Irreversible Commitment and Price Negotiation under Knightian Uncertainty: A Real Options Perspective

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Abstract

This paper tackles the problem of irreversible investment and price negotiation under Knightian uncertainty using the real options lens. We present a multiple-priors based formulation of utility in continuous-time that permits a distinction between risk and uncertainty in decision-making to study the impact of vagueness/ambiguity on bilateral price negotiation and investment. Specifically, we examine negotiation dynamics between a buyer and a seller in a dual options context (i.e., call for buyer and put for seller) to 1) derive thresholds for optimal commitment, 2) identify conditions under which mutual agreement is warranted (with and without bargaining power), and 3) estimate likelihood of agreement, all in an environment fraught with deep economic uncertainty. Besides generalizing risk uncertainty results found in previous research, our findings highlight the moderating effect of negotiators' perceived ambiguity (i.e., pessimism and optimism) on the process of negotiation and its related outcomes and provide insights into the formulation of robust optimal (buying/selling) strategies for negotiation under high uncertainty.

1 Introduction

Price negotiation is a fundamental and necessary part of doing business. Because of the many repercussions price and supply dynamics have across buyers and sellers' value chains, it is not surprising that risk and uncertainty affect/determine negotiation outcomes. This is more so for large businesses where agents' reservation profits need to be met in the context of complex and highly unpredictable supply chains. The influence of demand and supply side uncertainty coupled with irreversibility gives decision-makers little information about the consequences of price negotiation, making (ex-post) commitment and potential agreement difficult and risky. Added to this is the role played by ambiguity/vagueness in the negotiation process with buyers and sellers having their own perceptions of what a suitable price should be for each party, resulting in outcomes from mutual agreement to be even more uncertain. With incomplete information, risk, ambiguity and irreversibility characterizing the process of negotiation, buyers and sellers find themselves outweighing pros and cons of potential agreement under double sided uncertainty not knowing with confidence opponents' moves, influence and economic expectations. Faced with such a deep uncertainty, how should managers formulate their negotiation strategies and decide on when to buy (or sell) specific products or services in the presence of negotiation ambiguity? This paper addresses this problem using the real options lens.

Real option theory has offered a valuable theoretical framework for understanding decision-making under uncertainty. By charting options as a series of decision points under possible events, managers can understand the risks and rewards of decision-making, and more fully assess their opportunities. The real options logic is based on the modern financial options pricing theory of Black and Scholes (1973) and Merton (1973). Financial options give holders rights, without the obligation, to buy or sell underlying assets at pre-specified prices (i.e., strikes) on or before given expiry dates. Specific to the valuation and appraisal of real investment opportunities is the concept of real options or options written on/in real assets. The real options approach was developed and formalized by Tourinho (1979), McDonald and Siegel (1986), Trigeorgis (1988), Dixit and Pindyck (1994) and Trigeorgis (1996). In contrast to traditional views that managerial discretion is limited in the face of uncertainty or that organizational inertia dominates, real options theory maintains that firms can engage with uncertainty and benefit from it by exercising options to respond to uncertain futures.

There is an increasing interest in using real options to understand decision-making in supply chains. This can be explained by the various sources of uncertainty surrounding firms' operations and the need for supply chain flexibility in today's business environments (Cucchiella and Gastaldi, 2006). Thus, research on real options and supply chain management has been conducted by a number of authors such as Li and Kouvelis (1999), Kamrad and Siddique (2004), Alvarez and Stenbacka (2007), Tsai (2008), Hult et al. (2010) and Jiang et al. (2010). With respect to real options and negotiation in supply chains, Fotopoulos and Munson (2008) investigate supply contracts' design in an environment of risk uncertain prices. Jiang et al. (2010) utilize options pricing theory to study vendor firms' behavior during outsourcing arrangements. More in relation to our paper, is the work of Moon et al. (2011) who develop a bilateral negotiation model to derive optimal selling (buying) rules under risk uncertainty, propose the idea of an Implicit Zone of Possible Agreement (IZOPA) between buyers and sellers, and discuss the probability of negotiation agreement using the real options lens. We extend this specific research by revisiting the problem of price negotiation and its real options dynamics under a dimension of uncertainty that goes beyond risk. Specifically, we study the impact of vagueness/ambiguity or deep uncertainty on bilateral price negotiation and buyers/sellers' option exercise decisions.

