Wind Generators and Market Power: Does it matter who owns them?

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Abstract

Electricity production from wind generators holds significant importance in European Union's 20% renewable energy target by 2020. In this paper, I show that ownership of wind generators affects market outcomes by using both a Cournot oligopoly model and a real options model. In the Cournot oligopoly model, ownership of the wind generators by owners of fossil-fueled (peakload) generators decreases total peakload production and increases the market price. These effects increase with total wind generation and aggregate wind generator ownership. In the real options model, start up and shut down price thresholds are significantly higher when the monopolist at the peakload level owns both types of generators. Furthermore, when producing electricity with the peakload generator, the monopolist can avoid facing prices below marginal cost by owning a certain share of the wind generators.

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1 Introduction and Literature Review

Electricity production from wind generators holds significant importance in EU's 20% renewable energy target by 2020. On the upside, wind power is considered to be a low-cost, environmental-friendly and non-strategic way to produce electricity. On the downside, wind power production is stochastic and it may require high levels of operating reserves to maintain a certain level of security of supply.

In this paper, I show that ownership of wind generators affects the market outcomes using both a Cournot oligopoly model and a real options model. In the Cournot oligopoly model, ownership of the wind generators by owners of fossil-fueled (peakload) generators decreases total peakload production and increases the market price. These effects increase with total wind generation and aggregate wind generator ownership. In the real options model, start up and shut down price thresholds are significantly higher when the monopolist at the peakload level owns both types of generators. Furthermore, when producing electricity with the peakload generator, the monopolist can avoid facing prices below marginal cost by owning a certain share of the wind generators.

In the relevant literature (Senfuss *et al.* (2008), Green& Vasilakos (2009), Twomey& Neuhoff (2010) etc.), wind production is simply regarded as a negative shock to demand and it lowers the need for electricity from conventional fossil-fueled generators. However, this perspective greatly ignores the effects of ownership of wind generators. Those effects are important because ownership of the wind generators creates additional rents for the firms that exercise market power with their conventional generators. Therefore we may expect firms, that have wind generators in their generation portfolio, to produce less electricity with their conventional generators than those who do not own any wind generators at all. As a result, we may further expect different market outcomes for different ownership structures given the same level of wind production.

A number of papers focus on the short and long term impacts of high levels of wind/renewable power penetration in liberalized electricity markets. Senfuss *et al.* (2008) analyzes the impact of renewable electricity generation on the electricity market in Germany. Lamont (2008) investigates the system-wide effects of large-scale intermittent technologies in an electric generation system. Green and Vasilakos (2009) evaluates the impact of intermittent wind generation on hourly equilibrium prices and output. Bushnell (2010), models the impact of large amount of wind generation on the generation mix. Twomey and Neuhoff (2010) investigates how the relationship between wind production and market price is affected by market power.

This paper can be regarded as an extension of Twomey and Neuhoff (2010). For the

Cournot model, I keep their basic model structure and improve upon their paper by introducing different market competition scenarios based on the ownership of wind generators. They focus on the competition at conventional generator level and disregard the possibility of the ownership of wind generators by the existing conventional generator owners. In a Cournot setting, I calculate (expected) market prices to see the effects of different ownership scenarios of the wind generators.

The rest of the paper is organized as follows: In section 2, I set up the Cournot oligopoly model and provide discussion on the effects of wind generator ownership on welfare as well as start up and shut down decisions. In section 3, I provide a real options model for a monopolist at the peakload level and compare the start up and shut down thresholds depending on different wind generator ownership levels. In section 4, I give numerical results to the real options model. In section 5, I provide a brief conclusion on my findings.

2 The Cournot Oligopoly Model

In the short term, there are two ways to exercise market power for (peakload) firms in the electricity markets. First, firms can decrease the level of output by withholding capacity (Joskow & Kahn, 2002). Second, they expectedly operate their generators for a significantly shorter period of time by asking higher start up and shut down prices than the corresponding socially optimal case (Misir, 2012). Furthermore, Green and Vasilakos (2009) notes that a strategic generator that owns wind farms would wish to take their output into account when calculating the supply function from its thermal plants. In this section, following Green and Vasilakos (2009), I focus on the first case and set up a Cournot oligopoly model that incorporates different wind generator ownership scenarios. I aim to show how dispatch decisions of the peakload firms are affected by wind generator ownership.

The industry consists of two types of electricity generation technologies: *wind* (W) and conventional *peakload* (P) generation. I do not put restrictions on the number of generators available for each technology as I assume that peakload generators could be instantly started up and shut down without any costs. I further assume that wind generation is subject to exogenous shocks and zero marginal cost of production whereas peakload generation has constant marginal cost of production $c > 0$.

At time t, the industry output is determined by the sum of wind and peakload production: $Q(t) = Q_W(t) + Q_P(t)$. The instantaneous stochastic wind production is given by:

$$Q_W(t) = Q_{W,0} + \epsilon_t \geq 0 \quad (1)$$

where $Q_{W,0}$ is the average wind production level and ϵ_t is the exogenous shock to the wind production with $E[\epsilon_t] = 0$ and $Var[\epsilon_t] = \sigma^2$.

There are no side payments and regardless of the technology, all production is paid at the market clearing price. The market price fluctuates stochastically according to linear inverse demand function, $D : \mathbb{R}_+ \rightarrow \mathbb{R}$:

$$P(t) = D[Q_P(t) + Q_W(t)] = X - \gamma Q(t) \quad \text{with } \gamma > 0. \tag{2}$$

where $X > c$ is the constant demand intercept.

Below, I describe two different market scenarios. First, I investigate the case where there is a social planner (or perfect competition) in the industry as benchmark. Second, I investigate the case where there is oligopolistic Cournot competition at the peakload level. For the oligopolistic competition, I give the results of wind generator ownership on total production and market price. I further provide results on welfare implications as well as start up and shut down decisions of peakload generators.

2.1 Social Planning and Competitive Equilibrium

I set the benchmark by calculating the optimal production and market price in the case of the social planner. The social planner's objective is to maximize the total social surplus by deciding how much to produce by conventional peakload generators given the level of wind production. In order to maximize the total social surplus ($S[Q^{SP}(t)]$), the social planner will have to calculate the area under the demand curve for a given production level minus the total cost of production ($C[Q^{SP}(t)]$). As $Q^{SP}(t) = Q_P(t) + Q_W(t)$, we have:

$$\begin{aligned} S[Q^{SP^*}(t)] &= sup_{Q_P} \left\{ \left[\int_0^{Q^{SP}(t)} (X - \gamma q) dq \right] - C[Q^{SP}(t)] \right\} \\ &= sup_{Q_P} \left\{ X(Q_P + Q_W(t)) - \frac{\gamma (Q_P + Q_W(t))^2}{2} - cQ_P \right\} \end{aligned} \tag{3}$$

The first order condition entails:

$$Q_P(t) = \frac{X - c}{\gamma} - Q_W(t) \Longrightarrow Q^{SP}(t) = \frac{X - c}{\gamma} \Longrightarrow P^{SP}(t) = c. \tag{4}$$

It follows from **Equation 4** that the social planner fully internalizes the effects of wind generation and as a result, the total socially optimal level of production does not depend on total wind production. But if total wind production is higher than the total socially optimal level of production (i.e., $Q_W(t) > (X - c)/\gamma$), there will be no production coming from the peakload generators and the market price will be even lower than the marginal cost of peakload generation. On the other hand, for the case of perfect competition at the peakload level, one should start with the very last step in **Equation 4**. Since, perfectly competitive firms are price takers and equate price to marginal cost to find out how much to produce, the market outcome for the cases of social planner and perfect competition will be the same.

