PRODUCT DIFFERENTIATION AND OPTION GAMES: CLOSING THE GAP*

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ABSTRACT

Up to now, the Option Games literature has studied oligopolistic markets where firms compete *à la* Cournot, leaving aside other markets where firms use pricing policies as part of their market strategy.

To address this issue, Hotelling's linear city model is adapted, to include the tradeoff between preempting competitors under uncertainty or waiting to know where the city's customers are. Despite the simplicity of this model, it is one of the first to study the trade-off between flexibility and commitment under competition *à la* Bertrand.

The main result of this study shows that giving firms the freedom to endogenously determine their best entry strategy can lead to more than one entry pattern to arise as a Subgame Perfect Nash Equilibrium. This result can be extended to an adapted Cournot framework. Besides this point, the role of price competition *ex-post* once firms have chosen positions on the real axis lowers the commitment of the previous stages, lowering the relative value of their waiting option when compared to competition solely based on positions or *à la* Cournot. These results are then used to give plausible explanations to markets ranging from smartphones to political elections.

Keywords: *Option Games*, Hotelling, Cournot, Uncertainty

"Thus, what is of supreme importance in war is to attack the enemy's strategy"

Sun Tzu (Chinese General, 500 BC)

1. Introduction

Ever since Hotelling's "Stability in Competition" first appeared in 1929, there has been an increasing literature in economics that has come to realize that, without many exceptions, there are no two products that are equal.[1] And, even though two products can appear as identical, firms can appeal to other techniques such as marketing to differentiate themselves in the market. In fact, according to Porter (1980), incumbents have incentives to differentiate, because it "creates layers of insulation against competitive warfare because buyers have particular preferences and loyalties to particular sellers" (p. 36).

On the other hand, product differentiation does not come for free since, according to this same author, "investments in building a brand name (i.e. position in the market) are particularly risky, since they are unrecoverable" (p. 33, parenthesis added). Therefore, the decision of launching a product into a differentiated market cannot be taken lightly, because the firm faces: (1) the sunk cost of choosing a position into a particular market that cannot be changed, (2) the uncertainty of how customers will welcome this product and (3) how incumbents will react towards this action.

Up to date, for evaluating this type of investment opportunities, firms usually use the Net Present Value (from hereafter, NPV) criterion. Nevertheless, Del Sol and Ghemawat (1999) realize that this technique is known for its "failure to include option values and failure to take competition into account". Therefore, new evaluation techniques must be used to capture the true value of the entry opportunity of a firm into a differentiated market.

First, option pricing on real assets, based on the seminal work of Brennan and Schwartz (1985), tackles the issue of valuing irreversible decisions in an uncertain context. The general insight gained by this method is that having the option to modify a firm's actions or a project's design according to the outcomes of relevant uncertainties has value, and this methodology measures it under different contexts, such as deferring investment, expanding production capacity, opening/closing/mothballing current operations or storage options. Dixit and Pindyck (1994) contain an excellent survey of this line of research.

This first line of research, although extremely valuable and necessary to improve the NPV criterion, does not take into account the role of competition in investment opportunities.[2] Realizing that this effect can be significant, especially in oligopolistic markets, the line of research known as Option Games arises during the 90's. In this context, investigators use the mathematical tools presented by Fudenberg and Tirole (1985) for using non-cooperative game theory in a continuous-time framework and combined them with the continuous-time stochastic

[1] Commodities such as copper or oil are an excellent counterexample of homogeneous goods.

[2] To be fair, Dixit and Pindyck (1994) do tackle this issue in Chapters 8 and 9. However, in this case, they develop a simple model and recognize that "a more general analysis of this kind is a promising topic for future research".

processes used in option pricing to account for the role of competitors in the option value or optimal exercise strategies of the options embedded in real assets.

Revising some important contributions in the literature, Leahy (1993) analyzes a perfectly competitive market setting, where capacity expansions by firms affect the market price. He finds that, under certain assumptions, firms can be myopic when it comes to determining the optimal price for expanding, even though there is strategic interaction. Grenadier (1996) considers a dynamic duopoly where a firm's exercise of an expansion option affects its competitor's expansion option, leading to different Subgame Perfect Nash Equilibrium (from hereafter, SPNE) exercise strategies. He then applies this model to explain the paradox of real-estate booms in shrinking markets. Pawlina and Kort (2006) extend this model to the case where there are asymmetries in the competitor's project cash flows. Lambrecht and Perraudin (2003) analyze how the exercise strategies in the McDonald and Siegel (1986) model vary when there are two firms competing for it and their investment costs are unknown to their counterparts. Grenadier (2002) studies a repeated game of capacity expansion in a symmetrical Nash-Cournot framework of *n* firms, reaching the conclusion that a greater amount of competition erodes the option value of waiting to expand until prices improve. Aguerrevere (2003) relaxes the constant returns to scale assumption in Grenadier's model, reaching that the aggregate price process is mean-reverting, even though the stochastic demand shock follows a Geometric Brownian Motion (from hereafter, GBM) process. Novy-Marx (2007), on the other hand, relaxes the assumption that firms are symmetric, reaching the notable conclusion that the option value is not necessarily a decreasing function in the number of firms. Aguerrevere (2009) uses Grenadier's Cournot model to plausibly explain the asset returns of firms, which depend on the commodity price and the firm's expansion options and assets-in-place. Finally, even though this list goes on, it is important to note that this article is not a survey; to this end, the reviews of Azevedo and Paxson (2010) and Chevalier-Roignant *et al*. (2010) provide an excellent summary of the current state of the Option Games' literature.

2. Do waiting options in differentiated markets exist?

In spite of the improvements in the Option Games' literature in the last 10 years, the surveys mentioned above illustrate that all these games depict a world where the firms' payoff functions are either exogenous or determined by a downward sloping demand curve, weighed by a factor that follows a continuous-time stochastic process. Although this framework has the advantage of reaching elegant solutions for homogeneous products in a dynamic setting, it fails to capture the intuition in markets where differentiation is present, because the strategic interaction between firms is modeled using capacity instead of other variables, such as the price of each firm's product. This leads the price to clear out afterwards, as a result of the firms' decisions, instead of being part of a firm's strategy *per se*. This is a clear limitation of the Option

Games literature up to this point, because there are many markets where firms can influence their payoffs through pricing policies.[3]

This does not mean that there have not been attempts to extend the Option Games literature to model strategic interaction in frameworks other than Grenadier's Nash-Cournot equilibrium. Boyer *et al*. (2004) model a Bertrand duopoly where both firms compete for a fixed mass of consumers whose willingness to pay for a certain product varies in time according to a GBM process. Pawlina and Kort (2010), on the other hand, consider a differentiated market where two firms must select qualities to determine their revenues (that follows a GBM process as well), taking into account that network externalities affect the distribution of this cash flow among the firms. When revising the resolution of both models, these can be seen as an extension to Lambrecht and Perraudin (2003), where firms can now invest for a total or a partial amount of a cash flow stream that varies in time according to a GBM process. Hence, the underlying logic of Option Games' theory does not essentially change. And, what is even more important, both models are unrealistic from an Industrial Organization (from hereafter, IO) perspective:[4] Boyer *et al*. (2004) assume that the firms are capable of extracting the entire consumer surplus under Bertrand competition, while Pawlina and Kort (2010) assume that a customer's payment grows in time following a GBM process while the utility they get from the product remains constant.

Taking this into account, this article is a first attempt to successfully narrow the gap between the Option Games literature with markets where price (and not capacity) is the strategic variable at hand. To this end, an iconic model of differentiated markets in IO is used: Hotelling's linear city (1929). Lambertini's (1994) original setting is used, where a location-then-price duopoly game with quadratic transport costs[5] takes place and firms have the freedom to locate themselves anywhere on the real axis. However, there is uncertainty regarding the city's position in the location stage, and here firms have the option to wait until the city's exact position becomes known. Hence, a tradeoff arises between waiting to know where the market is and gaining a first mover advantage, which is known to exist in this type of models ever since Tabuchi and Thisse (1995).

Using this adapted model that incorporates uncertainty, the objective of this article is to analyze the entry pattern that emerges as a SPNE: either both firms enter before the uncertainty is resolved, after it is resolved or sequential entry occurs. The method for determining the endogenous entry pattern is done in the same fashion as Lambertini (1997), which analyzed the endogenous location and price patterns for the city in the [0,1] segment. Related to this point, as

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[3] When referring to pricing policies, this articles is restricted to the case where firms cannot discriminate consumers according to their willingness to pay, and therefore must charge the same price to all buyers.

[4] Jean Tirole (1988) says that "to study industrial organization is to study the functioning of markets" (p. 1). Hence, this branch of economics is *per se* the most appropriate one for analyzing differentiated markets. See Tirole (1988) for an excellent introduction to IO. Eaton and Lipsey (1989) is an excellent survey on product differentiation, while Anderson *et al*. (1992) establish the link between product differentiation models and Discrete Choice Theory.

[5] Hotelling's original game considered linear costs. However, D'Aspremont *et al*. (1979) realized that no NE in positions exists when firms are near each other and modified the game by using quadratic transport costs, ensuring a unique NE in positions.

a byproduct of the procedure to calculate the entry pattern that arises as a SPNE, the option value of waiting to know the city's location is calculated to see if these opportunities really exist under the context of product differentiation.

This result is important, because a secondary objective of this article is to compare the value of the waiting option to other benchmarks. First, the product differentiation model is adapted to represent Cournot competition. Doing this allows to establish a comparable link between the model analyzed here with the traditional results of the Option Games literature, since this model is not done in a continuous time framework. Secondly, the price setting stage of the model is removed to see the role prices play in the entry pattern that arises as a SPNE. This configuration without a price setting stage can be seen as introducing uncertainty into Down's model (1957) of political competition.

And, as a third objective, this article attempts to use the model developed here to plausibly explain the occurrences of a real market that does not fit the assumptions used in Option Games literature up to date. The smartphone industry is a contemporary example, since this type of phone has a series of differentiating features that make each of them different from its competitors, giving producers pricing power over their products.

This market is currently a flourishing one, with total sales increasing over 70% in the year 2010, as seen in Table 1:

Table 1: Worldwide smartphone sales to end users by operating system (OS) in 2010
Source: Pettey and Goasduff (2011)

Company	2010 Sales (000's Units)	2010 Market Share (%)	2009 Sales (000's Units)	2009 Market Share (%)
Symbian (Nokia)	111,576.7	37.6	80,878.3	46.9
Android (Google)	67,224.5	22.7	6,798.4	3.9
RIM (Blackberry)	47,451.6	16.0	34346.6	19.9
iOS (Apple)	46,598.3	15.7	24,889.7	14.4
Microsoft	12,378.2	4.2	15,031.0	8,7
Other OSs	11,417.4	3.8	10,432.1	6.1
Total	**296,646.6**	**100.0**	**172,376.1**	**100.0**

From this table, it is possible to see that not all operating systems (from hereafter, OS) were able to capture the same proportion of this new critical mass of smartphone users. Table 2 estimates the market shares of each OS considering the new mass of customers:[6]

[6] For calculating this result, the additional customers were calculated by simply subtracting 2010 sales with respect to 2009 sales. The market share was subsequently calculated by dividing these differences by total additional sales worldwide in 2010 (124,270,600 smartphones).

