $\delta_1^*(\delta_2) = \min\{\delta_1 | Q_{L,det}^{mult} = Q_{F,det}^{mult}, \delta_2\}$, and $\delta_2^*(\delta_1) = \min\{\delta_2 | Q_{L,det}^{mult} = Q_{F,det}^{mult}, \delta_1\}$.*

5.2 Accommodation strategy

For the additive demand function, in the presence of convex investment costs, we are able to derive analytical solutions for the accommodation strategy. Derivations are similar to the case discussed in Section 3. Proposition 8 summarizes the investment decisions when the leader uses the entry accommodation policy.

Proposition 8 *Suppose the additive inverse demand function equals $P(t) = X(t) - \eta Q(t)$, and investment costs are equal to $\delta_1 Q + \delta_2 Q^2$. The leader will consider the entry accommodation strategy whenever the current level of X is larger than or equal to $\underline{X^{acc}}$, with*

$$\underline{X_{add}^{acc}} = \frac{1}{A(\beta, \eta, \delta)} \left(\beta \delta_2 (\eta^2 + 6r\eta\delta_2 + 4r^2\delta_2^2)(r - \mu) \right), \tag{29}$$

where $A(\beta, \eta, \delta_2) = (\beta - 4)\eta^2 + 2r(3\beta - 8)\eta\delta_2 + 4r^2(\beta - 2)\delta_2^2$. The optimal investment threshold and corre-

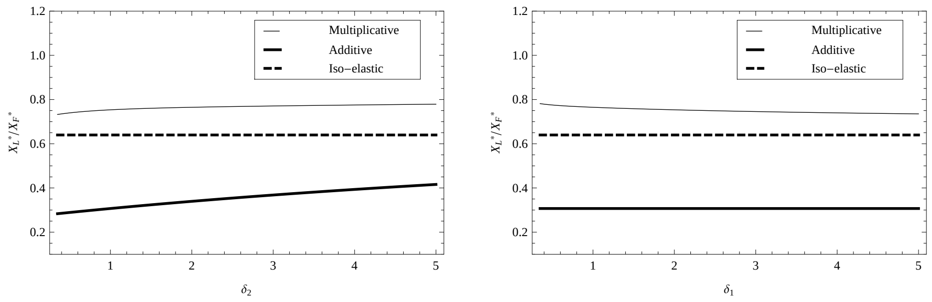

Figure 4: The ratios $\frac{X_{L,det}}{X_{F,det}}$, for a change investment parameter δ_1 and δ_2. Parameter values: r =0.1, μ =0.02, σ =0.18, η =0.5, η=0.2 and $\delta_1 = 2$ in the left graph, and δ_2 =1 in the right graph.

sponding capacity level of the leader for the entry accommodation strategy are given by

$$X^{add}_{L,acc} = \frac{\delta\beta(r-\mu)}{\beta-2}, \tag{30}$$

$$Q^{add}_{L,acc} = \frac{\delta r(\eta + 2r\delta_2)}{(\beta-2)(\eta^2 + 4r\eta\delta_2 + 2r^2\delta_2^2)}. \tag{31}$$

The assumptions $A(\beta,\eta,\delta_2) < 0$ and $\beta > 2$ are necessary to ensure that $X^{add}_{L,acc} > \underline{X^{acc}_{add}}$.

In case that $A(\beta,\eta,\delta_2) > 0$, it holds that $X^{add}_{L,acc} < \underline{X^{acc}_{add}}$. Investment will take place at the accommodation boundary $\underline{X^{acc}_{add}}$ with the corresponding optimal capacities for the leader and the follower:

$$Q^{add}_{L,acc}(\underline{X^{acc}_{add}}) = \frac{1}{A(\beta,\eta,\delta_2)}\left(2r\delta_1(\eta + 2r\delta_2)\right), \tag{32}$$

$$Q^{add}_{F,acc}(\underline{X^{acc}_{add}}) = \frac{1}{A(\beta,\eta,\delta_2)(\eta + r\delta_2)}\left(r\delta_1(\eta^2 + 6r\eta\delta_2 + 4r^2\delta_2^2)\right). \tag{33}$$

Considering convex investment costs, one cannot derive analytical expressions for the accommodation triggers X^{acc}_L and the lower bounds $\underline{X^{acc}}$, in case of multiplicative as well as iso-elastic demand.