Proposition 1 *Ownership structures of the wind generators do not have an impact on the aggregate peakload production and market price for the case of perfect competition at the peakload level.*

Proof. Perfectly competitive peakload firms observe the total wind production and decide how much to produce at the peakload level in order to reach the equilibrium production and price levels given in **Equation 4**. In other words, perfectly competitive peakload firms produce enough to set the market price to marginal cost. Therefore, we end up with the same market outcome regardless of the wind generator ownership. ■

2.2 Oligopolistic Cournot Competition

In this section, I investigate the market outcomes of the existence of oligopoly at the peakload level. I derive a general formula for the equilibrium peakload generation levels depending on the ownership of wind generators. I aim to show how wind generator ownership affects the individual firms' production levels and the industry outcomes. The results below show that all the peakload firms benefit from the aggregate wind generator ownership regardless of their individual wind generator ownership status.

There are n symmetric firms at the peakload level with the same constant marginal cost of peakload production, $c > 0$. I assume that $k \leq n$ firms equally own a share of wind generators and the rest of the firms at the peakload level do not own any wind generators at all. By assuming uniform production throughout wind generators, I consider that ownership of an equal share of wind generators results in an equal share of wind power production for each firm. Without loss of generality, for any firm $j \in J = \{k+1, k+2, ..., n\}$ who does not own any wind generators, the total production is just the individual peakload production, $Q_{P,j}(t)$. Consequently, for any firm $i \in I = \{1, 2, ..., k\}$ who owns an equal share of wind generators, the total individual production is the sum of peakload production and the share of total wind production. Namely;

$$Q_i(t) = Q_{P,i}(t) + \mathbf{A}Q_W(t) \tag{5}$$

$\mathbf{A} \in \mathbb{Q}$ is the share of total wind generators owned by each of the k peakload firms and it is formally defined by:

$$\mathbf{A} := \frac{\alpha}{k} \tag{6}$$

where $\alpha \in [0, 1]$ is the share of wind generators aggregately owned by the peakload firms. Specifically, $\alpha = 0$ when the peakload firms do not own any of the wind generators and $\alpha = 1$ when the peakload firms do own all of the wind generators.

Given this model, there are two different types of profit functions depending on a firm's wind generator ownership status. For any firm $i \in I$, with wind generator ownership, the profit function is:

$$\Pi_i(t) = [X - \gamma(Q_{P,i} + Q_{P,i'} + Q_{P,n-k} + Q_W(t))](Q_{P,i} + \mathbf{A}Q_W(t)) - cQ_{P,i} \tag{7}$$

where $Q_{P,i'}$ is the aggregate peakload production of the firms with wind generator ownership except for firm i and $Q_{P,n-k}$ is the aggregate peakload production of the firms without wind generator ownership.

Similarly for any firm $j \in J$, without wind generator ownership, the profit function is:

$$\Pi_j(t) = [X - \gamma(Q_{P,j} + Q_{P,j'} + Q_{P,k} + Q_W(t))]Q_{P,j} - cQ_{P,j} \tag{8}$$

where $Q_{P,j'}$ is the aggregate peakload production of the firms without wind generator ownership except for firm j and $Q_{P,k}$ is the aggregate peakload production of the firms with wind generator ownership.

Given the above profit functions, first order conditions entail (see **Appendix A.1**) the following equilibrium peakload production levels for firms i and j:

$$Q^*_{P,i}(t) = \frac{X - c}{\gamma(n+1)} - \frac{1-\alpha}{n+1}Q_W(t) - \mathbf{A}Q_W(t) \tag{9}$$

$$Q^*_{P,j}(t) = \frac{X - c}{\gamma(n+1)} - \frac{1-\alpha}{n+1}Q_W(t) \tag{10}$$

Looking at **Equation 9**, we see three different terms. The first term is the (symmetric) equilibrium level of individual production if the industry did not have any wind generators at all. The second term is the negative effect of unowned/un-internalized wind generation on the peakload production. The effect is as if there is a non-strategic firm that just produces $(1 - \alpha)Q_W(t)$ amount of electricity. The third term is the level of internalized wind generation for the peakload firms with wind generator ownership. The internalization results in a decrease in the level of individual peakload production exactly by the amount of the individual share of wind generation. Therefore, total (wind plus peakload) production for each peakload firm is the same but wind generator ownership provides a competitive advantage as total cost of production goes down due to zero-cost wind generation.

Definition 1 *For any firm $i \in I$ and $j \in J$, the **peakload production effect** of aggregate wind generator ownership is defined by:*

$$\Delta Q_{P,N} := Q_{P,N}[\alpha > 0] - Q_{P,N}[\alpha = 0], \;\; \textit{for} \;\; N = i, j. \tag{11}$$

Proposition 2 *For any peakload firm with wind generator ownership, aggregate wind generator ownership results in a decrease in the individual peakload production level. The **peakload production effect** increases with the level of wind production and aggregate wind generator ownership whereas it decreases with the total number of peakload firms with wind generator ownership.*

*For any peakload firm with no wind generator ownership, aggregate wind generator ownership results in an increase in the individual peakload production level. The **peakload production effect** increases with the level of wind production and aggregate wind generator ownership whereas it does not change with the total number of peakload firms with wind generator ownership.*

i.e., for any firm $i \in I$, we have $\Delta Q_{P,i} < 0$, $\partial \Delta Q_{P,i}/\partial \alpha < 0$, $\partial \Delta Q_{P,i}/\partial \epsilon_t < 0$ and $\partial \Delta Q_{P,i}/\partial k > 0$. Whereas for any firm $j \in J$, we have $\Delta Q_{P,j} > 0$, $\partial \Delta Q_{P,j}/\partial \alpha > 0$, $\partial \Delta Q_{P,j}/\partial \epsilon_t > 0$ and $\partial \Delta Q_{P,j}/\partial k = 0$.

Proof. From **Equations 9 & 10**, we have:

$$\Delta Q_{P,i} = \left(\frac{\alpha}{n+1} - \mathbf{A} \right) (Q_{W,0} + \epsilon_t) \tag{12}$$

and

$$\Delta Q_{P,j} = \frac{\alpha}{n+1} (Q_{W,0} + \epsilon_t) \tag{13}$$

It follows from the above equations that the properties given in **Proposition 2** are satisfied. ■

Equations 9&10 show that wind generation reduces the indivudual peakload production levels for all peakload firms. However, for positive values of α, all peakload firms increase their total production levels. Therefore, even if a peakload firm does not own any wind generators, it still benefits from the aggregate wind generator ownership. On the other hand, if a peakload firm owns a share of the wind generators, it decreases its peakload production exactly by the amount of its share of wind generation. As a result, the overall effect of aggregate wind generator ownership on the peakload production is negative for the firms with wind generator ownership since they provide a fraction of their production from zero-cost wind generation.

Proposition 3 *A redistribution of the wind generators amongst the firms at the peakload level, have no impact on the market outcomes. In other words, when the number of firms that owns wind generators (k) changes, the total peakload production and the market price stays the same.*

Proof. In order to prove the proposition, it is sufficient to show that the total peakload production does not depend on the parameter k.