Table 2: Worldwide smartphone additional sales in 2010 by operating system (OS)
Source: Based on Pettey and Goasduff (2011)

Company	2010 Additional Sales (000's Units)	2010 Additional Sales (%)
Symbian (Nokia)	30,698.4	24.7
Android (Google)	60,426.1	48.6
RIM (Blackberry)	13,105.0	10.5
iOS (Apple)	21,708.6	17.5
Microsoft	-2,652.8	-2.1
Other OSs	985.3	0.8
Total	**124,270.6**	**100.0**

This comes to show that the new customers are not divided equally among the firms, violating the Option Games' usual assumption that firms are symmetrical. Added to this, it is interesting to see that Google's OS, which is relatively new to the market, captured nearly half of the new mass of consumers. Hence, it is not true either that the new customers are divided proportionally among the existing firms' market share. Could this be due to the fact that Google's OS Android was a late follower that was capable to adapt better in this differentiated market under uncertainty? These facts are to be explained in Section 5.

The rest of this article is structured as follows: Section 2.2 presents the option game model based on Hotelling's linear city with uncertainty in the city's location and how this model is solved; Section 2.3 presents and solves the benchmark models of Cournot competition and product differentiation without price competition; Section 2.4 analyzes and compares the results found in the previous chapters; finally, Section 2.5 presents the most relevant findings of this paper and where future research must aim to continue closing the gap between product differentiation and the Option Games literature.

3. The model

3.1. Game structure

Regarding demand, the city is made up by a mass of price-taking consumers that are distributed uniformly over a linear city (represented by an interval on the real axis). This city is of fixed length (normalized at 1) that begins at k (which is greater than or equal to zero). Each consumer demands a unit of a differentiated good they value at v, which is assumed to be high enough so that the market is covered (i.e. all consumers purchase a unit of the good).

Regarding supply, there are two firms that select a location (also referred to as position or product type) on the real axis to satisfy demand. In this sense, both duopolists are not restricted to choose positions inside the linear city, emulating Lambertini (1994), because there is uncertainty regarding the city's position from their perspective. Additionally, besides the choice of location, this model also gives each firm the option to wait until this uncertainty is cleared out before positioning themselves on the real axis. This is the key difference between this article with past papers in the product differentiation literature that incorporate uncertainty in Hotelling's linear city: while this paper assumes that there is a trade-off between a first-mover advantage and waiting to reduce the city's uncertainty; the other papers up to date have compared the degree of differentiation between both duopolists with and without uncertainty in the city's position. Nevertheless, it can be anticipated that the insight given by this line of literature is present when determining the entry pattern that rises as a SPNE for the model, as will be seen in Subsection 2.3.

Related to the exercise of the entry option, it is important to mention that both firms do not have investment costs when choosing a location neither production costs when satisfying demand. However, firms do face an irreversible cost regarding the sacrificed earnings by choosing a position in the product space *ex-ante* that ends up being far away from the linear city once its position becomes known to both players, so the option to wait can be valuable.

Afterwards, once both firms choose positions, they compete simultaneously in prices. Here, both firms can use their pricing strategy to take advantage or compensate their location decision in the previous stages, leading their payoffs to become a non-linear function of both location (related to the option exercise strategy) and price (which is decided *ex-post* once both firms have entered). Therefore, this game has a non-trivial solution regarding the timing of entry for both firms into the product space.

Once both firms price their products, each consumer in the linear city selects the product that maximizes their utility. For consumer i, located at x_i, the utility achieved by selecting product j, located at x_j, is given by:

$$U(i,j) = v - p_j - t \cdot \left(x_i - x_j\right)^2 \tag{1}$$

Here, t is a constant that represents the disutility caused by quadratic transport costs between the buyer and seller. For simplicity, this value is normalized to 1.

To summarize the structure of this option game together with the strategy space available to both duopolists, Table 3 shows each of the stages in their chronological order. Seeing this table, it is clear that this model is more closely related to the IO literature because, unlike other option games, there is only one period where firms compete for consumers (stage 6), which is separated from the periods where the firms must exercise their entry options (stages 1-4). Finally, the price setting period (stage 5) is decided after the firm's locations are picked, reflecting the difference between long term competition versus short term competition, an aspect which has been ignored up to date in the Option Games literature.

Table 3: Summary of the Entry Option Game into Hotelling's Linear City under Uncertainty

Stage	Description	Chosen Variables
1	Firms a and b decide whether to exercise their entry option into the linear city or not.	$X_i = \{Entry, deferral\}\ \forall\ i = \{1,2\}$
2	Firms that selected entry decide their position in the linear city.	$x_1 = a$ ($\epsilon\ \mathbb{R}$) or $x_2 = b$ ($\epsilon\ \mathbb{R}$) if and only if they entered in previous stage
3	Uncertainty over the city's location is cleared out.	None
4	Firms that deferred entry exercise their entry option.	$x_1 = a$ ($\epsilon\ \mathbb{R}$) or $x_2 = b$ ($\epsilon\ \mathbb{R}$) if and only if they deferred in stage 1
5	Firms select prices for products	$P_1, P_2\ \epsilon\ \mathbb{R}^+$
6	Customers select products and firms collect profits.	None

Up to now, the uncertainty in the city's location has been mentioned rather ambiguously. To clarify how this uncertainty, known hereafter as *locational uncertainty*, is modeled, the city's position on the real axis is given by the interval $[k, k+1]$, where k is an uncertain variable that follows a Bernoulli distribution: the city can begin either at 0, occupying the [0,1] segment, with probability $(1-p)$ or at θ, occupying the $[\theta, \theta+1]$ segment, with probability p. It is important to note that, due to the uncertainty's symmetry, the valid range for p lies in $[0, \frac{1}{2}]$,[7] while the possible values for θ are determined in such a way that vertical differentiation is not possible.[8] This uncertainty emulates the uncertainty seen in previous articles in the product differentiation literature, such as Casado-Izaga (2000) and Meagher and Zauner (2004). However, the Bernoulli distribution was used in this case because the relationship between p and θ is easy to manipulate achieving any expected value for the city's location, while keeping the model traceable enough to reach closed mathematical expressions.

3.2.Game resolution

As in all games with a finite number of stages, the appropriate method for solving this model implies using backward induction. To make the results derived in this paper comparable to others in the product differentiation literature, Hotelling's original convention is used, where firms 1 and 2 select positions a and b, respectively. Position a is measured from the point $x = 0$ on the real axis towards the right; position b, on the other hand, is measured from the point $x = 1$ towards the left.

[7] This due to the fact that for values of p between $[\frac{1}{2}, 1]$, the same phenomenon can be reproduced by simply changing the point of reference to $x = \theta$ and reversing the axis' direction.

[8] In the IO literature, this implies that "all consumers agree over the most preferred mix of characteristics and, more generally, over the preference ordering" (Tirole, 1988, pág. 96). Therefore, this model avoids the case where a firm can capture the entire market.

In the final stage, assuming that firms have already chosen positions and prices, and the city's characteristics – symbolized by the vector $\bar{\omega}$ –has been revealed,[9] it is possible to calculate each firm's profits by determining the city's indifferent customer positioned at x_{indif}, which would gain the same utility buying from either firm. This stage is straightforward and can be seen in references such as D'Aspremont *et al*. (1979) or Tirole (1988).

For choosing prices both firms, knowing their competitor's position (a and b) and the city's characteristics ($\bar{\omega} \epsilon \Omega$), select a price that maximizes their profits, giving its competitor no incentive to change its price unilaterally. This system of equations can be expressed as follows:

$$\begin{aligned} P_a^* &= argmax_{P_a}\{\pi(P_a, P_b^*, a, b, \bar{\omega})\} \\ P_b^* &= argmax_{P_b}\{\pi(P_a^*, P_b, a, b, \bar{\omega})\} \end{aligned} \tag{2}$$

Solving this system of equations, prices will be a function of the city's characteristics $\bar{\omega}$ and the firms' positions a and b.

Afterwards, when firms choose positions, this will depend on the entry pattern at hand: either both firms enter after the uncertainty was resolved, before the uncertainty was resolved or if sequential entry occurs.

The first case, where both wait until the uncertainty is cleared, has both firms maximizing their profits for a city with known characteristics $\bar{\omega}$, giving their competitor no incentive to change unilaterally their position. The system of equations is given by:

$$\begin{aligned} a^* &= argmax_a\{\pi(P_a^*(a, b^*, \bar{\omega}), P_b^*(a, b^*, \bar{\omega}), a, b^*, \bar{\omega}\} \\ b^* &= argmax_b\{\pi(P_a^*(a^*, b, \bar{\omega}), P_b^*(a^*, b, \bar{\omega}), a^*, b, \bar{\omega})\} \end{aligned} \tag{3}$$

For the second case, where both firms enter immediately, it is assumed that both firms will maximize their expected profits considering all possible outcomes of ω together with their probability of occurrence $q(\omega)$. The NE problem for this stage can be seen as:

$$\begin{aligned} a^* &= argmax_a\left\{\sum_{\omega \,\epsilon\, \Omega} q(\omega) \cdot \pi(P_a^*(a, b^*, \omega), P_b^*(a, b^*, \omega), a, b^*, \omega)\right\} \\ b^* &= argmax_b\left\{\sum_{\omega \,\epsilon\, \Omega} q(\omega) \cdot \pi(P_a^*(a^*, b, \omega), P_b^*(a^*, b, \omega), a^*, b, \omega)\right\} \end{aligned} \tag{4}$$

[9] Although this article introduces uncertainty only in the city's location, other attributes such as the city's length or density can also act as random variables. Hence, a general vector $\bar{\omega}$ is used to represent this general case, which is part of the subspace Ω of all feasible combinations of city attributes.

For the third case, when sequential entry occurs, it is assumed (without a loss in generality) that firm 2 enters after firm 1. Hence, the follower maximizes its profits already knowing where the city is and the leader's location, as shown:

$$b^*(a,\overline{\omega}) = argmax_b\{\pi(P_a^*(a,b,\overline{\omega}), P_b^*(a,b,\overline{\omega}), a, b, \overline{\omega})\} \tag{5}$$

The leader, on the other hand, must maximize its expected profits considering all possible outcomes for the city's characteristics ω together with their probability of occurrence $q(\omega)$ and the fact that its position influences the follower's location $b^* = b^*(a,\omega)$.

$$a^* = argmax_a\left\{\sum_{\omega\,\epsilon\,\Omega} q(\omega)\cdot\pi(P_a^*(a,b^*,\omega), P_b^*(a,b^*,\omega), a, b^*, \omega)\right\} \tag{6}$$

Finally, once the optimal positions are determined for each of these scenarios, it is possible to construct a 2x2 matrix with the payoff functions in equilibrium for both players considering all the possible combinations of the firms' decisions of entering before or after the city's characteristics are revealed, in the same fashion as Lambertini (1997), which sought to determine the endogenous order of entry under a deterministic setting.