Proposition 9 shows that for the additive and multiplicative demand the result of Proposition 7 is robust against the introduction of quadratic costs. For iso-elastic demand function, we can only show numerical results, therefore Figure 5 illustrates numerically that also for the iso-elastic demand function it holds that $Q^{acc}_{L,iso}(\underline{X^{acc}}) > Q^{acc}_{F,iso}(Q^{acc}_{L,iso}(\underline{X^{acc}}))$, for a chosen set of parameters.

Proposition 9 *Suppose additive inverse demand function $P(t) = X(t) - \eta Q(t)$, or multiplicative inverse demand function $P(t) = X(t)(1 - \eta Q(t))$, and costs $\delta_1 Q + \delta_2 Q^2$. Given that firm roles are exogenously determined, when the leader uses the accommodation strategy, it will invest in a larger capacity compared to the follower, i.e. $Q^{acc}_{L,add}(\underline{X^{acc}}) > Q^{acc}_{F,add}(Q^{acc}_{L,add}(\underline{X^{acc}}))$ for $A(\beta,\eta,\delta_2) > 0$, $Q^{acc}_{L,add}(X^{acc}_{L,add}) > Q^{acc}_{F,add}(Q^{acc}_{L,add}(X^{acc}_{L,add}))$ for $A(\beta,\eta,\delta_2) < 0$, and $Q^{acc}_{L,mult}(X) > Q^{acc}_{F,mult}(Q^{acc}_{L,mult}(X))\ \forall\ X$.*

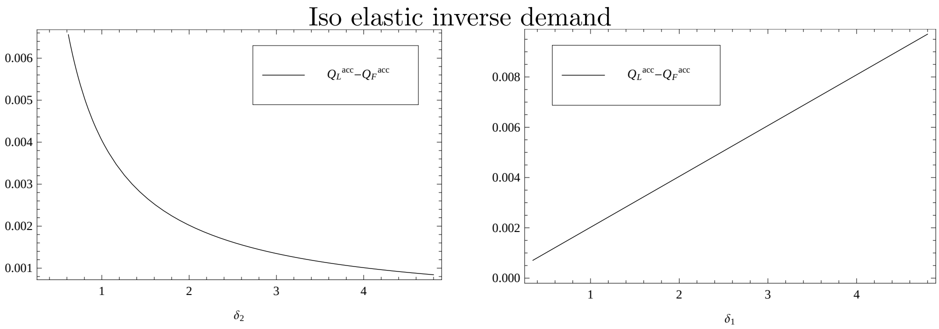

Figure 5: An illustrative example that $Q^{acc}_{L,iso} > Q^{acc}_{F,iso}$ for the iso elastic demand model. Take r =0.1, μ =0.02, σ =0.1, η =0.5, γ =0.1, $\delta_2 = 1$ and $\delta_1 = 2$.

6 Conclusion

This paper discusses three types of demand functions, additive, multiplicative, and iso-elastic demand, in a real options context. The optimal investment decisions of two firms in the corresponding markets are analyzed for these demand functions separately. The three demand models are not comparable in absolute terms, therefore we look at the relative differences in optimal capacity, timing and values between the two firms in the market. This relative information can be compared among the demand functions.

We assume that the leader can use two strategies. Applying the deterrence strategy, gives it a period of monopoly profits before the entry of the followers. However, when initial demand in the market is very high, the follower will invest immediately after the entry of the leader. This is the accommodation strategy.