So, taking the previously derived expressions for $Q_{P,i}$ and $Q_{P,j}$ into account, the total peakload production is:

$$Q_P(t) = kQ_{P,i}(t) + (n-k)Q_{P,j}(t) = \frac{n}{n+1}\frac{X-c}{\gamma} - \frac{n+\alpha}{n+1}Q_W(t) \tag{14}$$

which does not depend on k. ■

Following **Equation 14**, total production and market price are given by:

$$Q(t) = \frac{n}{n+1}\frac{X-c}{\gamma} + \frac{1-\alpha}{n+1}Q_W(t) \quad \text{and} \quad P(t) = \frac{X+nc}{n+1} - \frac{\gamma(1-\alpha)}{n+1}Q_W(t) \tag{15}$$

Remark 2 *It follows from **Equations 14&15** that when $\alpha = 1$, total peakload production decreases exactly by the amount of the total wind production and as a result, total industry production and market price do not depend on the level of wind production. This is simply because, when all of the wind generators are owned by the peakload firms, the effects of wind generation are entirely internalized by the peakload firms. Furthermore, as $0 < \alpha < 1$, $\partial P/\partial\alpha > 0$. In other words, if the aggregate ownership of the wind generators by the peakload firms increases, so does the market price. This result shows that market power of peakload firms increases with wind generator ownership.*

Following **Equation 15**, **Figure 1** shows the effect of aggregate wind generator ownership on the demand curve. When $\alpha = 0$, none of the wind generators are owned by the peakload firms and the demand curve shifts to the left exactly by the amount of the current level of wind generation. When $0 < \alpha < 1$, the residual demand curve shifts closer back to the original demand function and when $\alpha = 1$, the effect of wind generation vanishes and the residual demand and original demand curves coincide.

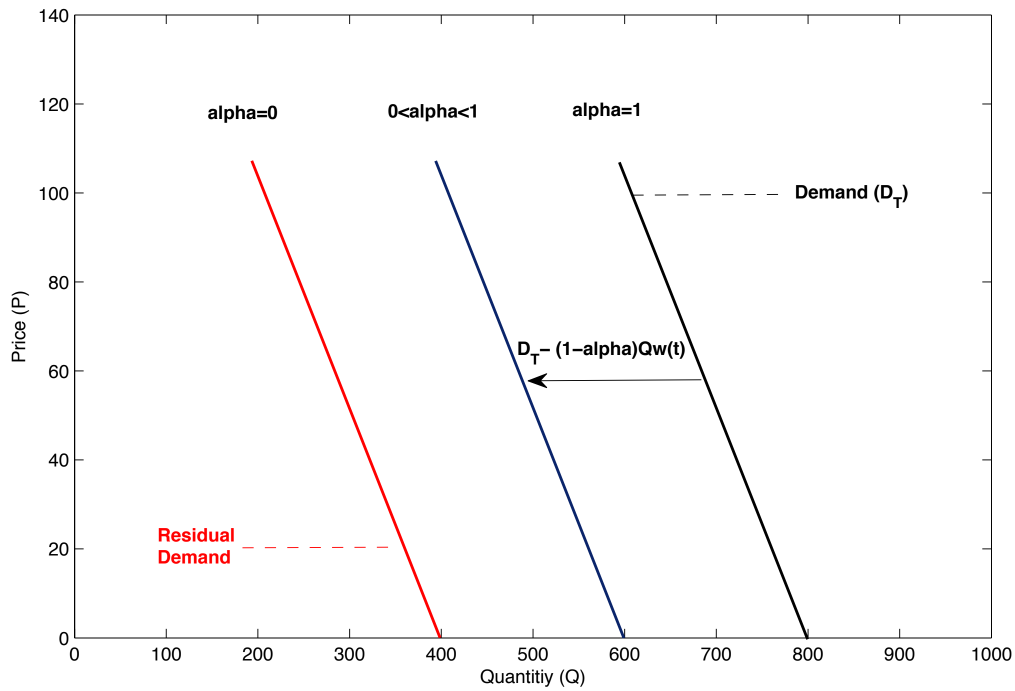

Figure 1: Effect of aggregate wind generator ownership on the demand for $Q_W(t) = 400$.

Definition 3 *The **price effect** of aggregate wind generator ownership is defined by:*

$$\Delta P := P[\alpha > 0] - P[\alpha = 0].$$

Proposition 4 *The **price effect** is always positive and it increases with the level of wind production and aggregate wind generator ownership whereas it decreases with the total number of peakload firms.*

i.e., $\Delta P > 0$, $\partial \Delta P / \partial \alpha > 0$, $\partial \Delta P / \partial \epsilon_t > 0$ *and* $\partial \Delta P / \partial n < 0$*. Furthermore,* $\Delta P \to 0$ *as* $n \to \infty$*.*

Proof. It follows from **Equation 15** that:

$$\Delta P = \frac{\gamma \alpha}{n+1}(Q_{W,0} + \epsilon_t).$$

As a result we obtain, $\Delta P > 0$, $\partial \Delta P / \partial \alpha > 0$, $\partial \Delta P / \partial \epsilon_t > 0$, $\partial \Delta P / \partial n < 0$ and $\Delta P \to 0$ as $n \to \infty$. ■

Wind generator ownership provides a higher level of market power to the peakload firms and the difference in market price increases with the aggregate wind power ownership (α).

Also as the number of peakload firms approaches infinity, the increase in market power resulting from owning the wind generators becomes infinitesimal. Hence, the price effect of aggregate wind generator ownership vanishes. On the other hand, the counter-intuitive result in **Proposition 4** is that the price effect gets bigger with the increased wind power production. I call this result counter-intuitive because market price decreases as the wind production increases. But the decrease in the market price under wind generator ownership is lower due to internalization of wind production. Hence, we get the increase in the price effect.

2.3 Welfare Implications

As it was mentioned before, when wind generation is high enough, there will not be any need for peakload generation. Therefore, peakload firms will not have any strategic actions to take when facing sufficiently large wind generation levels. As a result, aggregate wind generator ownership does not have an impact on consumer and producer surplus when peakload generators are idle. In this section, I calculate the following derivations by assuming that wind generation is so low that all the firms in the industry find it profitable to produce electricity with their peakload generators.

In this section I provide the effects of wind generator ownership on producer surplus, consumer surplus and total social surplus for the models given in the previous section. I specifically focus on the relationship between aggregate wind generator ownership and the volatility of wind generation. The derivations below show that aggregate wind generator ownership does not have an impact for the case of social planner but it reduces the effects of volatility for the oligopoly case.