3.3. Solution under locational uncertainty

Following the game resolution process described above, the vector of city characteristics is reduced to the city's location ($\omega = k$) with all possible outcomes restricted to two positions as seen in subsection 3.1 ($\Omega = \{0,\theta\}$) with their respective probabilities of occurrence ($q(0) = 1 - p$ and $q(\theta) = p$). To derive the entry pattern that rises as a SPNE, the optimal positions and prices of both firms are determined for all possible entry patterns. Afterwards, the value functions evaluated at these positions and prices are compared using Lambertini's 2x2 matrix to determine which pattern rises as a SPNE.

However, considering that both the probability of displacement p and the displacement factor θ are parameterized, it is possible to vary these variables to achieve different "amounts" of uncertainty in the city's location. Hence, the entry pattern that rises as a SPNE is clearly a function of these variables and results in a 2x2 matrix for each combination of p and θ.[10] To determine the SPNE for all valid (p,θ) pairs, the value function of preempting and waiting is found for each pair assuming that its competitor either preempts or waits for the city's location to be revealed. This determines the reaction function for the firm. The same process is then done for the other firm and the intersection of these reaction functions gives the SPNE for all (p,θ) pairs.

Before determining the position equilibriums that arise for each entry pattern, the price selection stage must be solved. This stage is common to all entry patterns, because the location

[10] This problem is the general case of Lambertini (1997), since this reference only analyzes the lines $p = 0$ and $\theta = 0$ in the (p,θ) space, which is the case without locational uncertainty.

of the city's consumers is already known to both firms. Here, prices are determined simultaneously from the following equation system, considering a general city location parameterized at k:

$$\begin{gathered}\frac{d}{dP_a}\left(P_a \cdot (x(P_a, P_b, a, b) - k)\right) = 0 \\ \frac{d}{dP_b}\left(P_b \cdot \left(k + 1 - x(P_a, P_b, a, b)\right)\right) = 0\end{gathered} \tag{7}$$

Where x is the indifferent consumer, which is a function solely of both firms' strategic variables. Proposition 1 determines the equilibrium in prices for this type of uncertainty.

Proposition 1: The equilibrium prices that arise under locational uncertainty are given by:

$$\begin{gathered}P_a^* = (1 - b - a) \cdot \left(1 + \frac{a - b}{3} - \frac{2k}{3}\right) \\ P_b^* = (1 - b - a) \cdot \left(1 + \frac{b - a}{3} + \frac{2k}{3}\right)\end{gathered} \tag{8}$$

Proof: Formulating the maximization problem for the firms' profits in equation (2), under this type of uncertainty, leads to the system in (7). Solving this system of first order conditions leads to this solution, which satisfies the second order conditions that ensure that this equilibrium maximizes the profits of both firms. Q.E.D.

As said before, these prices arise after both firms choose locations and, therefore, are consistent for all entry patterns seen under locational uncertainty and are not dependent on the probability of displacement p, but are a function of the firms' and the city's locations.

Once prices are determined, the NE in positions for each entry pattern can be calculated. First, for simultaneous entry after the city's location is known (a SPNE known hereafter as mutual waiting), the NE in positions that arises is the traditional result seen in Lambertini (1994), where firms 1 and 2 locate themselves at $a^* = k - \frac{1}{4}$ and $b^* = k + \frac{5}{4}$, respectively. Since the degree of differentiation between both firms is not a function of the city's position *ex-post*, both players gain the known profit of $\pi_1 = \pi_2 = \frac{3}{4}$. Hence, the upside a firm can gain by waiting when their competitor also waits is limited.

Secondly, assuming that both firms' locations are to be determined simultaneously before the city's location is revealed (a SPNE known hereafter as mutual preemption) leads firms to maximize expected profits, as shown:

$$\frac{d}{da}\Big((1-p)\cdot P_a^*(a,b,0)\cdot(x(P_a^*(a,b,0),P_b^*(a,b,0),a,b)-0)+(p)\cdot P_a^*(a,b,\theta)\cdot(x(P_a^*(a,b,\theta),P_b^*(a,b,\theta),a,b)-\theta)\Big)=0$$
$$\frac{d}{db}\Big((1-p)\cdot P_b^*(a,b,0)\cdot\big(1-x(P_a^*(a,b,0),P_b^*(a,b,0),a,b)\big)+(p)\cdot P_b^*(a,b,\theta)\cdot\big(\theta+1-x(P_a^*(a,b,\theta),P_b^*(a,b,\theta),a,b)\big)\Big)=0 \quad (9)$$

Solving this system, the position of firms 1 and 2 are a non-linear function of both the probability p that the city shifts from its initial point 0 and the distance θ of this shift, as seen in Proposition 2. However, to understand the intuition behind the results seen in this section, these will be expressed in terms of the uncertain variable's mean ($E(k)=p\theta$) and variance ($Var(k)=p\cdot(1-p)\cdot\theta^2$).

Proposition 2: The equilibrium locations that arise when both firms enter before the city's locational uncertainty is cleared are defined by:

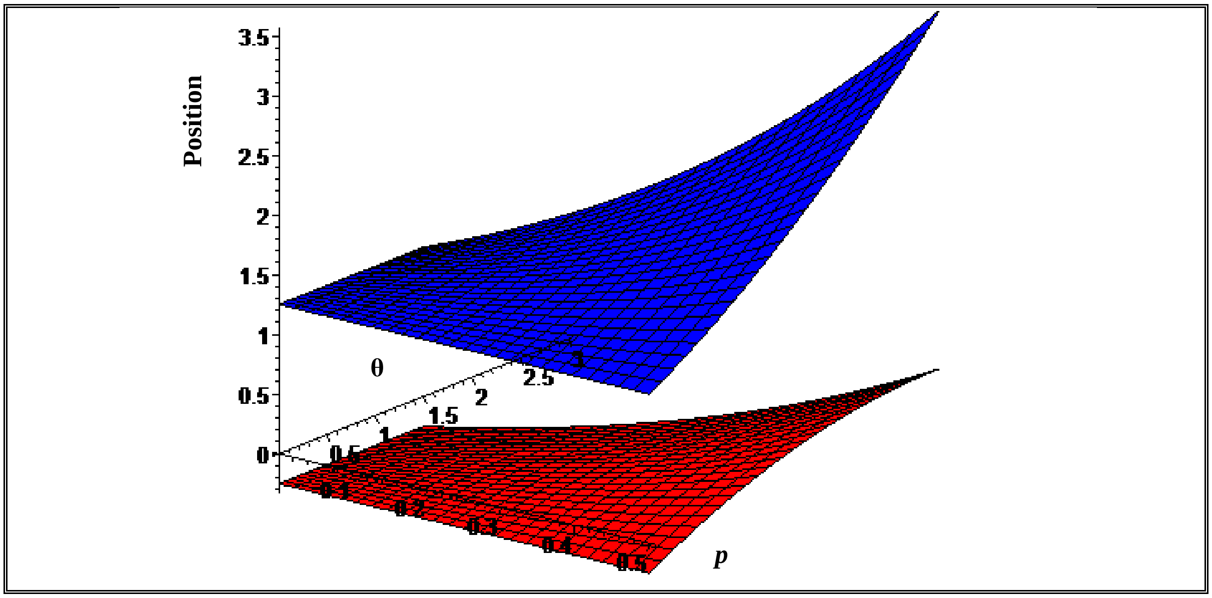
$$a^*=-\frac{1}{4}+E(k)-\frac{1}{3}\cdot Var(k)$$
$$b^*=\frac{5}{4}+E(k)+\frac{1}{3}\cdot Var(k) \quad (10)$$

Proof: Formulating the maximization problem for the firms' expected profits, seen in equation (4), leads to the system in (9). Solving this system of first order conditions leads to this solution, which satisfies the second order conditions that ensure that this equilibrium maximizes the expected profits of both firms. Q.E.D.

These positions can be seen graphically as follows for all relevant (p,θ) pairs,[11] where the convention is that firm 1 (position *a*) is in red and firm 2 (position *b*) is in blue:

[11] It can be shown that, for prices to remain positive and/or to avoid vertical differentiation, $\theta\le\frac{3}{2\sqrt{1-p}}$ must be true (assuming that $p\,\epsilon\,\left[0,\frac{1}{2}\right]$).

Figure 1: Firms' positions under simultaneous entry before solving locational uncertainty



Here, considering that the range for p leads the city's most probable location to be at $x = 0$ causes firm 1 to avoid moving from the [0,1] segment, which is related to the fact that the city's mean and variance act as opposing forces in determining firm 1's position. On the other hand, firm 2's position grows in a non-linear manner, which is related to the fact that the city's mean and variance both act as a differentiating force in determining firm 2's position. In the limit, when there is maximum uncertainty (for $[p, \theta] = \left[\frac{1}{2}, 3\right]$), the resulting city will be attended by only one firm. Hence, it is interesting to see in Proposition 3 that the value of entering immediately rises (i.e. the relative value to postponing entry until the city is revealed to both firms falls) when locational uncertainty rises, which is expressed solely in terms of the city location's variance, not depending whatsoever on the expected location of the city's customers. Nevertheless, this assumes necessarily that both firms enter before the city's uncertainty is resolved.

Proposition 3: The value of both firms under mutual preemption is given by the following expression:

$$\pi_1 = \pi_2 = \frac{4}{27} \cdot (Var(k))^2 + \frac{2}{3} \cdot Var(k) + \frac{3}{4} \tag{11}$$

Proof: The results obtained from the firms' maximization problem – the optimal prices for each resulting city position in (8) and the optimal firm positions in (10) – are replaced in the firms' objective function in (9), which correspond to the expressions inside the derivatives. Q.E.D.

When analyzing this result, it becomes clear that the expected value of the firms rise when inserting themselves before the city's position is known. This is due to the results found in the product differentiation literature, which has determined that locational uncertainty is a differentiation force. As Meagher and Zauner (2004) put it: under locational uncertainty "the move away from the competitor has the positive effect of weakened price competition (…) while the negative effect of losing market share will not be as dramatic, since the firm captures a greater market share when realizations of the uncertainty place it closer to consumers". Hence, restricting firms to a joint entry strategy will lead them to mutual preemption, since it is a strictly dominating strategy over waiting for the city location to be revealed to the firms.