The standard practice of real option analysis consists of laying out a vision of future events precise enough to be captured in a probability distribution (risk uncertainty), while assuming that agents have perfect confidence in their probability judgments. Of course, that approach serves companies well in relatively stable or risky business environments. But when there is greater uncertainty about the future, it is at best marginally helpful. Underestimating uncertainty can lead to decisions that neither protect against threats nor take advantage of opportunities (Courtney, 1997). In reality, heightened concerns about uncertainty make agents generally not confident about the likelihoods of specific events. This ambiguity tends to affect decision-making judgment and as a result alters decision outcomes.

When a buyer or seller determines when to negotiate a supply contract, the presence of vagueness in probability judgments can be critical. The decision to buy or sell a product or service incurs sunk costs and is at least partly irreversible. Revenues and costs are always uncertain as they are affected by many risk factors, and hence are difficult to forecast accurately. This dimension of uncertainty, characterized by not a single probability measure but a set of probability measures for prediction, is frequently referred to as ambiguity or Knightian uncertainty in economics and decision theory (Nishimura and Ozaki, 2007).

Therefore when studying the problem of price negotiation, the standard real options models under risk uncertainty need further extension and development. Hence, here we adopt the multiple-priors utility model (e.g., Gilboa and Schmeidler, 1989) to analyze the dynamics of price negotiation under uncertainty and highlight the impact of ambiguity on the call and put option exercise decisions of buyers and providers. For consistency we refer to the standard approach of decision-making under normal uncertainty as the risk uncertainty case, while the more general case of ambiguity is referred to as decision-making under ambiguity or Knightian uncertainty[1] .

This distinction between risk uncertainty and Knightian uncertainty was first highlighted by Knight (1921) and Keynes (1937) and has been further explored by Ellsberg (1961) and Bewley (1986). The Ellsberg Paradox demonstrates that people prefer to bet on events with known probability (risk) prospects rather than events with unknown probability outcomes (ambiguity). Ellsberg-type behavior contradicts the Bayesian paradigm or the existence of a single probability measure underlying choices (Basili, 2006). In the series of papers referring to ambiguity and its decision theoretic properties, the Choquet expected utility theory by Schmeidler (1989) and the multiple-priors utility in a min-max way by Gilboa and Schmeidler (1989) are the most prominent. Worst-case robust appraisal has been widely recognized as the standard attitude towards uncertainty in economics, financial markets and engineering. However, since the maxmin criterion only captures pessimistic attitudes towards ambiguity, it leads to very conservative choices. Ambiguity loving features should also be considered in decision-making and prospects analysis. This viewpoint is demonstrated by Heath and Tversky (1991) and Kilka and Weber (2001). Therefore, when assessing choices and their consequences under uncertainty, it is more realistic to model the full set of probability distributions and ambiguity attitudes of decision-makers.

In this paper, we rely on the α-maxmin expected utility - which is a generalization of the “maxmin” model of Gilboa & Schmeidler (1989) - proposed by Marinacci (2002), Olszewski (2007) and Schröder (2011), in order to model preferences that display ambiguity aversion and ambiguity loving attitudes towards incomplete information. This specification provides a natural way of broadening the spectrum of agents’ behavioral traits (i.e. ambiguity attitude) towards uncertainty and recommends evaluating an act by taking a

[1]The case of fundamental uncertainty with infinite variance or complete ignorance is not considered here.

convex combination (with weight α) of the utility of its worst possible result and of the utility of its best possible outcome (Ghirardato et al., 2008). Given the potential dollar value of price negotiation in supply chains, understanding vagueness in probability judgments and studying the influence of agents' uncertainty preferences during negotiation can be of significant theoretical and practical importance.

Thus, we present a formulation of utility in continuous-time that permits a distinction between risk and uncertainty, ambiguity loving/seeking and ambiguity aversion attitudes, study the impact of ambiguity on bilateral price negotiation, and present conditions under which it is optimal to buy/sell or commit to a given product or service. This paper accounts for the ambiguity surrounding the probability measures related to seller cost and buyer revenue and the attitudes of decision-makers towards them in the presence of incomplete information.