In the first part of our paper we consider linear investment costs, and additive and multiplicative demand functions are compared. Due to the restrictive upper bound on the multiplicative demand function, there is a fixed market size that has to be shared among the two firms. On the other hand, in the additive demand function, a higher level of uncertainty parameter X creates a bigger market size. This difference in the structure of the demand function also explains the relative difference in capacity and timing of investment between the two firms. The use of a multiplicative demand function makes the leader the firm with the largest capacity in the market, because this is the firm that chooses its optimal capacity first. For the follower that invests later, only a small part of the market size is left, and it will invest in a smaller capacity than the leader. The use of the additive demand function makes the follower the firm with the largest capacity in the market. Now, the follower has the ability to delay investment until it can optimally invest in a large capacity. Consequently, the late investment of the follower gives the leader a long period of monopoly profits. To know what type of demand function a leader would prefer, we compare these two options, i.e a large period of monopoly profits, but facing a strong competitor in the future, or dominating the market with the largest capacity but enjoying a shorter period of monopoly profits. We show that the leader would rather be in a market with a large period of monopoly profits and endure the entrance of a strong competitor in the future (i.e. the use of the additive demand function).

In the second part of our paper we consider convex investment costs, which allows us to compare the iso-elastic demand function with the two linear demand functions. The total market size in the iso-elastic demand function is, just like the additive demand function, not restricted. Therefore we also find that the use of this demand model gives the follower the largest capacity in the market. Furthermore, we find that the introduction of convex costs violates the previous multiplicative result with linear investment costs. When the quadratic part of the total investment costs dominates the total investment costs, the follower will also in this model be the firm with the largest capacity. High convex costs directly cause that optimal capacity of the leader and follower will be small, however the small optimal capacity of the leader will indirectly increase the followers optimal capacity. Consequently the follower will have a larger capacity than the leader, when the quadratic part of investment costs dominate the total investment.

7 Appendix

7.1 Proof of propositions

Proof of Proposition 1

The follower's profit in a model with additive demand and linear costs is equal to

$$\pi_{F,t} = (X_t - \eta(Q_{F,t} + Q_L))Q_{F,t} - \delta_1 Q_{F,t}. \tag{34}$$

$V(X, Q_F)$ denotes the follower's discounted expected value that invests in capacity amount, Q_F, and is equal to:

$$V_F(X, Q_F, Q_L) = E\left(\int_t = 0^{\infty}(X_t - \eta(Q_{F,t} + Q_L))Q_{F,t}\,\mathrm{d} - \delta_1 Q_F t\right) = \frac{XQ_F}{r-\mu} - \frac{Q_F^2 + Q_L Q_F}{r} - \delta_1 Q_F. \tag{35}$$

Differentiating (35), with respect to Q_F results into the followers optimal capacity for a given X and Q_L:

$$Q_F^*(X, Q_L) = \frac{Xr - (r-\mu)(r\delta + Q_L\eta)}{2\eta(r-\mu)}. \tag{36}$$

Substitution of (36) into (35) gives the value function of the follower after investment:

$$V_F(X, Q_L) = \frac{(Xr - (r-\mu)(r\delta + Q_L\eta))^2}{4r\eta(r-\mu)^2}. \tag{37}$$

The value of the firm before investment, i.e. the value of waiting, takes the form $f(X) = AX^{\beta}$, where A is a constant to be determined and β the positive root of the quadratic polynomial

$$\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0. \tag{38}$$

Denote with $X_F^*(Q_F)$ the investment moment of the follower. We value match and smooth paste the follower's value of waiting to its value function after investment. This results in solving the following equations for $X_F * t(Q_F)$ and constant A:

$$AX_F^{*\beta} = V_F(X_F^*, Q_L),$$
$$\beta AX_F^{*\beta-1} = \frac{\partial V_F(X_F^*, Q_L)}{\partial X}.$$

We find that

$$A_F^*(Q_L) = \left(\frac{r(\beta-2)}{\beta(r\delta+Q_L\eta)(r-\mu)}\right)^{\beta}\left(\frac{(r\delta+Q_L\eta)^2}{(\beta-2)^2 r\eta}\right), \tag{39}$$

$$X_F^*(Q_L) = \frac{\beta(r\delta+Q_L\eta)(r-\mu)}{r(\beta-2)}, \tag{40}$$

so that the optimal followers capacity is equal to

$$Q_F^*(Q_L) = Q_F^*(X_F^*(Q_L), Q_L) = \frac{r\delta+Q_L\eta}{r(\beta-2)}. \tag{41}$$