2.3.1 Social Planner

As stated in **Proposition 1**, ownership structures of the wind generators do not have an effect on the socially optimal level of production. Then, by using the conventional definitions (see **Appendix A.2**), expressions for consumer surplus as well as producer surplus and total social surplus for the case of social planner (or perfect competition) are:

$$CS^{SP} = \frac{(X-c)^2}{2\gamma} \quad , PS^{SP} = c(Q_{W,0} + \epsilon_t) \quad , S^{SP} = \frac{(X-c)^2}{2\gamma} + c(Q_{W,0} + \epsilon_t) \tag{16}$$

Therefore, expected values are given by

$$E[CS^{SP}] = \frac{(X-c)^2}{2\gamma} \quad , E[PS^{SP}] = cQ_{W,0} \quad , E[S^{SP}] = \frac{(X-c)^2}{2\gamma} + cQ_{W,0} \tag{17}$$

Due to internalization of the entire wind generation; $E[CS^{SP}]$, $E[PS^{SP}]$ and $E[S^{SP}]$ does not depend on the volatility, σ^2. Furthermore, $\partial CS^{SP}/\partial\epsilon_t = 0$, $\partial PS^{SP}/\partial\epsilon_t > 0$ and $\partial S^{SP}/\partial\epsilon_t > 0$. In other words, producer surplus and total social surplus increases with wind production whereas consumer surplus does not change. There is such a result since producer surplus is simply the cost savings due to internalized wind generation. Therefore, producer surplus and hence total social surplus are positively affected by the level of wind generation.

2.3.2 Cournot Oligopoly

In this section, I investigate the effects of wind generator ownership on the (expected) consumer surplus, producer surplus and total social surplus for the oligopoly case. By using the conventional definitions (see **Appendix A.2**) I derive expected consumer surplus ($E[CS^{\alpha}]$) as well as producer surplus ($E[PS^{\alpha}]$) and total social surplus ($E[S^{\alpha}]$) for α share of aggregate wind generator ownership.

$$E[CS^{\alpha}] = \frac{n^2}{(n+1)^2}\frac{(X-c)^2}{2\gamma} + \frac{n(1-\alpha)(X-c)}{(n+1)^2}Q_{W,0} + \frac{\gamma(1-\alpha)^2}{2(n+1)^2}(Q_{W,0}^2+\sigma^2) \tag{18}$$

$$E[PS^{\alpha}] = \frac{n}{(n+1)^2}\frac{(X-c)^2}{\gamma} + \left[\frac{(1-n)(1-\alpha)(X-c)}{(n+1)^2} + c\right]Q_{W,0} - \frac{\gamma(1-\alpha)^2}{(n+1)^2}(Q_{W,0}^2+\sigma^2) \tag{19}$$

$$E[S^{\alpha}] = \frac{n(n+2)}{(n+1)^2}\frac{(X-c)^2}{2\gamma} + \left[\frac{(1-\alpha)(X-c)}{(n+1)^2} + c\right]Q_{W,0} - \frac{\gamma(1-\alpha)^2}{2(n+1)^2}(Q_{W,0}^2+\sigma^2) \tag{20}$$

There are three immediate results of **Equations 18-20**. First, it follows from **Proposition 3** that the total peakload production and hence the total social surplus is independent of k. Second, the volatility of wind generation (σ^2) has no impact on the expected social surplus when all of the wind generators are owned by the peakload firms (i.e., $\alpha = 1$). This is again because, in both of those cases, the effects of wind generation are fully internalized by the peakload firms. Third, $E[CS^{\alpha}]$ increases whereas $E[PS^{\alpha}]$ and $E[S^{\alpha}]$ decrease with σ^2. **Equation 20** further shows that the negative effect of volatility on the producer surplus outweighs the positive effect on the consumer surplus. Furthermore, the effect of the volatility on the expected consumer, producer and total social surplus decreases with α.

Definition 4 *The* ***expected social effect*** *of aggregate wind generator ownership is defined by:*

$$\Delta S := E[S^{\alpha=0}] - E[S^{\alpha>0}].$$

Proposition 5 *The* ***expected social effect*** *of aggregate wind generator ownership decreases with the volatility of wind generation. i.e.,* $\partial \Delta S/\partial\sigma^2 < 0$.

Proof. It follows from **Equation 20** that:

$$\Delta S = \frac{\alpha(X-c)}{(n+1)^2} Q_{W,0} - \frac{\gamma(2\alpha - \alpha^2)}{2(n+1)^2}(Q_{W,0}^2 + \sigma^2)$$

As a result we obtain, $\partial \Delta S / \partial \sigma^2 < 0$. ■

The volatility of wind generation (σ^2), negatively affects the expected social surplus. This negative effect is lower in the wind generator ownership case as the internalization of the wind generator dampens the effect of σ^2.

Definition 5 *The* ***expected producer effect*** *of aggregate wind generator ownership is defined by:*

$$\Delta PS := E[PS^{\alpha>0}] - E[PS^{\alpha=0}].$$

Proposition 6 *The* ***expected producer effect*** *is always positive and it increases with the aggregate wind generator ownership and volatility.*

i.e., $\Delta PS > 0$, $\partial \Delta PS / \partial \alpha > 0$ *and* $\partial \Delta PS / \partial \sigma^2 > 0$.

Proof. It follows from **Equation 19** that:

$$\Delta PS = \frac{\alpha(n-1)(X-c)}{(n+1)^2} Q_{W,0} + \frac{\gamma(2\alpha - \alpha^2)}{(n+1)^2}(Q_{W,0}^2 + \sigma^2)$$

As a result we obtain, $\Delta PS > 0$, $\partial \Delta PS / \partial \alpha > 0$ and $\partial \Delta PS / \partial \sigma^2 > 0$. ■

It follows from **Proposition 6** that producers are better off with wind generator ownership. Furthermore, wind generator ownership provides a protection against the negative effects of the volatility of wind generation by internalization of the wind generation.

2.4 Start up and Shut down Decisions

In **Section 2.2**, I calculated the equilibrium peakload production levels depending on the status of wind generator ownership. Since it is not possible to have negative peakload generation, equilibrium peakload production levels given in **Equations 9&10** must be non-negative. Given the model structure, the peakload firms first observe the level of wind generation. Afterwards, when the level of wind generation is high enough, the peakload generators are shut down and when the wind generation is low enough, the peakload generators are started up.[1] As there are no start up or shut down costs in the Cournot model, there will be a single threshold for both start up and shut down decisions for a specific firm.

[1] In this context, *start up* means positive peakload generation whereas *shut down* means zero peakload generation.

Looking at the corresponding equilibrium peakload production levels in **Equations 9&10**, the start up and shut down thresholds are different for the firms with zero and positive wind generator ownership. When $\alpha = 0$, all peakload firms have the same start up and shut down threshold. As $0 < \alpha < 1$, for any firm i that owns **A** share of total wind generators, the start up and shut down threshold is given by:

$$\overline{Q}_i = \frac{1}{1 - \alpha + \mathbf{A}(n+1)} \frac{X - c}{\gamma} \tag{21}$$

For any firm j that does not own any wind generators, the start up and shut down threshold is given by:

$$\overline{Q}_j = \frac{1}{1 - \alpha} \frac{X - c}{\gamma} \tag{22}$$

$\mathbf{A}(n+1) > 0$ since $\alpha > 0$. Then, it follows from **Equations 21&22** that, $\overline{Q}_i < \overline{Q}_j$. In other words, as peakload firms own wind generators, they start up and shut down their peakload generators at a lower wind generation level. Consequently, start up and shut down price thresholds are higher in that case. This result shows that with wind generator ownership, market power increases and peakload generators are expected to produce electricity for a shorter period of time. Furthermore, we have $\partial\overline{Q}_i/\partial\alpha < 0$ and $\partial\overline{Q}_j/\partial\alpha > 0$. In other words, as the aggregate ownership of wind generators (α) increases, the difference between start up and shut down thresholds for firm i and j increases as well. In the following section, I give a continuous-time real options model to investigate the impact of wind generator ownership on start up and shut down decisions of a peakload firm.