However, there is a third entry pattern to consider, where firms enter sequentially. In this case, it is assumed without a loss in generality that firm 1 enters before the city's location is revealed, while firm 2 waits, because the resulting equilibrium is the same when permuting the firms' order of entry. Therefore, firm 2 determines its position knowing both the city's location and firm 1's position, as shown:

$$\frac{d}{db}\Big(P_b^*(a,b,k)\cdot\big(k+1-x(P_a^*(a,b,k),P_b^*(a,b,k),a,b)\big)\Big)=0\ \ \forall\ (a,k) \tag{12}$$

Using the solution to this first order condition, which delivers a function $b^*(a,k)$, firm 1 chooses its optimal position, knowing that its decision depends on both the locational uncertainty and its influence upon its competitor's position. The problem the leader solves is:

$$\begin{aligned}\frac{d}{da}\big((1-p)\cdot P_a^*(a,b^*(a,0),0)\cdot(x(P_a^*(a,b^*(a,0),0),P_b^*(a,b^*(a,0),0),a,b^*(a,0))-0)\\ +(p)\cdot P_a^*(a,b^*(a,\theta),\theta)\cdot(x(P_a^*(a,b^*(a,\theta),\theta),P_b^*(a,b^*(a,\theta),\theta),a,b^*(a,\theta))-\theta)\big)=0\end{aligned} \tag{13}$$

Solving this equation leads to another non-linear function in terms of the expected location of the city's customers $E(k)$ and the displacement factor's variance $Var(k)$, as described in Proposition 4.

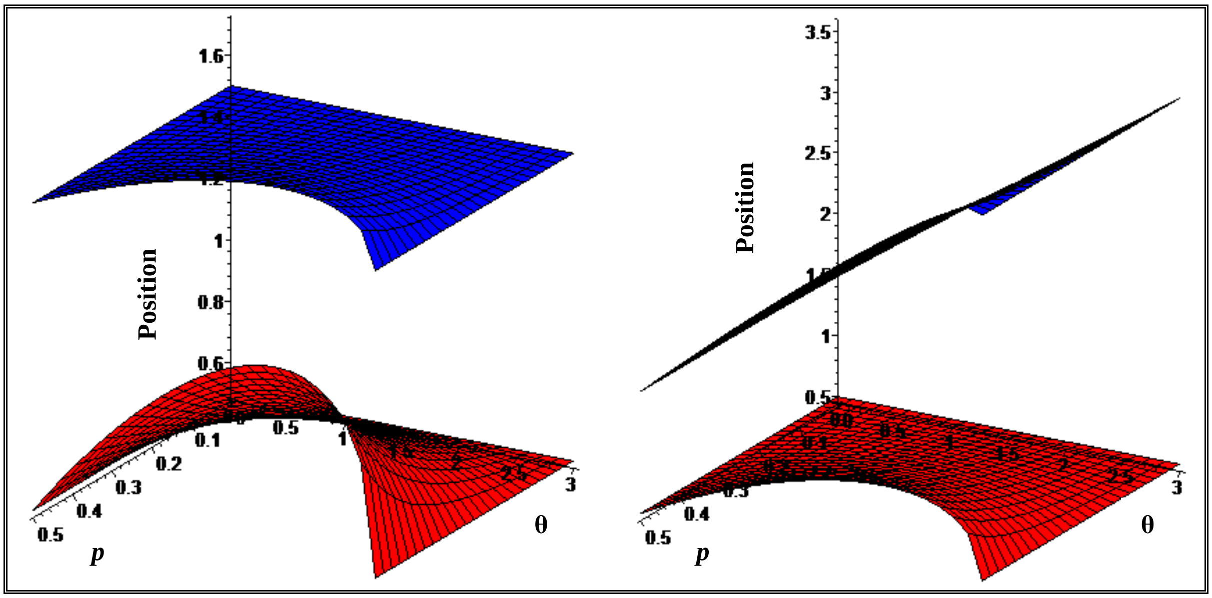
Proposition 4: The equilibrium locations that arise when firms enter sequentially into the linear city under locational uncertainty are defined by:

$$\begin{aligned}b^*(a,k)&=\frac{1}{3}\cdot(a+2k+4)\\ a^*&=E(k)-1+\frac{\sqrt{9-4\cdot Var(k)}}{2}\end{aligned} \tag{14}$$

Proof: Following the method of backward induction, the maximization of profits is first solved for firm 2 as shown in equation (5), which leads to the first order condition shown in equation (12). Solving this condition leads to the optimal position of firm 2, as shown in the first line of equation (14). Using this result, firm 1 maximizes its expected profits as shown in equation (6), which leads to the first order condition shown in equation (13). Solving this condition for firm 1's position leads to the result seen in the second line of equation (14). Both solutions satisfy their respective second order conditions that ensure they maximize each firm's objective function. Q.E.D.

It is important to mention at this point that, while firm 1 (in red) has a unique location, firm 2 (in blue) has the flexibility to vary its location according to the city's position. For each of the two possible outcomes in the city's location, the respective NE in positions is shown in Figure 2. Here, the left figure considers when the city is located at 0, while the right one considers when the city is located at θ.

Figure 2: Firm's locations under sequential entry for possible outcomes of locational uncertainty



In this case, firm 1, which acts as the market leader, intends to take advantage of this position under all possible outcomes. For small displacements (in terms of θ), it can focus on gaining a Stackelberg advantage in the [0,1] segment, knowing that it will also be useful in the $[\theta, \theta + 1]$ segment. For medium displacements, this firm sacrifices the ideal position in the [0,1] segment (at $x = 0.5$) to gain an advantage if the city locates itself at the $[\theta, \theta + 1]$ segment. Finally, for large displacements, the leader realizes that it cannot gain the upper hand in all scenarios and is sacrifices the least probable position (at θ) to gain the best position at the most probable city location, which is why at $\theta = 3$ it locates itself at $a = 0.5$ for all possible values of p (for higher values of θ, vertical differentiation occurs and is beyond the scope of this paper).

When calculating the expected value of the leader and follower, a complex expression in terms of $E(k)$ and $Var(k)$ is obtained, as seen in Proposition 5.

Proposition 5: The expected value of both firms under sequential entry is given by:

$$\begin{aligned} \pi_1 &= C + \frac{4}{27}\sqrt{9 - 4 \cdot Var(k)} + \frac{4}{9} \\ \pi_2 &= C - \frac{26}{27}\sqrt{9 - 4 \cdot Var(k)} + \frac{28}{9} \\ C &= \frac{16}{243} \cdot Var(k) \cdot \left(\theta - 2 \cdot E(k) - \sqrt{9 - 4 \cdot Var(k)}\right) \end{aligned} \tag{15}$$

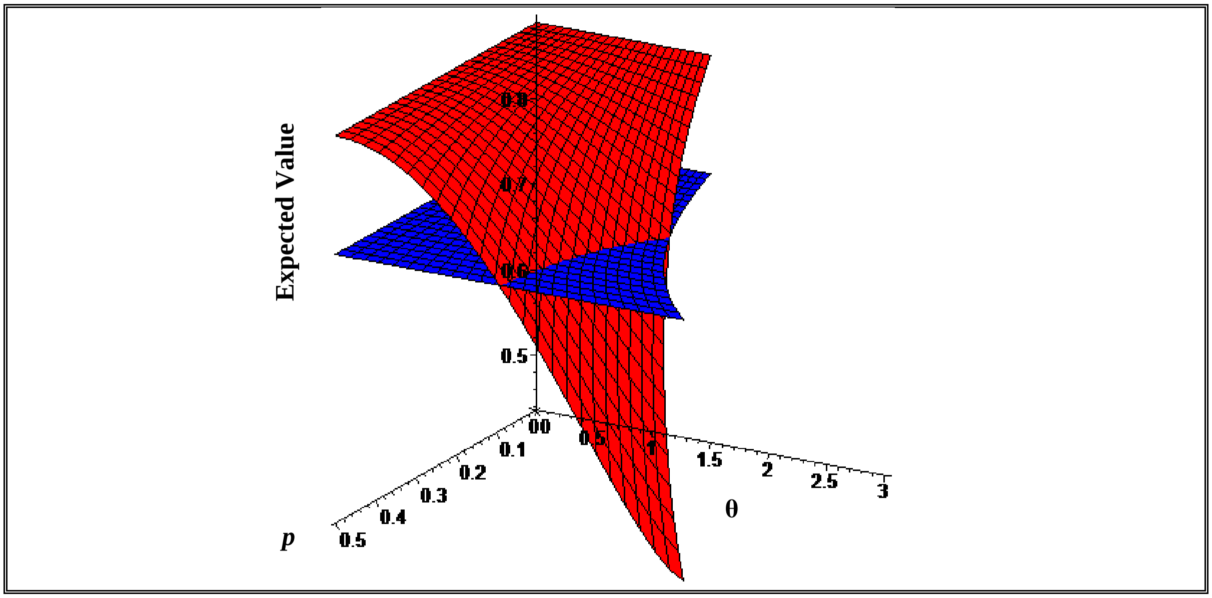
Proof: The results obtained from the firms' maximization problem – the optimal prices for each resulting city position in (8) and the optimal firm positions in (14) – are replaced in the firms' objective function in (12) and (13) for the follower and leader, respectively. These expressions correspond to the expressions inside the derivatives. Q.E.D.

From equation (15), it is clear that there is an asymmetry between the firms' profit functions, due to the exogenous entry pattern assumed here. However, differing from the Tabuchi and Thisse (1995) logic, it is not always worthwhile to be the Stackelberg leader in this game, because the additional profits of being the leader (given by the expression $\pi_1 - \pi_2$) is decreasing in the city location's variance $Var(k)$, which leads to two distinct regions. First, there is a region of "high" uncertainty, given by the inequality $Var(k) > \frac{81}{100}$, where the leader has a lower expected value than the follower. This occurs due to the fact that the leader, for high displacements of the linear city, prefers to sacrifice these gains and to locate itself at the middle of the [0,1] segment. On the other hand, the follower has the flexibility for choosing a privileged position if the city is located in the $[\theta, \theta + 1]$ or being a Stackelberg follower in the [0,1] segment. Therefore, the follower would have a higher expected value in this case. Secondly, if $Var(k) \leq \frac{81}{100}$, the level of uncertainty is "low" enough for the leader to gain from both city scenarios.

Once all the entry pattern equilibriums are calculated separately, it is possible to compare them to derive the endogenous entry pattern which must arise by calculating the reaction functions of each firm. These functions should be symmetrical, considering that both firms have no differences *ex-ante* before deciding which entry pattern to follow.

Assuming that a firm's competitor defers entry, the firm's value when entering immediately (in red) and waiting (in blue) is shown in Figure 3. Here, only the most stringent restriction for the (p, θ) space was considered, which corresponds to the restriction $\theta < \frac{3}{2 \cdot \sqrt{1-p}}$ under simultaneous entry before solving the location uncertainty.