□

Proof of Proposition 2

The value function after investment, of the leader that uses the deterrence strategy is equal to

$$V_L^{det}(X, Q_L) = \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta Q_L - (\frac{\eta Q_L Q_F^*(Q_L)}{r})(\frac{Xr(\beta-2)}{\beta(\delta r+Q_L\eta)(r-\mu)})^{\beta}. \tag{42}$$

Substitution of (41) into (42) gives

$$V_L^{det}(X) = \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta Q_L - (\frac{Q_L(\delta r+Q_L\eta)}{r(\beta-2)})(\frac{Xr(\beta-2)}{\beta(\delta r+Q_L\eta)(r-\mu)})^{\beta}. \tag{43}$$

In order to find the optimal leaders capacity Q_L^{det}, we need to take the first order derivative of this value with respect to Q_L, and set it equal to zero, which results in the following condition:

$$\frac{\partial V_L(X, Q_L)}{Q_L} = \frac{X}{r-\mu} - \frac{2Q_L\eta}{r} - \delta + (\frac{Q_L(\beta-2)\eta - r\delta}{r(\beta-2)})(\frac{Xr(\beta-2)}{\beta(\delta r+Q_L\eta)(r-\mu)})^{\beta} = 0. \tag{44}$$

The envelope theorem shows that

$$\frac{\partial V_L(X, Q_L(X))}{\partial X} = \frac{\partial V_L(X, Q_L)}{\partial X} + \frac{\partial V_L(X, Q_L)}{Q_L}\frac{\partial Q_L}{\partial X} = \frac{\partial V_L(X, Q_L)}{\partial X}, \tag{45}$$

because $\frac{\partial V_L(X,Q_L)}{Q_L}$ is zero, as is shown in condition (44). Therefore the value matching and smooth pasting conditions are given by

$$AX^{\beta} = \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta Q_L - (\frac{Q_L(\delta r+Q_L\eta)}{r(\beta-2)})(\frac{Xr(\beta-2)}{\beta(\delta r+Q_L\eta)(r-\mu)})^{\beta}, \tag{46}$$

$$\beta AX^{\beta-1} = \frac{Q_L}{r-\mu} - \frac{q_L}{r-\mu}(\frac{Xr(\beta-2)}{\beta(\delta r+Q_L\eta)(r-\mu)})^{\beta-1}, \tag{47}$$

and result in the following leader's investment threshold

$$X_L^{det}(Q_L) = \frac{\beta(r\delta_1 + Q_L\eta)(r-\mu)}{r(\beta-1)}. \tag{48}$$

Solving (48) simultaneously with (44), results in the optimal leader's investment trigger X_L^{det}, with corresponding optimal capacity Q_L^{det}:

$$X_L^{det} = \frac{\delta\beta(r-\mu)}{\beta-2}, \tag{49}$$

$$Q_L^{det} = \frac{\delta r}{(\beta-2)\eta}. \tag{50}$$

To find the boundaries on X, i.e. lower bound $\underline{X^{det}}$ and upper bound $\overline{X^{det}}$, we have to substitute $Q_L = 0$ and $Q_L = \underline{Q_L}$, respectively into condition (44), and solve for X. Substitution of $Q_L = 0$ into equation (44) results in:

$$\psi(X) = \frac{\underline{X^{det}}}{r-\mu} - \delta - \frac{\delta}{\beta-2}(\frac{\underline{X^{det}}(\beta-2)}{\delta\beta(r-\mu)})^{\beta} = 0. \tag{51}$$