3 The Real Options Model

In **Section 2**, I give a Cournot oligopoly model that incorporates different wind generator ownership scenarios. I show that wind generator ownership gives higher market power and competitive advantage to peakload firms. I also give a simple comparison of start up and shut down trigger levels depending on different wind generator ownership cases. Because of the absence of start up and shut down costs for peakload generators, there is a single trigger level for both decisions in the Cournot model. In this section, I build on the results in **Section 2.4** with the use of real options analysis. The aim of this section is to give a more realistic set up by incorporating operational characteristics (start up, shut down costs and peakload capacity) in a continuous-time real options model. As a result, I will be able to calculate distinct values for start up and shut down trigger levels.

I keep the basic model set up as before and assume that the industry consists of two types of electricity generation technologies: *wind* (W) and conventional *peakload* (P) generation. For simplification, I assume that there is only one peakload generation unit which is characterized by (K_P, I_P, E_P, c_P). The peakload generator has fixed production capacity K_P, start up cost I_P, shut down cost E_P and constant marginal cost of production c_P. I assume fixed peakload production as I want to simplify the model to focus primarily on the start up and shut down decisions.

In this model, I take wind generators acting as baseload. One can think of this extreme case as if all the baseload generation units are replaced by the wind turbines. Furthermore, I assume that wind generation is stochastic with zero marginal cost of production. There is no possible strategic action (i.e., capacity withholding) at the wind generator level and total industry production will depend on the optimal operation (start up and shut down) of the peakload generator. There are no transmission or maintenance costs and generators have infinite lifetime.

In contrast with the oligopoly case in the previous section, I assume to have a monopolist at the peakload level. Aside from the peakload generator, the monopolist also owns $\mathbf{A} \in [0,1]$ share of the wind generators. Therefore, the total production of the monopolist consists of peakload production plus its share of the wind production. Namely:

$$Q^M(t) = Q_P^M(t) + \mathbf{A} Q_W(t) \tag{23}$$

As before, the market price of electricity fluctuates stochastically according to linear inverse demand function, $D : \Theta \text{ x } \mathbb{R}_+ \rightarrow \mathbb{R}$:

$$P(t) = D[Q(t)] = X - \gamma Q(t) \quad \text{with } \gamma > 0. \tag{24}$$

where $Q(t) = Q_P^M(t) + Q_W(t)$ is the total industry production at time t and $X > c$ is the constant demand intercept.

The only source of uncertainty in the model is stochastic wind production, $Q_W(t)$, following a *Geometric Brownian Motion*:

$$dQ_W(t) = \alpha Q_W(t)dt + \sigma Q_W(t)dz \tag{25}$$

where α is the drift paramater, σ is the volatility parameter, dt is the increment of time and dz is the increment of a Wiener process.

The state of the industry is characterized by $[Q_W(t), \omega]$ as ω is an indication of the state of the peakload generator where $\omega = 1$ when the peakload is active, and $\omega = 0$ when it is not. In general, the peakload generator will be started when the wind generation is low and it will be shut down when the wind generation is high enough. Specifically, when $Q_W(t) \leq Q_H$ (when wind production is low enough) the start up cost I_P is incurred and peakload generator is started up. Afterwards, when $Q_W(t) \geq Q_L$ (when wind production is high enough) the shut down cost E_P is incurred and the peakload generator is shut down.

3.0.1 Wind-Only Generation

Wind-only generation takes place when wind production is high enough and there is no need for peakload production. For wind-only generation, when $\omega = 0$, the profit function of the monopolist is:

$$\Pi^W[Q_W(t)] = [X - \gamma Q_W(t)]\mathbf{A}Q_W(t) \tag{26}$$

3.0.2 Wind and Peakload Generation

According to the monopolist's objective, the peakload generator will not be started unless it yields *positive additional* value to the total investment portfolio. Therefore, peakload generation takes place when wind production is low enough. In this case, the profit function of the monopolist is:

$$\Pi^{W+P}[K_P + Q_W(t)] = [X - \gamma(K_P + Q_W(t))](K_P + \mathbf{A}Q_W(t)) - c_P K_P \tag{27}$$

3.0.3 Real Options Set-up

Given the initial state of the economy, $[Q_W(t), \omega = 0]$, let us denote $V^0[Q_W(t)]$ as the expected net present value of the total investment when the peakload generator is idle with optimal future strategies. Similarly, $V^1[Q_W(t), K_P]$ is the expected net present value of the total investment when the peakload generator is active with optimal future strategies. Using standard real options techniques, $V^0[Q_W(t)]$ will be the solution to the ordinary differential equation:

$$\frac{1}{2}\sigma^2 Q_W(t)^2 V^0_{Q_W Q_W} + \alpha Q_W(t) V^0_{Q_W} - rV^0 + \Pi^W[Q_W(t)] = 0 \tag{28}$$

Similarly, , $V^1[Q_W(t), K_P]$ will be the solution to the ordinary differential equation:

$$\frac{1}{2}\sigma^2 Q_W(t)^2 V^1_{Q_W Q_W} + \alpha Q_W(t) V^1_{Q_W} - rV^1 + \Pi^{W+P}[K_P + Q_W(t)] = 0 \tag{29}$$

Depending on the different states of the industry, we have the following value functions:

$$V^0 = \begin{cases} C_2 Q_W^{\beta_2} + \dfrac{X\mathbf{A}Q_W}{r-\alpha} - \dfrac{\gamma\mathbf{A}Q_W^2}{r-2\alpha-\sigma^2} & \text{if } Q_W > Q_H^M \\ V^1 - I_P & \text{if } Q_W \leq Q_H^M \end{cases}$$

Similarly,

$$V^1 = \begin{cases} V^0 - E_P & \text{if } Q_W \geq Q_L^M \\ D_1 Q_W^{\beta_1} + \dfrac{(X\mathbf{A} - \gamma K_P(\mathbf{A}+1))Q_W}{r-\alpha} - \dfrac{\gamma\mathbf{A}Q_W^2}{r-2\alpha-\sigma^2} + \dfrac{(X - c_P - \gamma K_P)K_P}{r} & \text{if } Q_L^M \geq Q_W \end{cases}$$

where formal definitions of the trigger levels Q_H^M and Q_L^M are given by:

$$Q_H^M = sup\left\{Q_W(t)|\omega = 0 \wedge V^1[Q(t)] - I_P \geq V^0[Q_W(t)], \forall t\right\} \tag{30}$$

$$Q_L^M = inf\left\{Q_W(t)|\omega = 1 \wedge V^0[Q_W(t)] - E_P \geq V^1[Q(t)], \forall t\right\} \tag{31}$$

In the value functions above, $C_2 Q_W^{\beta_2}$ is the option value of starting up the peakload generator whereas $D_1 Q_W^{\beta_1}$ is the option value of shutting down the peakload generator and the rest of the terms are the discounted value of the profit streams for the corresponding state of the industry. It is evident that the share of the wind generators owned by the monopolist ($\mathbf{A}$) affects those profit streams. Therefore parameter $\mathbf{A}$ will have an effect on the start up and shut down trigger levels (see **Appendix A.3**). In the oligopolistic Cournot model, I showed that market power increases with the value of $\mathbf{A}$. So, we may expect to have higher start up and shut down trigger levels for higher values of $\mathbf{A}$ in the real options model as well. In the following section, I provide numerical results for the effects of $\mathbf{A}$ as well as the other model parameters on the start up and shut down trigger levels.