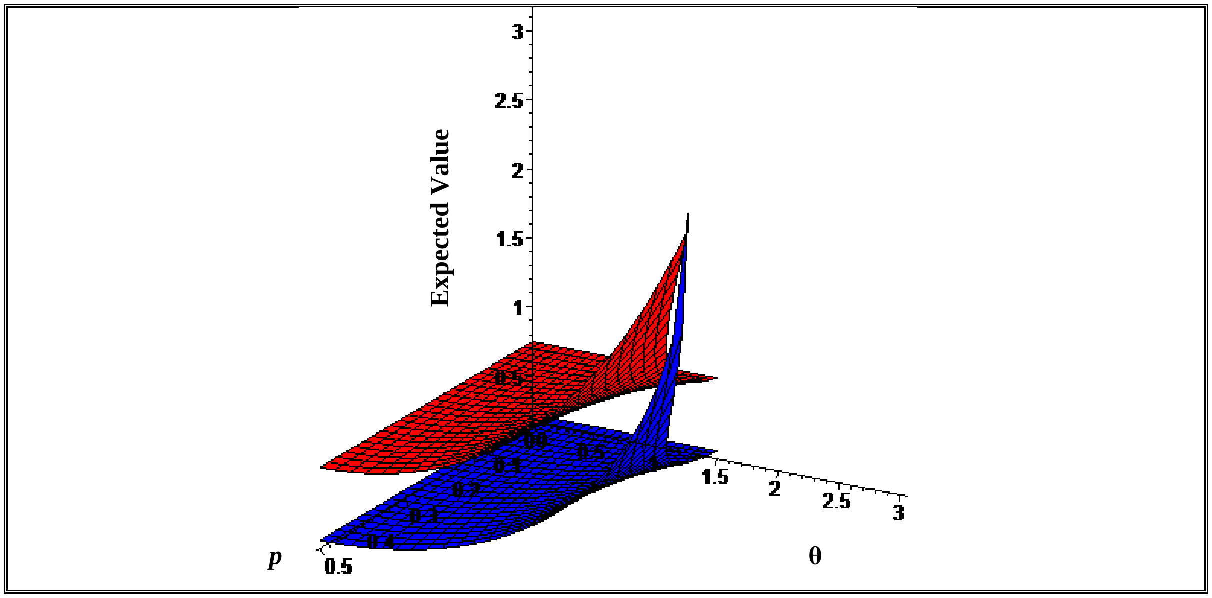
Figure 3: Firms' expected value for preempting and waiting, assuming that the other firm waits under locational uncertainty



It is clear that there is a boundary for "high" (p, θ) pairs where it is worthwhile to wait as well rather than acting as a Stackelberg leader. This boundary is not readily solved analytically, but can easily be determined using numerical methods. This result, although seen previously in the literature, is new, considering that there is no investment or production cost whatsoever that the leader commits when entering under uncertainty. It is rather the sacrificed profits that the leader ceases to gain when the city's location ends up at θ what moves him to wait until the uncertainty is resolved. Hence, for high levels of locational uncertainty, a SPNE where both firms defer investment is possible. On the other hand, when uncertainty is low, it can easily be proved that the firm can gain at most an extra 18% in value by entering early while its competitor waits.

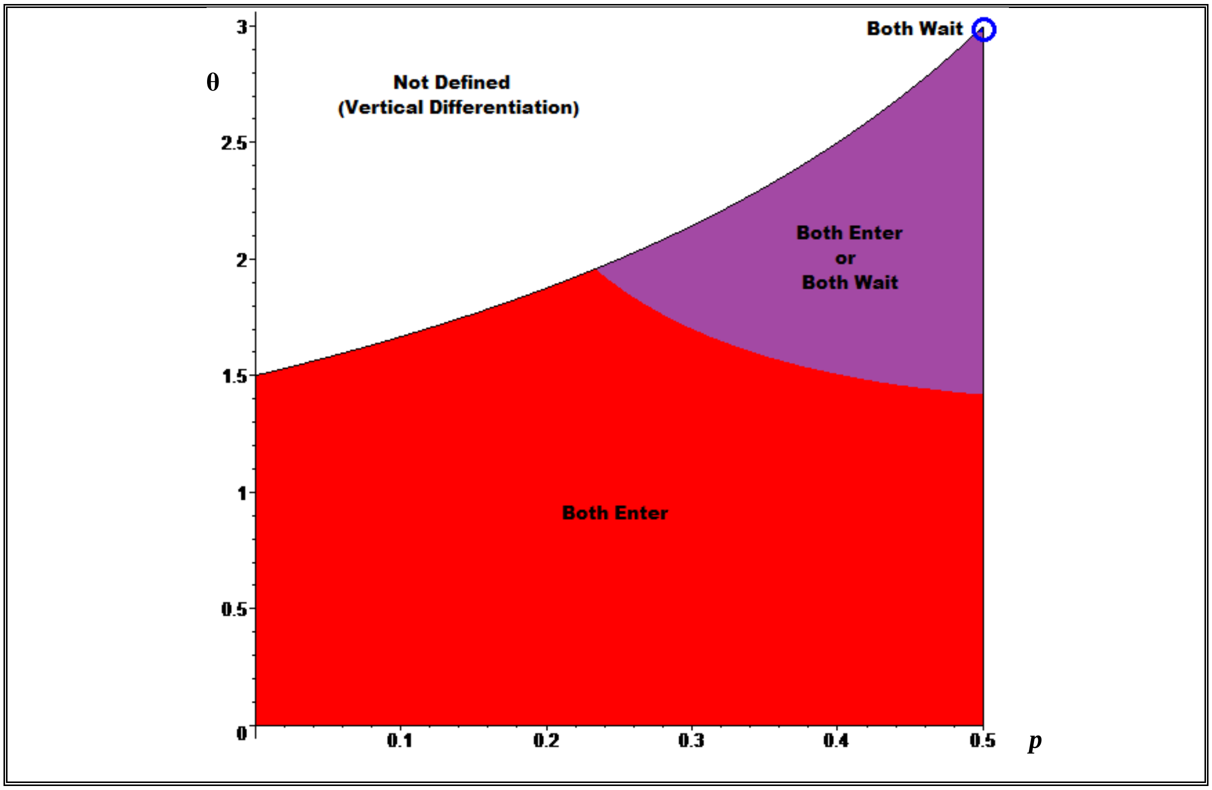
Assuming now that the firm's opponent enters before the uncertainty is resolved; Figure 4 shows that, unless under extreme conditions of uncertainty, the firm will emulate its opponent and enter as well (in red), due to the higher expected value that comes from increased differentiation combined with locational uncertainty, which increments the strategic effect for sure and the demand effect for both firms with a positive probability (either p or $1 - p$). Hence for most of the (p, θ) space, simultaneous entry before the uncertainty is solved is also a SPNE.

Figure 4: Firms' expected value when preempting and waiting, assuming that the other firm preempts under locational uncertainty



In summary, locational uncertainty can sustain two different entry patterns. First, the equilibrium where both firms enter before the city's location is clear can be sustained for practically all relevant (p, θ) pairs, except under extreme uncertainty (given by the blue circle in Figure 5), which would lead to vertical differentiation. This, as said before, is in line with current research of product differentiation under uncertainty. However, after a certain threshold, an equilibrium where both firms wait can also be obtained, which is more in line with the Option Games literature, despite the fact that there are no investment or production costs. Hence, although locational uncertainty can increase a firm's profits by enhancing the strategic effect, if the magnitude or probability of the city's displacement is high enough, both firms can also wait as well. This result demonstrates that introducing locational uncertainty into Hotelling's model does not necessarily lead to both firms preempting each other from entering. In this case, additional information is needed to determine which equilibrium will emerge for the region in purple, as shown in Figure 5.

Figure 5: Entry patterns for all feasible (p, θ) pairs under locational uncertainty



4. Benchmark comparison

The previous section described an option game model that, up to the author's knowledge, is the first serious attempt to successfully represent the tradeoff between preempting competitors and waiting for better information under a price competition context. However, as said before in Subsection 2.1., this has been done sacrificing the dynamic nature of competition in the Option Games literature, by making both duopolists compete only once for consumers in Hotelling's linear city.

To comprehend the results obtained in the previous section, two related models are presented. First, the differentiation model is adapted to represent a Cournot framework. Only in this way it is possible to compare the results with the Options Games logic, since this framework sacrifices the repeated market interaction with a greater range of exercise policies for the firms to choose from. This is why more than one entry pattern can arise as a SPNE, since firms can choose different location and price strategies according to the entry pattern at hand, while the Option Games literature necessarily assumes an exogenous rule.

Secondly, the price setting stage in the locational uncertainty model is removed to avoid short term competition. This leads firms to compete only in the option exercise stage. This exercise is done to determine how *ex-post* competition can affect the waiting option value and, therefore, the resulting entry pattern that emerges as a SPNE.

4.1.Nash-Cournot competition

Now, firms 1 and 2 choose capacities q_1 and q_2 to cover market demand, which is linear and has the following functional form:

$$D(\tilde{a}, q_1, q_2) = \tilde{a} - (q_1 + q_2) \tag{16}$$

Where $\tilde{a}$ is the demand intercept that is related to the market size, which is a random variable and is modeled according to a Bernoulli distribution, just like the locational uncertainty case: demand can be "low" (normalized at 1) with probability $(1 - p)$ or "high" (parameterized to δ greater than 1) with probability p.[12] Both firms hold the option of waiting to invest in capacity until the demand curve becomes known and, to maintain the similarities between both models, the investment and production costs are assumed to be nil. Hence, each firm must make a trade-off between investing right away to gain a first mover advantage, although demand remains uncertain; or to postpone investment until demand becomes known, to avoid flooding the market with excess production. This is due to the assumption that firms are faced with the irreversible commitment of producing at full capacity.

To summarize the game's structure and how it changes compared to the previous game, Table 4 shows the game's stages in chronological order with the strategic variables available to the firms at each point of the game. There are two key differences compared to the previous game: (1) firms compete now in capacities instead of positions on the real axis, and (2) there is now no *ex-post* competition, since both firms charge a common price which is cleared in the market according to the firm's installed capacities.