Since

$$\begin{aligned}
\psi(0) = &\quad -\delta_1 < 0,\\
\psi(0) = &\quad \frac{\delta_1}{\beta-2} > 0,\\
\frac{\partial\psi(X)}{\partial X} = &\quad \frac{1}{r-\mu}(1-(\frac{(\beta-2)X}{\delta_1\beta(r-\mu)})^{\beta-1}) > 0 \qquad \text{for } X \in (0, X_F^*(0)),
\end{aligned}$$

it holds that $\underline{X^{det}}$ exists. Furthermore $\underline{X^{det}} < \underline{X^{acc}}$, because

$$\begin{aligned}
\frac{\partial\psi(X)}{\partial X} = &\quad \frac{1}{r-\mu}(1-(\frac{(\beta-2)X}{\delta_1\beta(r-\mu)})^{\beta-1}) = 0 \qquad \text{for } X = X_F^*(0),\\
\frac{\partial\psi(X)}{\partial X} = &\quad \frac{1}{r-\mu}(1-(\frac{(\beta-2)X}{\delta_1\beta(r-\mu)})^{\beta-1}) < 0 \qquad \text{for } X \in (X_F^*(0), \infty),\\
\frac{\partial\psi(\underline{X^{acc}})}{\partial X} = &\quad \frac{1}{r-\mu}(1-(\frac{\beta-2}{\beta-4})^{\beta-1}) < 0,
\end{aligned}$$

which indicates that $\underline{X^{acc}} > X_F^*(0) > \underline{X^{det}}$ for $\beta > 4$. When $\beta < 4$, $\underline{X^{acc}}$ negative and therefore dispensable.

Next we will show that $\overline{X^{det}}$ does not exist. Solving condition (44) for Q_L can lead to either a minimum or a maximum value. Substitution of $\underline{Q_L}$ into (44) leads to a unique value $\bar{X} = \frac{\delta\beta(r-\mu)}{2(\beta-2)}$ that makes $\underline{Q_L}$ the capacity choice that corresponds to a *minimum* leader's value. Solving condition (44) numerically for the $Q_L(X)$ that leads to a maximum leader's value, gives as result that the optimal capacity will always be bigger than the minimum boundary for Q_L, i.e. $Q_L^*(X) > \underline{Q_L}\ \ \forall X$. Therefore, there is no upper bound for the deterrence strategy, considering the additive demand function. □

Proof of Proposition 3

The value function after investment, of the leader that uses the accommodation strategy is equal to

$$V_L^{acc}(X, Q_L) = \frac{XQ_L}{r-\mu} - \frac{\eta((Q_L)^2 + Q_LQ_F)}{r} - \delta Q_L. \tag{52}$$

Substitution of (36) into (52) and maximizing with respect to Q_L gives the optimal capacity size of the leader, as a function of X

$$Q_L^{acc}(X) = \frac{r(X - (r-\mu)\delta_1)}{2\eta(r-\mu)}. \tag{53}$$

Substitution of (53) and (36) into (52) leads to

$$V_L(X) = \frac{r(X - (r-\mu)\delta_1)^2}{8\eta(r-\mu)}. \tag{54}$$

Solving for the value matching and smooth pasting conditions leads to the following optimal investment trigger and the corresponding leader's capacity level:

$$X_L^{acc} = \frac{\delta\beta(r-\mu)}{\beta-2}, \tag{55}$$

$$Q_L^{acc} = \frac{\delta r}{(\beta-2)\eta}. \tag{56}$$

The accommodation strategy can only occur for level of $X > \underline{X^{acc}}$. The leader can only consider the accommodation strategy if the optimal leader's capacity Q_L^{acc} leads to immediate investment of the follower, i.e. for $\underline{X^{acc}}$ it should hold that

$$X_F^*(Q_L^{acc}(\underline{X^{acc}})) \leq \underline{X^{acc}}. \tag{57}$$

Substitution of (56), (40) into (57) gives

$$\underline{X^{acc}} = \frac{\delta\beta(r-\mu)}{\beta-4}. \tag{58}$$

□

Proof of Proposition 4

Follows directly from Proposition 4. □

Proof of Proposition 5

For the additive demand function, the monopoly period is given by

$$\frac{X_{L,add}^{det}}{X_{F,det}^{add}} = \frac{\frac{\delta\beta(r-\mu)}{\beta-2}}{\frac{\delta(\beta-1)\beta(r-\mu)}{(\beta-2)^2}} = \frac{\beta-2}{\beta-1} = \frac{(\beta-2)(\beta+1)}{(\beta-1)(\beta+1)} = \frac{\beta^2-\beta-2}{(\beta-1)(\beta+1)}. \tag{59}$$