4 Numerical Results

In this section, I give results to the theoretical real options derivations in the previous section. We can only derive the solutions to the corresponding value functions numerically. In that regard, I take values for the model parameters as; $\alpha = 0.01$, $r = 0.05$, $\sigma = 0.1$, $\gamma = 0.1$, $c_P = 100$, $I_P = 1000$, $E_P = 1000$ and $X = 150$. As a result, we have the following numerical results for start up shut down trigger levels.

Table 1: Trigger Levels for Fixed Peakload Production

K_P	**A**	Q_H^M	Q_L^M	P_H^M	P_L^M
400	0	84.0987	116.059	141.5902	98.3941
400	0.1	76.4530	105.508	142.3547	99.4492
400	0.16059	72.462	100	142.7538	**100**
400	0.5	56.0658	77.3727	144.3934	102.2627
400	0.9	44.2624	61.0837	145.5737	103.8916
400	1	42.0493	58.0295	145.795	104.197

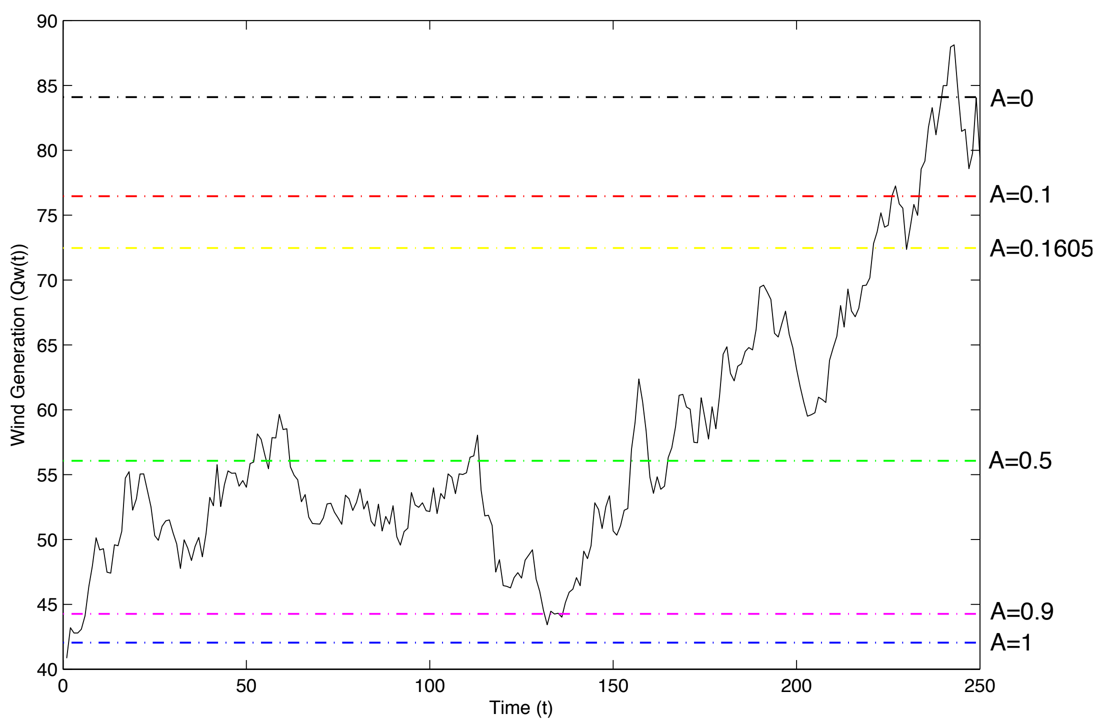

Figure 2: Startup Thresholds for $X = 150$, $K_P = 200$, $I_P = 1000$ and $E_P = 1000$.

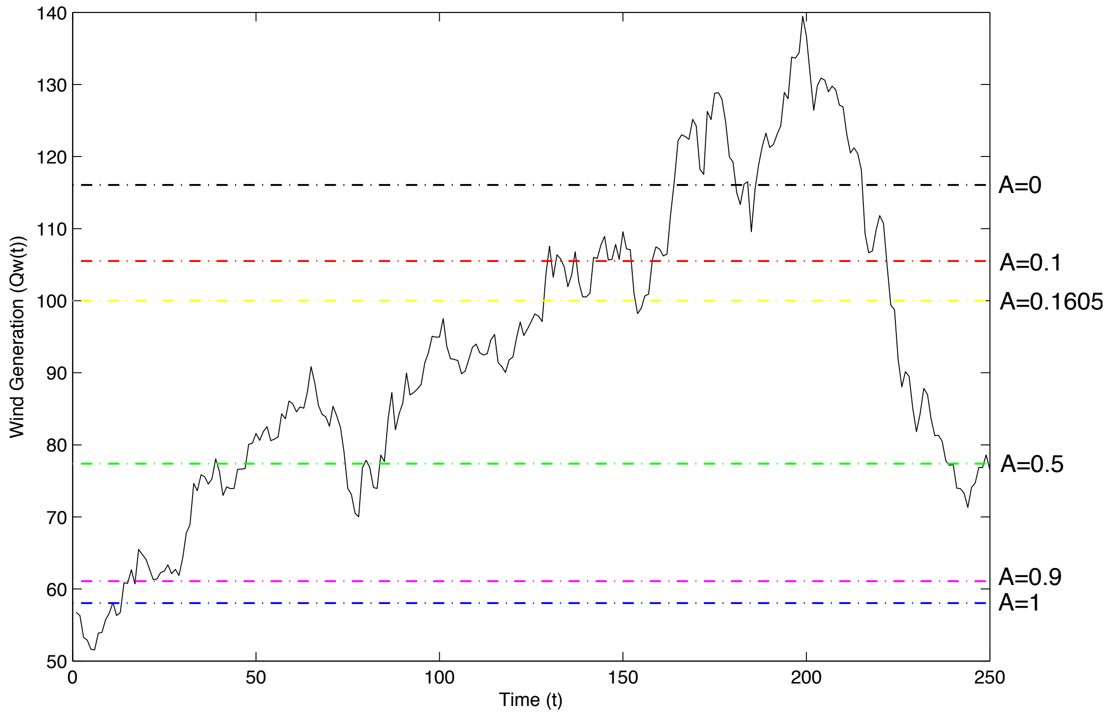

Figure 3: Shutdown Thresholds for $X = 150$, $K_P = 200$, $I_P = 1000$ and $E_P = 1000$.

P_H^M is the trigger price prior to starting up the peakload generator whereas P_L^M is the trigger price prior to shutting down the peakload generator. The trigger prices are formally given as:

$$P_H^M = D[Q_H^M] \text{ and } P_L^M = D[Q_L^M + K_P] \quad (32)$$

First, **Table 1** shows that shut down wind generation levels (Q_L^M) are higher than the start up levels (Q_H^M) as we need sufficiently low wind production levels to start up and sufficiently high wind production levels to shut down the peakload generator. Second, higher values of **A**, results in higher start up and shut down price levels. In other words, for higher values of **A**, the monopolist gain a higher level of market power as well.