Table 4: Summary of the Entry Option Game into a Cournot Market with Demand Intercept Uncertainty

Stage	Description	Chosen Variables
1	Firms a and b decide whether to exercise entry option into Cournot market	$X_i = \{Entry, deferral\}\ \forall\ i = \{1,2\}$
2	Firms that selected entry decide their capacity.	$x_1 = q_1\ (\epsilon\ \mathbb{R}^+)$ or $x_2 = q_2\ (\epsilon\ \mathbb{R}^+)$ if and only if they entered in stage 1
3	Uncertainty over the demand size is cleared.	The intercept in the linear demand curve is determined (parameter $\tilde{a}$ in $P(Q) = \tilde{a} - Q$).
4	Firms that deferred entry exercise their entry option.	$x_1 = q_1\ (\epsilon\ \mathbb{R}^+)$ or $x_2 = q_2\ (\epsilon\ \mathbb{R}^+)$ if and only if they deferred in stage 1
5	Market price is cleared from demand curve and firms collect profits	None

To determine the entry pattern that rises as a SPNE in this game, the profit functions must be determined for all possible entry patterns. And, as in the previous subsection, the results

[12] Now, there is no symmetry between the possible outcomes of the demand intercept. Hence, the relevant range for p is from 0 to 1. On the other hand, it is possible to prove that, to avoid negative prices in the sequential entry pattern, δ must lie in the range $[1, (1 + p)/p]$.

derived here will be expressed in terms of the demand intercept's expected value ($E(\tilde{a}) = (1-p) \cdot 1 + p \cdot \delta$) and variance ($Var(\tilde{a}) = p \cdot (1-p) \cdot (\delta - 1)^2$) to understand the intuition on how demand uncertainty affects de SPNE in each case and to make the results comparable to the locational uncertainty model.

First, when both firms decide to wait until the uncertainty is resolved, the traditional Cournot game is reached, where both firms produce (for a general intercept a of the demand curve) $a/3$ to receive a profit of $a^2/9$. Hence, the profit function for this entry pattern is:

$$\pi_1 = \pi_2 = \frac{1}{9} \cdot \left(\left(E(\tilde{a}) \right)^2 + Var(\tilde{a}) \right) \tag{17}$$

Secondly, when both firms decide to invest before the demand curve is fully known, firms lose their flexibility to invest according to demand conditions and must maximize expected profits. Considering this change in the objective function, both firms play the traditional Cournot game using the expected value of the demand curve intercept $E(\tilde{a})/3$. Hence, the expected profits of both firms under this scenario are:

$$\pi_1 = \pi_2 = \frac{1}{9} \cdot \left(E(\tilde{a}) \right)^2 \tag{18}$$

When comparing both results, it is clear that the value of each firm is maximized when both firms wait instead of investing beforehand, since it is easy to prove that subtracting expression (18) from (17) leads to $Var(\tilde{a})/9$, which is positive. This comes to confirm that the firm's profits are a convex function – just like any option in general – with respect to total demand, since the expected value of profits under all possible demand levels is greater than the profits under the expected demand scenario. And, what is even more interesting, this differs from the previous model, where both firms preferred to enter before the location of city's customers is known.

Now, when a sequential investment pattern occurs, the leader emulates the traditional Stackelberg equilibrium under the expected market demand level, which is taken into account by the follower. The capacities chosen by each firm (assuming that firm 1 is the leader) in this case is shown in equation (19). Here, the follower's investment is a function of the demand level parameterized at a.

$$\begin{gathered} q_1 = \frac{1}{2} \cdot E(\tilde{a}) \\ q_2(a) = \frac{a}{2} - \frac{q_1}{2} = \frac{a}{2} - \frac{1}{4} \cdot E(\tilde{a}) \end{gathered} \tag{19}$$

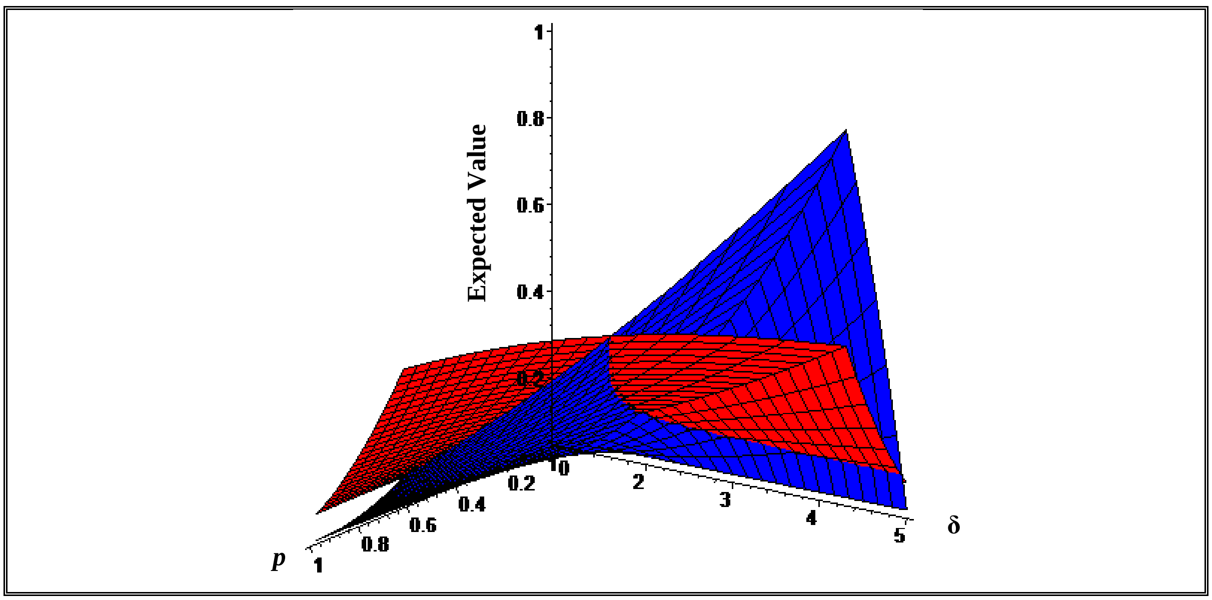
Under this investment equilibrium, there is an asymmetry between both firms' values, as shown in equation (20).

$$\pi_1 = \frac{1}{8} \cdot \left(E(\tilde{a})\right)^2$$
$$\pi_2 = \frac{1}{16} \cdot \left(E(\tilde{a})\right)^2 + \frac{1}{4} \cdot Var(\tilde{a}) \tag{20}$$

Here, the same phenomenon seen in the previous model also occurs, where if the uncertainty is high enough, the follower has a higher expected value than the leader. However, unlike the locational uncertainty model, here the additional leader's profits (given by $\pi_1 - \pi_2$) decreases linearly in terms of uncertainty's variance, instead of the square root of the variance as in Hotelling's model under uncertainty.

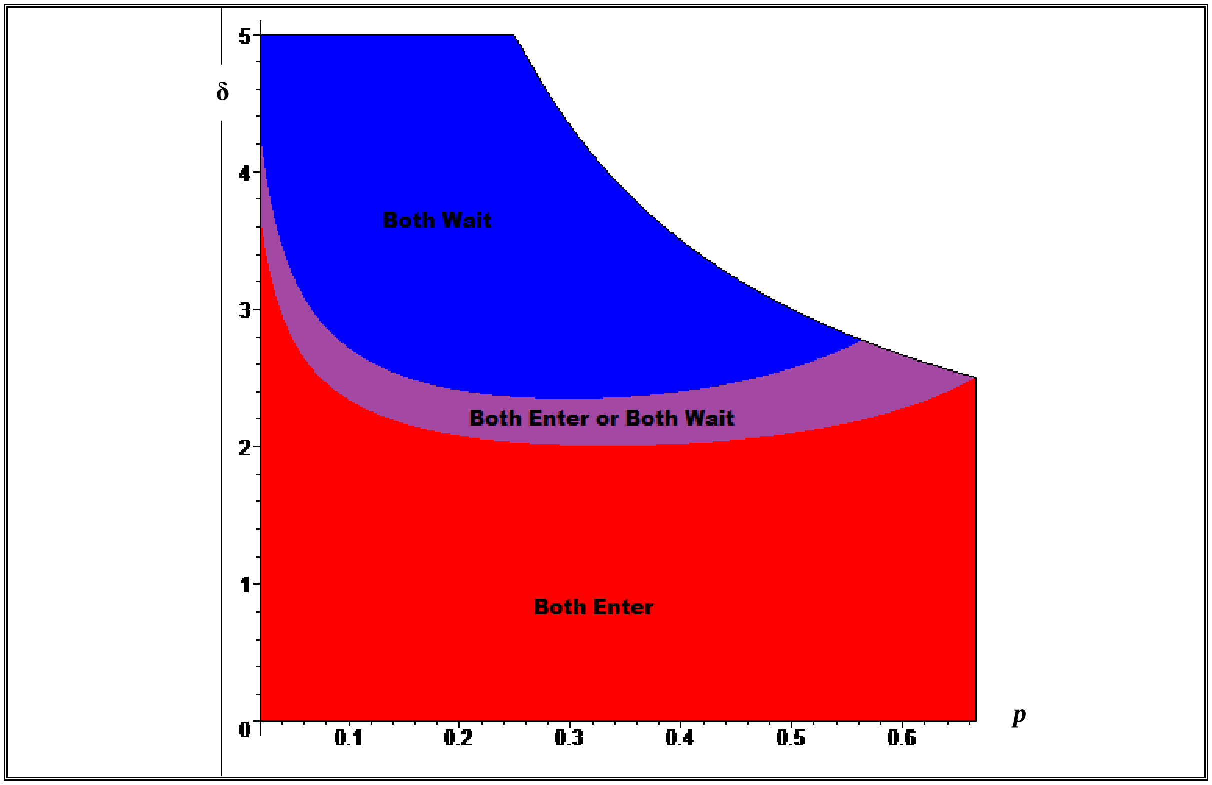
When plotting these results, as seen in Figure 6, it is possible to see that for the range $\delta \in \left[\frac{5 \cdot p \cdot (1-p) + 2 \cdot \sqrt{p \cdot (1-p)}}{p \cdot (4 - 5 \cdot p)}, \frac{1+p}{p}\right]$ for each p, the follower (in blue) has a higher expected value than the Stackelberg leader (in red), even though there are no investment or production costs. This is due to the flexibility that the follower has to adapt to the less likely event of a large market. The upside of this unlikely event is, in this case, large enough to make the follower's expected value higher than the leader's.

Figure 6: Expected Value functions for leader and follower under sequential entry in a Cournot framework



Now that all the value functions for each entry pattern are calculated, these are compared to determine the entry pattern that arises as SPNE for all feasible (p, δ) pairs – that avoid negative prices under all entry pattern equilibriums. Doing this leads to three distinct regions, as seen in Figure 7. It is possible to prove that, for values of δ in the red region, which corresponds to the inequality $\delta \leq \frac{9 \cdot p \cdot (1-p) + 2\sqrt{2 \cdot p \cdot (1-p)}}{p \cdot (8 - 9 \cdot p)}$, the dominant equilibrium pattern is for both firms to enter simultaneously before the uncertainty is resolved. And, when $p > \frac{2}{3}$, this is the sole SPNE entry pattern that can be sustained, because the high demand state is likely enough for the leader to preempt the follower's entry.[13] On the other hand, for high values of δ, satisfying the inequality $\delta \geq \frac{43\cdot p\cdot(1-p)+6\sqrt{7\cdot p\cdot(1-p)}}{p\cdot(36-43\cdot p)}$ and valid for all $p \leq \frac{9}{16}$, the dominant equilibrium pattern is for both firms to simultaneously wait until the uncertainty is cleared. Here, both firms prefer to wait just in case the high demand state is reached, instead of gaining moderate additional profits from preempting in a low demand state. Finally, the region between both curves allows both types of equilibrium to occur.