For the multiplicative demand function, the monopoly period is given by

$$\frac{X_{L,det}^{mult}}{X_{F,mult}^{det}} = \frac{\frac{(\beta+1)\delta(r-\mu)}{\beta-1}}{\frac{(\beta+1)^2\delta(r-\mu)}{\beta(\beta-1)}} = \frac{\beta}{\beta+1} = \frac{\beta^2-\beta}{(\beta-1)(\beta+1)}. \tag{60}$$

□

Proof of Proposition 6

The value of the leader at its deterrence trigger, in a model with additive demand, is equal to

$$V_{L,add}(X_{L,add}^{det}) = \frac{r\delta^2((\beta-2) - (\beta-2)^\beta(\beta-1)^{1-\beta})}{(\beta-2)^3\eta}. \tag{61}$$

The value of a monopolist at its investment trigger, in a model with additive demand, is equal to

$$V_{M,add}(X_M) = \frac{r\delta^2}{(\beta-2)^2\eta}. \tag{62}$$

Therefore, we find that

$$\frac{V_{L,add}(X^{det}_{L,add})}{V_{M,add}(X_M)} = 1 - (\frac{\beta-2}{\beta-1})^{\beta-1}. \tag{63}$$

Similar, for a model with multiplicative demand we find

$$V_{L,mult}(X^{det}_{L,mult}) = \frac{(1-(\frac{\beta}{1+\beta})^{\beta})\delta}{(\beta^2-1)\eta}, \tag{64}$$

and

$$V_{M,mult}(X_M) = \frac{\delta}{(\beta^2-1)\eta}. \tag{65}$$

Therefore

$$\frac{V_{L,mult}(X^{det}_{L,mult})}{V^{mult}_M(X_M)} = 1 - (\frac{\beta}{\beta+1})^{\beta}. \tag{66}$$

We know that $\infty > \beta > 2$. For $\beta = 2$ it holds that

$$\frac{V_{L,add}(X^{det}_{L,add})}{V_{M,add}(X_M)} = 0 < 2 = \frac{V_{L,mult}(X^{det}_{L,mult})}{V_{M,mult}(X_M)}.$$

Furthermore, the theory of convergence shows that when $\beta \to \infty$, it holds that $\frac{\beta-2}{\beta-1} \to 1$. This leads to:

$$\lim_{\beta\to+\infty} \frac{V_{L,add}(X^{det}_{L,add})}{V_{M,add}(X_M)} = \lim_{\beta\to+\infty} 1 - (\frac{\beta-2}{\beta-1})^{\beta-1} = 1 - (\frac{\beta-2}{\beta-1})^{\beta}.$$

Consequently,

$$\lim_{\beta\to+\infty} \frac{V_{L,add}(X^{det}_{L,add})}{V_{M,add}(X_M)} = 1 - (\frac{\beta-2}{\beta-1})^{\beta} > 1 - (\frac{\beta}{\beta+1})^{\beta} = \lim_{\beta\to+\infty} \frac{V_{L,mult}(X^{det}_{L,mult})}{V_{M,mult}(X_M)},$$

since $(\frac{\beta-2}{\beta-1})^{\beta} = (\frac{\beta^2-\beta-2}{(\beta-1)(\beta+1)})^{\beta} < (\frac{\beta^2-\beta}{(\beta-1)(\beta+1)})^{\beta} = (\frac{\beta}{\beta+1})^{\beta}$. □

Proof of Proposition 7

Follows directly from Proposition 7. □

Proof of Proposition 8

This concept is analogous to the proof of Proposition 3. □

Proof of Proposition 9

For the multiplicative demand function, I find that the optimal capacities as a function of X are given by:

$$Q^{acc}_{L,mult}(X) = \frac{X\eta_2\delta_2(r-\mu))(X-\delta_1(r-\mu))}{2X^2\eta^2 + 8X\eta\delta_2(r-\mu) + 4\delta_2^2(r-\mu)^2}, \tag{67}$$

$$Q^{acc}_{F,mult}(X) = \frac{X - \delta_2(r-\mu) - \frac{X\eta_2\delta_2(r-\mu))(X-\delta_1(r-\mu))}{2X^2\eta^2+8X\eta\delta_2(r-\mu)+4\delta_2^2(r-\mu)^2}}{2(X\eta + \delta_2(r-\mu))}. \tag{68}$$