Looking further into **Table 1**, we see that shut down price trigger level is equal to the marginal cost when $\mathbf{A} = 0.16059$. This observation is important since, when **A**=0, shut down price trigger levels are expected to be always below the marginal cost. But as $\mathbf{A} > 0$, the monopolist attains a higher level of market power which results in higher shut down trigger price levels. And for $\mathbf{A} \geq 0.16059$, the monopolist can even avoid facing prices below marginal cost while operating the peakload generator.

Figure 4 shows the comparison of start up and shut down thresholds for the extreme

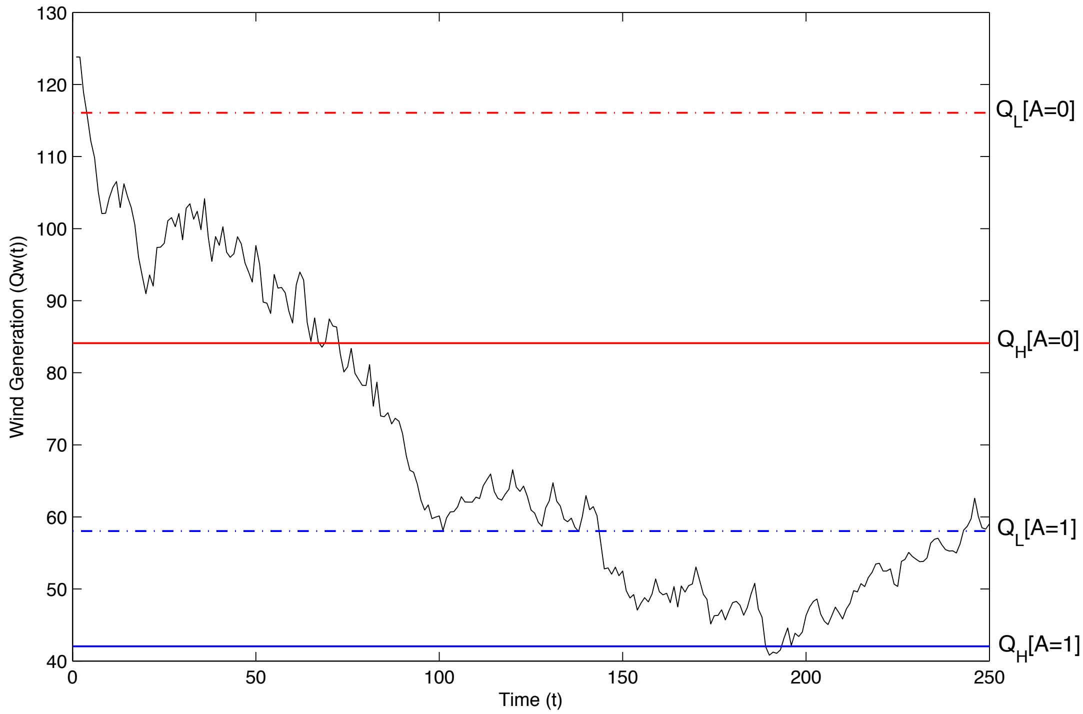

Figure 4: Comparison of the Start up and Shutdown Thresholds for $\mathbf{A} = 0$ and $\mathbf{A} = 1$.

cases $\mathbf{A} = 0$ and $\mathbf{A} = 1$. At $t = 0$, the peakload generator is idle for the both cases. As the sample evolution of the stochastic wind generation shows, the peakload generator is started up first when $\mathbf{A} = 0$. On the other hand, when $\mathbf{A} = 1$, the peakload generator is started up later and shut down earlier. Therefore, the peakload generator ends up being operated for a significantly shorter period of time when all of the wind generators are owned by the monopolist.

In **Table 2**, I present the effect of uncertainty (σ) on the start up and shut down trigger levels for a monopolist owning only the half of the wind generators. For this case, an increase in σ also increases the start up trigger price level but decreases the shut down price level. Therefore, with increasing σ, the wedge between start up and shut down trigger levels increases as well. Hence, operational time for peakload generator is expected to be longer for higher values of σ. These findings are in line with the real options literature (Dixit&Pindyck 1994, Chp. 7).

In **Table 3**, I present the effect of demand intercept (X) on the start up and shut down trigger levels for a monopolist owning only the half of the wind generators. By definition, as X increases, the demand curve shifts to the right. Therefore, for higher values of X, consumers are willing to pay higher prices for the same amount of electricity. In that regard,

Table 2: Effect of Uncertainty (σ)
[For $\alpha = 0.01$, $r = 0.05$, $\gamma = 0.1$, $K_P = 400$, $c_P = 100$, $I_P = 1000$, $E_P = 1000$ and $X = 150$]

A	σ	Q_H^M	Q_L^M	P_H^M	P_L^M
0.5	0.1	56.0658	77.3727	144.3934	102.2627
0.5	0.2	51.7923	85.65	144.82.07	101.435
0.5	0.3	48.4559	93.2258	145.1544	100.6774
0.5	0.4	45.713	100.595	145.2874	99.405
0.5	0.5	43.3849	107.934	145.615	99.2066

Table 3 shows that as X increases both start up and shut down wind generation (price) thresholds increases as well.

Table 3: Effect of the Demand Intercept (X)
[For $\alpha = 0.01$, $r = 0.05$, $\sigma = 0.1$, $\gamma = 0.1$, $K_P = 400$, $c_P = 100$, $I_P = 1000$ and $E_P = 1000$]

A	X	Q_H^M	Q_L^M	P_H^M	P_L^M
0.5	150	56.0658	77.3727	144.3934	102.2627
0.5	160	116.642	150.108	148.3358	104.9892
0.5	170	178.202	221.87	152.1798	107.813
0.5	180	240.312	293.087	155.9688	110.6913

In **Table 4**, I present the effect of absolute value of the slope of the inverse demand function (γ) on the start up and shut down trigger levels for a monopolist owning only the half of the wind generators. As the parameter γ increases, the demand intercept X stays the same and the demand function rotates to the left. In other words, as γ increases the demand function becomes steeper. As a result, we see in **Table 4** that we end up with higher price thresholds for higher values of γ.

Table 4: Effect of the Slope of the Inverse Demand Function (γ)
[For $\alpha = 0.01$, $r = 0.05$, $\sigma = 0.1$, $K_P = 400$, $c_P = 100$, $I_P = 1000$, $E_P = 1000$ and $X = 150$]

A	γ	Q_H^M	Q_L^M	P_H^M	P_L^M
0.5	0.08	130.531	169.576	136.9469	93.0424
0.5	0.09	89.0275	118.484	141.0972	98.1516
0.5	0.10	56.0658	77.3727	144.3934	102.2627
0.5	0.11	29.4485	43.3927	147.0551	105.6607
0.5	0.12	7.9684	14.4073	149.2031	108.5592

5 Conclusion

In this paper, I investigate the effects of different wind generator ownership scenarios on the market outcomes. In that regard, I first give a Cournot oligopoly model to show the effects on the total production and market price. Then I provide a real options model to show the effects on the start up and shut down decisions of the peakload generator. In both models, ownership of the wind generators provides a higher level of market power to peakload firms. Hence, we end up with significantly lower total (peakload) production and higher market price in the Cornout model whereas we end up with higher start up and shut down price trigger levels in the real options model. Given the theoretical evidence, policy makers and regulators should consider the outcomes of the possible ownership structures in the electricity markets as important as the investment in renewable electricity production technologies if they want to convey the potential benefits of renewable generation to consumers.