Figure 7: Entry patterns for relevant (p, δ) pairs under demand intercept uncertainty in a Cournot framework



In summary, the freedom given to the firms to determine their optimal capacity size alongside their entry strategy also leads to sustain two separate SPNE patterns. However, unlike the previous model, there is a significant region where the only SPNE entry pattern that arises is for both firms to wait, as seen in the previous figure. This occurs because preemptive action of a market leader is limited here, as introducing extra capacity does not lead to differentiate a firm's production from its competitor's production. Hence, the follower is capable of adding production without regretting that it will not be consumed, since the market always clears out all production

[13] Remember that maximum uncertainty is given, for any value of δ, at $p = 0.5$.

by both firms. On the other hand, the flexibility gained by waiting in this case ensures a greater reward if the market's potential size is large and firms believe that this event is not likely to occur. The potential earnings a follower can gain in the large, but unlikely, market compensate the limited gains it will have in the more likely event of a smaller market, leading the follower to have a higher expected value than the leader in this case: it is more worthwhile to be a Stackelberg follower in a Cournot market instead of Hotelling's city with price competition.

4.2.Locational competition

To analyze the role of price competiton in this setting, we alter the locational uncertainty model seen in subsection 3.3 by eliminating the price stage. This leaves the payoff functions of both firms dependent only on the firms' positions, where they attempt to maximize their market share. This type of model – without locational uncertainty – has been extensively used in political science to model the rationale behind elections.

Considering the case without uncertainty, the well known "Median Voter Paradox" derived by Black (1948) and Downs (1957) is obtained, where both firms (also known as candidates in a political setting) locate themselves at the position of the median voter to capture half of the customers (also known as the electorate in a political setting) each, reaching a tie.[14] This necessarily is the NE in positions, because neither candidate has incentives to deviate – because they prefer to have a chance in winning than a certain loss. Another way of viewing this result is to consider that eliminating the price competition stage leads to eliminating the strategic effect and both firms now have no incentives to separate themselves. This result is crucial, because maximizing market share limits the payoff a firm can achieve: in a political setting: candidates cannot expect to achieve more votes than the universe of voters.

When both candidates enter before knowing the location of the electorate, the essential result that the expected outcome is a tie must be maintained. Nevertheless, there are now infinite NE in positions that can be sustained. Assuming that under both outcomes (with the city beginning at zero or at θ) both candidates capture part of the electorate, it is possible to calculate each candidates' position as follows. First, both candidates expected support must be equal, as seen in equation (21):[15]

$$(1-p)\cdot(x-0)+p\cdot(x-\theta)=(1-p)\cdot(1-x)+p\cdot(\theta+1-x)$$
$$\Rightarrow x=\frac{1}{2}\cdot(1+2\cdot\theta\cdot p) \qquad (21)$$

On the other hand, since there is no price stage, the indifferent voter must locate itself at the midpoint between both candidates. Clearing the right hand side as a function of the second firm's position, it is possible to express the infinite NE in parametric form, as seen in equation (22).

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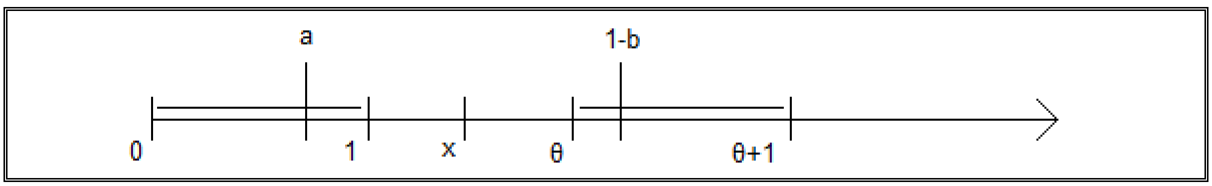
14 For an intuitive demonstration of this result, refer to Anderson *et al*. (1992).

15 The result in equation (21) can also be reached by assuming that either player's expected amount of voters is $\frac{1}{2}$.

$$x = \frac{1}{2} \cdot (a + 1 - b)$$
$$\Rightarrow 1 - b^* = 2 \cdot x - a^* = 1 + 2 \cdot \theta \cdot p - a^* \quad \forall\, a^* \epsilon \left[0, \frac{1}{2} \cdot (1 + 2 \cdot \theta \cdot p)\right] \tag{22}$$

This result is graphically shown in Figure 8. This solution comes to show that, without the possibility of both firms to compete in prices after exercising their entry option leads to no extra gain when compared to waiting until after the locational uncertainty is cleared out. This makes sense, considering that the firm's strategic variables allow them to capture market share in a city of limited size: they do not have the tools to alter the payment received by each "customer" – each voter can deliver (at most) their vote. However, as a side note, it is extremely valuable for voting theory literature to see that introducing locational uncertainty into Down's model can lead politicians to reach any desired amount of differentiation without violating this author's assumptions regarding the expected outcome of the election: both candidates are expected to tie.

Figure 8: SPNE visualization for locational uncertainty model without price competition under simultaneous entry



Using this same type of logic can lead to prove that, under a sequential entry pattern, it will never be beneficial for the leader to enter first. The NE in locations in this case is for the leader to locate himself at the median voter of the most likely location of the electorate, while the follower has the flexibility to choose the same location as the leader in the locational uncertainty's most likely outcome (reaching a tie) or to choose a privileged location in the less likely case, thereby achieving an expected value of over half the electorate.

Hence, the action of waiting until the locational uncertainty is cleared is the (weakly) dominant strategy for both firms for all (p, θ) pairs, because firms have no incentive whatsoever to preempt their competitor. This is radically different than the locational uncertainty model with price competition, where both firms preempt in equilibrium, and demonstrates that eliminating *ex-post* competition through pricing policies converts firms from innovative agents seeking first-move opportunities into politicians, who seek to reveal their stances on many issues until only after the election has been carried out.

5. Analysis

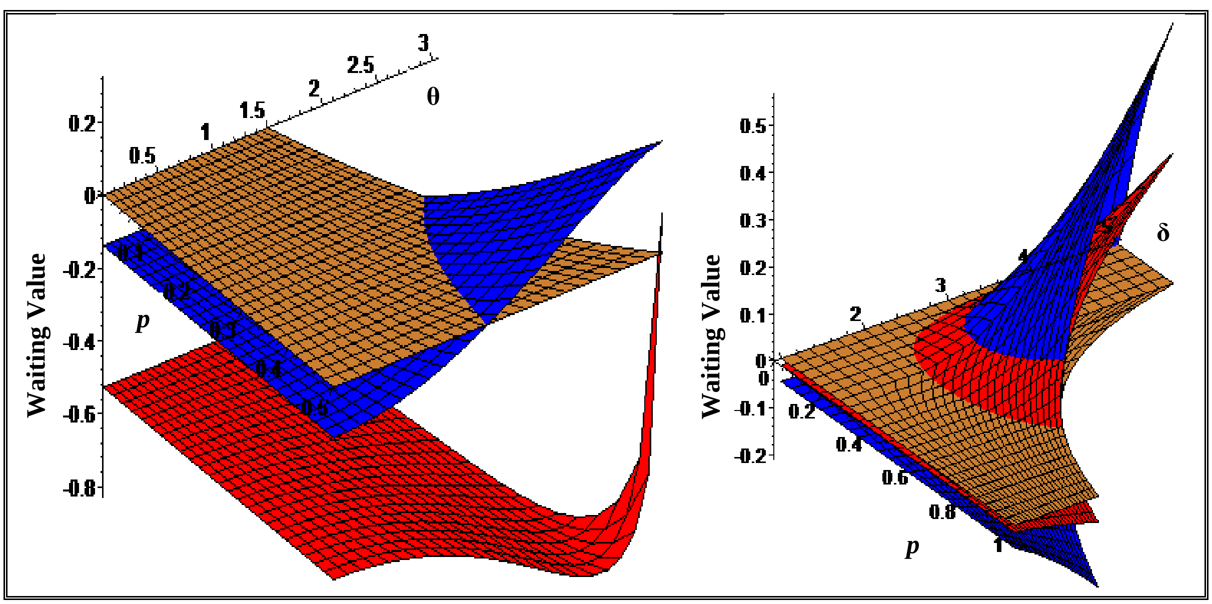
When comparing the results from the models in Chapters 3 and 4, a series of differences from the Option Games literature arise that are worth considering in greater detail.

Why are there multiple equilibriums?

Both the price competition model with locational uncertainty and the adapted Cournot model have regions where both mutual waiting and mutual preemption are possible. This phenomenon has not been seen up to date in the Option Games literature. Past papers, such as Pawlina and Kort (2006), have focused on determining how the entry pattern that rises as a SPNE varies when introducing asymmetries in the firms' investment costs and/or cash flows in models such as Grenadier's (1996).

The main difference when comparing these models is that, while Grenadier's or Pawlina and Kort's models consider an exogenous cash flow stream and must only determine the optimal entry trigger, the models seen here have to determine both the optimal entry trigger and another variable, such as capacity or position on the real axis. This is what leads to multiple SPNE in this case, although they would be identified solely as a joint entry equilibrium under Pawlina and Kort's perspective. To prove this point, Figure 9 displays the value to wait until the uncertainty is cleared for both the locational uncertainty model (left) and the adapted Cournot model (right). The different curves represent whether the firm's competitor preempts (in red) or waits as well (in blue), and are calculated by subtracting the firm's value when it waits with respect to the case when it enters (for each of the competitor's entry patterns mentioned before). The gold curve is the barrier where the alternative to wait becomes positive.

Figure 9: Value of waiting in Hotelling's linear city and in a Cournot market under uncertainty



This graph must be interpreted as follows: when the waiting value falls below zero, it is worthwhile for the firm to enter immediately, since it is not an obligation for the firm to wait if it is less profitable (in expected value) than entering; the *waiting option* in this case is worth zero. On the other hand, when the waiting value is greater than zero, the firm will wait, since it delivers a higher expected value in this case; the waiting option in this case is worth the additional value delivered by waiting when compared to preempting its competitor. Taking this into account, it is possible to determine the SPNE entry pattern from this graph: if the waiting value is greater than zero for both curves, a mutual waiting equilibrium will occur; if both curves are negative, a mutual preemption equilibrium will occur; and if the preemption curve (in red) is negative and the waiting curve (in blue) is positive, both equilibriums are possible. In this case, it is said that the compelling equilibrium is the one that has a higher absolute value, since both firms can expect to gain more from this equilibrium.