It holds that $Q^{acc}_{L,mult}(X) = Q^{acc}_{F,mult}(X)$, at $X = 0$ and $X = \delta_1(r-\mu)$. Furthermore $Q^{acc}_{L,mult} = Q^{acc}_{F,mult} = 0$ at $X = \delta_1(r-\mu)$, and we know that $\frac{\partial Q^{acc}_{L,mult}}{\partial X} > \frac{\partial Q^{acc}_{F,mult}}{\partial X} > 0$. For an $\epsilon > 0$, it holds that $Q^{acc}_{L,mult}(\delta_1(r-\mu)+\epsilon) >$

$Q^{acc}_{F,mult}(\delta_1(r-\mu)+\epsilon)$. Therefore $Q^{acc}_{L,mult}(X) > Q^{acc}_{F,mult}(X)$ for $X > \delta_1(r-\mu)$.

For the additive demand function we have to consider two cases. First consider that $A(\beta,\eta,\delta_2) > 0$, here it holds that

$$Q^{acc}_{L,add}(\underline{X^{acc}}) = (2(\eta+2r\delta_2))\left(\frac{r\delta_1}{A(\beta,\eta,\delta_2)}\right), \tag{69}$$

and

$$Q^{acc}_{F,add}(Q^{acc}_{L,add}(\underline{X^{acc}})) = \left(\frac{(\eta^2+6r\eta\delta_2+4r^2\delta_2^2)}{(\eta+r\delta_2)}\right)\left(\frac{r\delta_1}{A(\beta,\eta,\delta_2)}\right). \tag{70}$$

Since $2(\eta+2r\delta_2) = \frac{2(\eta+2r\delta_2)(\eta+r\delta_2)}{\eta+r\delta_2} = \frac{2\eta^2+8r\eta\delta_2+4r^2\delta_2^2}{\eta+r\delta_2} > \frac{\eta^2+6r\eta\delta_2+4r^2\delta_2^2}{\eta+r\delta_2}$, it holds that $Q^{acc}_{L,add}(\underline{X^{acc}}) > Q^{acc}_{F,add}(\underline{X^{acc}})$.

Next, we consider the case where $A(\beta,\eta,\delta_2) < 0$, where it holds that

$$Q^{acc}_{L,add} = \left(\frac{r\delta_1}{\beta-2}\right)\left(\frac{\eta+2r\delta_2}{\eta^2+4r\eta\delta_2+2r^2\lambda^2}\right), \tag{71}$$

and

$$Q^{acc}_{F,add} = \left(\frac{r\delta_1}{\beta-2}\right)\left(\frac{r\delta_2}{\eta^2+4r\eta\delta_2+2r^2\lambda^2}+\frac{0.5}{\eta+r\delta_2}\right). \tag{72}$$

For $Q^{acc}_{L,add} > Q^{acc}_{F,add}$ it should hold that

$$\frac{\eta+r\delta_2}{\eta^2+4r\eta\delta_2+2r^2\lambda^2} > \frac{0.5}{\eta+r\delta_2} \tag{73}$$

$$\frac{(\eta+r\delta_2)^2}{\eta^2+4r\eta\delta_2+2r^2\lambda^2} > \frac{1}{2} \tag{74}$$

$$\frac{2\eta^2+4r\eta\delta_2+2r^2\lambda^2}{2(\eta^2+4r\eta\delta_2+2r^2\lambda^2)} > \frac{\eta^2+4r\eta\delta_2+2r^2\lambda^2}{2(\eta^2+4r\eta\delta_2+2r^2\lambda^2)}. \tag{75}$$

This is the case, therefore $Q^{acc}_{L,add} > Q^{acc}_{F,add}$ for $(\beta-2)(\eta^2+4r\eta\delta_2+2r^2\delta_2^2) < 0$. □

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