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A Appendix Additional Model Details and Results

A.1 Oligopolistic Cournot Equilibrium

Profit function for any firm $i \in I$ is given:

$$\Pi_i(t) = [X - \gamma(Q_{P,i} + Q_{P,i'} + Q_{P,n-k} + Q_W(t))](Q_{P,i} + \mathbf{A}Q_W(t)) - cQ_{P,i}$$

Therefore, firm i's objective is to maximize the profit function:

$$max_{Q_{P,i}}\{\Pi_i(t)\} \tag{33}$$

s.t. $Q_{P,i} \geq 0$.

Then, the first order condition is given by:

$$Q_{P,i} = \frac{1}{2}\left[\frac{X-c}{\gamma} - Q_{P,i'} - Q_{P,n-k} - (\mathbf{A}+1)Q_W\right]$$

By taking $Q_{P,i'} = (k-1)Q_{P,i}$, best-response function for firm i is given by:

$$Q_{P,i} = \frac{1}{k+1}\left[\frac{X-c}{\gamma} - Q_{P,n-k} - (\mathbf{A}+1)Q_W\right]$$

Similarly for any firm $j \in J$, we have:

$$\Pi_j(t) = [X - \gamma(Q_{P,j} + Q_{P,j'} + Q_{P,k} + Q_W(t))]Q_{P,j} - cQ_{P,j}$$

Hence, firm j's objective is to maximize the profit function:

$$max_{Q_{P,j}}\{\Pi_j(t)\} \tag{34}$$

s.t. $Q_{P,j} \geq 0$.

Then the first order condition entails:

$$Q_{P,j} = \frac{1}{2}\left[\frac{X-c}{\gamma} - Q_{P,j'} - Q_{P,k} - Q_W\right]$$

By taking $Q_{P,j'} = (n-k-1)Q_{P,j}$, best-response function for firm j is given by:

$$Q_{P,i} = \frac{1}{n-k+1}\left[\frac{X-c}{\gamma} - Q_{P,k} - Q_W\right]$$

By inserting $Q_{P,j}$ into $Q_{P,i}$, we get the equilibrium quantities:

$$Q^*_{P,i}(t) = \frac{X-c}{\gamma(n+1)} - \frac{1-\alpha}{n+1}Q_W(t) - \mathbf{A}Q_W(t)$$

$$Q^*_{P,j}(t) = \frac{X-c}{\gamma(n+1)} - \frac{1-\alpha}{n+1}Q_W(t)$$

A.2 Welfare Implications

I use the conventional definition for consumer surplus as the difference in area under the demand curve for a given production level (Q(t)) minus the total market value of purchasing that level of output. Therefore, for our linear inverse demand function, we have:

$$CS[Q(t)] = \left[\int_0^{Q(t)} (X - \gamma q)dq\right] - (X - \gamma Q(t))Q(t) = \frac{\gamma Q(t)^2}{2} \tag{35}$$

Given the total production level in **Equation 12**, we have:

$$CS^\alpha[Q_W(t)] = \frac{n^2}{(n+1)^2}\frac{(X-c)^2}{2\gamma} + \frac{n(1-\alpha)(X-c)}{(n+1)^2}Q_W(t) + \frac{\gamma(1-\alpha)^2}{2(n+1)^2}Q_W(t)^2 \tag{36}$$

Producer surplus for a given prodution level (Q(t)) is given by the total revenue minus the total cost of production. Hence:

$$PS[Q(t)] = (X - \gamma Q(t))Q(t) - cQ_P(t) \tag{37}$$

Again taking **Equation 12** into account, we have:

$$PS^\alpha[Q_W(t)] = \frac{n}{(n+1)^2}\frac{(X-c)^2}{\gamma} + \left[\frac{(1-n)(1-\alpha)(X-c)}{(n+1)^2} + c\right]Q_W(t) - \frac{\gamma(1-\alpha)^2}{(n+1)^2}Q_W(t)^2 \tag{38}$$

For the calculation of the social surplus we can either use the objective function in **Equation 3** or alternatively the sum of the consumer and producer surplus. Then total social surplus is given by:

$$S^\alpha[Q_W(t)] = \frac{n(n+2)}{(n+1)^2}\frac{(X-c)^2}{2\gamma} + \left[\frac{(1-\alpha)(X-c)}{(n+1)^2} + c\right]Q_W(t) - \frac{\gamma(1-\alpha)^2}{2(n+1)^2}Q_W(t)^2 \tag{39}$$

A.3 Monopolist At the Peakload Level

Given the corresponding value functions, by using value matching and smooth pasting conditions (Dixit & Pindyck 1994), we end up with following system of non-linear equations:

$$C_2Q_H^{\beta_2}+\frac{X\mathbf{A}Q_H}{r-\alpha}-\frac{\gamma\mathbf{A}Q_H^2}{r-2\alpha-\sigma^2}=D_1Q_H^{\beta_1}+\frac{(X\mathbf{A}-\gamma K_P(\mathbf{A}+1))Q_H}{r-\alpha}-\frac{\gamma\mathbf{A}Q_H^2}{r-2\alpha-\sigma^2}+\frac{(X-c_P-\gamma K_P)K_P}{r}-I_P \tag{40}$$

$$\beta_2C_2Q_H^{\beta_2-1}+\frac{X\mathbf{A}}{r-\alpha}-\frac{2\gamma\mathbf{A}Q_H}{r-2\alpha-\sigma^2}=\beta_1D_1Q_H^{\beta_1-1}+\frac{(X\mathbf{A}-\gamma K_P(\mathbf{A}+1))}{r-\alpha}-\frac{2\gamma\mathbf{A}Q_H}{r-2\alpha-\sigma^2} \tag{41}$$

$$C_2Q_L^{\beta_2}+\frac{X\mathbf{A}Q_L}{r-\alpha}-\frac{\gamma\mathbf{A}Q_L^2}{r-2\alpha-\sigma^2}=D_1Q_L^{\beta_1}+\frac{(X\mathbf{A}-\gamma K_P(\mathbf{A}+1))Q_L}{r-\alpha}-\frac{\gamma\mathbf{A}Q_L^2}{r-2\alpha-\sigma^2}+\frac{(X-c_P-\gamma K_P)K_P}{r}+E_P \tag{42}$$

$$\beta_2C_2Q_L^{\beta_2-1}+\frac{X\mathbf{A}}{r-\alpha}-\frac{2\mathbf{A}\gamma Q_L}{r-2\alpha-\sigma^2}=\beta_1D_1Q_L^{\beta_1-1}+\frac{(X\mathbf{A}-\gamma K_P(\mathbf{A}+1))}{r-\alpha}-\frac{2\gamma\mathbf{A}Q_L}{r-2\alpha-\sigma^2} \tag{43}$$

where $\beta_1 > 1$, $\beta_2 < 0$ are solutions for the following quadratic equation:

$$\frac{1}{2}\sigma^2\beta^2+\left(\alpha-\frac{1}{2}\sigma^2\right)\beta-r=0. \tag{44}$$

Using above equations, we can numerically solve for C_2, D_1, Q_H and Q_L (Dixit 1989). Numerical solutions are given in **Section 4**.