It is interesting to mention at this point that no sequential equilibrium can occur in this model, because both firms are symmetrical. Therefore, if a firm has incentives to preempt or to wait, necessarily its competitor will have the same incentives and a symmetric equilibrium must necessarily occur. This limitation exists because both models have only one stage of competition for consumers; hence, a sequential entry equilibrium does not give the follower the chance to gain the upper hand later on, unlike other typical option games seen in the literature. Nevertheless, this does not mean that the sequential entry equilibrium does not play a role here, because the threat of this equilibrium is what leads both entry patterns to be possible if the uncertainty at hand is “high” enough. Take, for example, Hotelling’s linear city under locational uncertainty (left), where for a configuration of “high” uncertainty – such as the point $(p, \theta) = (0.5, 2.5)$ – both firms would prefer to preempt. However, if one of the duopolists gives any prior signals that it will wait (or has an internal restriction to enter) its competitor no longer has any incentives to preempt, because the follower will have the flexibility of adapting to the city’s outcomes. In contrast, if an exogenous rule (such as Grenadier’s assumption of a symmetrical equilibrium) is forced in this case, mutual preemption will be the only SPNE.

Hence, the models presented here recognize that the waiting option’s value depends not only on the uncertainty at hand, but on the entry strategy that firms follow. This somewhat obvious fact has been forgotten in recent papers within the Option Games literature, which has focused on showing that a particular entry strategy can arise as a SPNE, instead of determining whether it is the best SPNE that can be reached.

Prices versus capacities in an Option Games context

From Figure 9, it is possible to see that the value of waiting to gather more information about demand is quite different for each of these models. Nevertheless, as a word of caution, no conclusions can be extrapolated in terms of comparing the values of both graphs, since these are different games and the waiting option’s values are not directly comparable.

In Hotelling’s linear city under locational uncertainty, it is extremely odd to see that, for a significant portion of all valid (p, θ) pairs – to imitate a competitor’s strategy proves to increase the firm's expected value as uncertainty increases, either in terms of p or θ (or both). The mutual preemption equilibrium makes sense, because this uncertainty acts as a differentiation force, as emphasized in Subsection 3.3. However, the waiting option also increments its value, even though the value of mutual waiting *does not change*, limiting the upside a firm can gain under this equilibrium. In this case, a prisoner's dilemma occurs, because – even though mutual preemption is the more compelling equilibrium – knowing that its competitor will wait does not give the leader the capacity to preempt under high levels of uncertainty, and forces him to wait as well to avoid the downside of being a leader who could not correctly anticipate where the city would be. Hence, the waiting option has value because of the strategic interaction between firms, instead of the uncertainty *per se*.

In contrast, for the Cournot model the value of waiting rises with the level of uncertainty,[16] no matter what its competitor does. This can be seen in Figure 9, as the waiting curves rise in value. This demonstrates that the Cournot leader has fewer arguments to preempt followers. However, unlike the previous model, in this case the waiting option's value is not maximized at $p = 0.5$. It can easily be proved that the probability where the waiting option is maximized is a monotonically increasing function of the high state of demand δ and can reach, at most, a value of $p = \frac{4}{9}$ when $\delta \to \infty$. Hence, unlike the Option Pricing literature, the waiting option does not monotonically increase its value with the probability of displacement. Instead, this option attains its greatest value when there is a high demand potential that is relatively unlikely to occur; in that way, the follower can take advantage that the leader will not accommodate its strategy to this unlikely event and gain monopolistic profits in this case, which surpass the sacrificed earnings of becoming a Stackelberg follower in the more likely event of a smaller market. In this case, the uncertainty in the demand intercept is what explains the waiting option's value, not the strategic interaction.

Finally, on a different note, it is worthwhile to emphasize that monetary costs are not a necessary condition for a waiting option to have value. In both games, there are neither investment costs nor production costs; however, if there were no threat of preemption, firms would wait to reap monopolistic rents once the demand uncertainty (either in terms of the city's location or the demand curve's intercept) is dissipated. Nevertheless, there still must be a commitment of some sort implied in the waiting option: in Hotelling's linear city, the firm's positions cannot be changed; in the adapted Cournot model, the firm's must produce at capacity. This is a necessary condition for the option to have value.

Hotelling's city under locational uncertainty: an Option Games anomaly?

Up to now, there has been no questioning in the product differentiation literature of the fact that uncertainty increments the expected value of firms when they enter *ex-ante*. Unlike the Option Pricing literature, they preach that uncertainty can increase the value of preemption

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[16] Remember once more that the valid range of p goes from 0 to 1, but the maximum level of uncertainty is reached at $p = 0.5$ (maintaining δ constant).

instead of waiting. However, this has not been seen from an Option Games perspective up to date.

Actually, this setting implicitly assumes that firms have an embedded option that improves their profits no matter what the final outcome of the city is: product prices. This stage in the game precisely leads the firms to achieve a convex profit function in Hotelling's linear city under locational uncertainty, by limiting the downside when the city's location end up being far away from their own and incrementing the upside they can reach when the uncertainty favors them.

Taking this stage away is what alters the entry pattern that emerges as a SPNE completely, as seen in Subsection 3.3. Hence, the role of short-term competition in prices also serves to diminish the value of waiting options. However, this is not done by altering the position stage, but by introducing flexibility *ex-post*, which ends up mitigating the irreversibility of selecting a position before knowing the city's location.

Reflecting once more on the Smartphone Industry

Revising the surge in smartphone sales under this new light illustrates some interesting facts. It is important to remember first that smartphones are anything but a homogeneous product. This is especially true when considering Apple's iPhone, which has consistently captured around 15% of world smartphone sales belonging to the "high end" of the market, according to Wall Street Journal (WSJ) correspondent Andrew Dowell (2010). This is due to non-price features, such as its sleek design, it high quality photos and audio capabilities. Hence, this producer in particular has been capable of sustaining a competitive advantage of higher quality and rivals have not been able to question its predominance among the higher end of the market. This lack of (potential) competition up to date may come to explain the fact that Apple waited over 7 years before determining it was time to launch its product, according to Dolan (2006). It is precisely these rules that escape the framework described before that can lead to a equilibrium where a firm (or firms) wait until the market is ripe enough to launch a product even when it is not the compelling SPNE.[17]

Secondly, Dowell (2010) recognizes that the latest battle has been over the market share of the lower end of smartphone consumers. And, unlike Apple's example, WSJ correspondent Christopher Lawton (2010) argues that late competitors such as Research in Motion (Blackberry manufacturers), Nokia and Motorola are the ones rushing to grasp market share by launching a "price war in midrange smartphones this holiday season".[18] The question in this case is: why attack this segment only 3 years after the iPhone's entry in 2007?

This answer is related to one of the newer trends that have risen during the last year: the rise of "Apps" in the smartphone business. Dowell (2010) defines these as "cheap" and "silly"

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[17] Or, from an Option Pricing perspective, Apple's iPhone can be seen as a McDonald and Siegel (1986) type model where only the optimal timing to invest is to be determined, since no competitors can preempt them of this business opportunity.

[18] This Wall Street Journal article was written on Christmas Eve in 2010.

computer programs that have turned phones into "game rooms, barcode scanners and photo manipulators". And, as Table 2 shows, the OSs that have best supported these features (Google's Android and Apple's iPhone) are the ones that have conquered 2010 sales, capturing around 70%.

Hence, the question raised in the previous paragraph can be answered plausibly using the locational uncertainty model defined in Subsection 3.3. The lagging firms, having lost their edge towards the iPhone, dared not to venture into other segments considering that the future trends of the industry were highly uncertain, leading into a mutual waiting equilibrium in years 2008 and 2009.[19] Then, with the rise of Apps, the uncertainty in the position of the lower end of the market was partially dissipated, as these firms now knew what low end customers were expecting: affordable phones with internet connectivity to download applications. This change in market conditions hassled firms to compete in this segment – according to Lawton (2010) – leading to a mutual preemption equilibrium in Hotelling's city (with a "small" amount of locational uncertainty) during 2010. This has led to the subsequent price war observed up to date; because the market actors' entries occurred only after the city's characteristics became relatively clear. This has finally caused "a dilution in manufacturer's profits because of their lower margins", according to this same source.

6. Conclusions

Economists have come a long way in understanding the roles that competition and uncertainty play when analyzing firm behavior in markets. Option Pricing has first identified that the traditional NPV criterion rule (invest when NPV > 0) is not an optimal strategy under uncertainty. The recent stream of Option Games has then introduced imperfect competition into Option Pricing models to see how firms balance the flexibility given by options with the commitment needed in oligopolistic markets to gain a first mover advantage. Nevertheless, up to date current research has focused primarily in Cournot markets for homogeneous goods, leaving price competition aside to be analyzed by IO.

This paper is one of the first – up to the author's knowledge – to tackle the Option Games predicament of commitment versus flexibility for price competition, although it models this tradeoff using Hotelling's linear city under quadratic transport costs, a framework that leans towards the product differentiation literature in IO.

In this paper, mutual preemption proves to be the most compelling SPNE in Hotelling's linear city under locational uncertainty. Nevertheless, this equilibrium (which is not unique) and the option value to wait is directly dependant on the strategies of the firm's competitors. This fact, although obvious to game theorists, has been left aside in the Option Games literature, because an option exercise strategy is devised first and is subsequently proven as a SPNE. Here, on the other hand, no restrictions were imposed on determining the optimal position and prices

[19] Even though it is not the compelling NE, other elements – primarily investment costs – gave foot towards this entry pattern.

that firms play under each entry pattern. This additional freedom in determining the optimal exercise rule comes at a cost, since this game has sacrificed the continuous time dynamics and restricted itself to a discrete six stage game.

However, the stability of the mutual preemption equilibrium is also due to the nature of price competition, since the price stage carried out after the firms have located themselves on the real axis acts an option *per se,* by letting the firms take advantage when the city is near and to minimize the downside when the city end up far from their location. Taking the price competition stage away from the game leads firms to act as politicians, who have no incentives to preempt entry because they are expected to lose the election in this case. In this same line, when adapting the game's structure to model Cournot competition, the sole SPNE (independent of the competitor's strategy) is for both firms to wait if the uncertainty is "high" enough, because this setting also has no flexibility *ex-post* for preemptors to adapt their strategy once they build capacity.

Finally, it is important to mention that this framework was not built solely as a theoretical exercise, but to help explain actual market behavior. Despite its simplicity, it is capable of giving a plausible explanation of the recent events which have led to the smartphone boom among lower-end users in the mobile phone market. However, this model is just the first step to explain this type of phenomenon. To close the gap between the product differentiation and Option Games literature, new improvements must be introduced, either through introducing repeated interaction of the firms with the customers in a continuous-time framework or by including additional improvements in the field of Product Differentiation, such as multiple players or other types of uncertainty in Hotelling's linear city: the possibilities of tending bridges between these fields are boundless.

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