The paper is organized as follows. In Section 2, we explain the model set-up for both buyer and seller, derive put and call option values and obtain thresholds for optimal option exercise under ambiguity/Knightian uncertainty. Section 3 defines an implicit zone of achievable agreement (IZOAA) and studies the condition for the existence of IZOAA and the impact of Knightian uncertainty on this condition. Section 4 presents an analysis for negotiation agreement probability under ambiguity/Knightian uncertainty. Section 5 extends these findings to a model incorporating each party's bargaining power under ambiguity/Knightian uncertainty and studies the condition for the existence of IZOAA with negotiation power and the impact of Knightian uncertainty on this condition. The final section concludes with a summary of findings and research implications. Proofs are covered in the Appendix sections.

2 The optimal buying time and selling time under (Knightian) uncertainty

Most purchases by institutions, government agencies, and commercial businesses are negotiated (Reeder, 1987). Negotiation is typically framed as a one buyer-one provider situation. Bilateral negotiations are important mechanisms to achieve distributed conflict resolution and to meet the common interest of the various parties. Studies related to the negotiation model in this paper mainly come from the following topics: negotiation range and cooperative bargaining game models and their applications to supply chain relationships.

From the literature on negotiation range, Walton and Mckersie (1965), Raiffa (1982), Sebenius (1992), define the range, "Zone of Possible Agreement" (ZOPA), which is a zone of reservation prices in a negotiation that will be acceptable to both parties. Fudenberg and Tirole (1983) examine the effects of changes in bargaining costs, the size of the "contract zone", and the length of the bargaining process on such aspects of the solution as the probability of impasse and the likelihood of concessions. Moon et al. (2011) present an Implicit Zone of Possible Agreement (IPZOA) under risk uncertainty with and without negotiation power and study the negotiation agreement probability using real options. Our paper examines these issues in a Knightian uncertain environment. Our approach to the bilateral price negotiation problem here differs from the analysis under risk uncertainty of Moon et al. (2011) in that we incorporate the uncertainty preference of negotiators and ambiguity in probability distributions to analyze the price negotiation problem and its commitment dynamics.

Game theoretic models of bargaining have generally been classified as either cooperative ("axiomatic") or non-cooperative ("strategic"). In cooperative bargaining games, the parties have a shared interest, whereas in noncooperative bargaining games (e.g., Rubinstein, 1982; Chattejee and Samuelson, 1987) they have distinctly opposing interests. Nash (1950, 1953) laid down the framework for the axiomatic Nash bargain solution[2]. In this paper, we focus on cooperative bargaining games and their application to supply chain management. Examples of studies in this area include Dukes and Gal-Or (2003) on the optimal design of exclusive advertising contracts, Gurnani and Shi (2006) on dealing with supplier (un)reliability under asymmetric information and Nagarajan and Sosic (2008) on coalition formation in supply chains. More in relation to our study, Mieghem (1999) develops a game-theoretic investment model considering the role of transfer prices and bargaining power in supply chain capacity contracts. Bernstein and Marx (2006) examine bargaining over the wholesale price within supply chains using the Nash bargaining model. Moon et al. (2011) study supply contract negotiation when buyer revenue and seller cost are risk uncertain and obtain the optimal buying and selling strategies incorporating negotiation powers. We cover the specific issue of bargaining power under ambiguity in Section 5 of this contribution.

In this paper, a buyer and a seller bargain over a product/service and negotiate at the same time. The price of the product or service might not be defined completely yet. The buyer has a call option which gives them the option to buy the underlying asset before a given date at a given price. The seller has a put option which gives the seller the right to sell the underlying asset before a given date at a given price. The buyer and the provider trade at a contract price X, which is assumed to be constant over time. The contract price X connects the buyer and the seller when constructing an IZOAA where call and put prospects intersect.

2.1 The price negotiation problem under Knightian uncertainty in continuous time

The conceptual framework underlying our proposed model is based upon the IZOPA risk uncertainty framework of Moon et al. (2011). We extend the authors' findings to the Knightian uncertainty case using multiple-priors and the α-maxmin expected utility. This is a valuable way of capturing the vagueness in judging probability distributions of uncertain factors and the ambiguity attitudes of negotiators, and can yield robust buying/selling strategies for negotiation under scenarios of deep uncertainty.

Geometric Brownian motions are frequently used to model prices as well as costs. Let S_1 and S_2 denote the cost and the revenue for a seller and a buyer. The ambiguity in seller cost (S_1) and buyer revenue

[2] Further analysis of cooperative bargaining games can be found in the works of, among others, Roth (1979) and Muthoo (1996).

(S_2) are introduced through a set of geometric Brownian motions. The set of probability measures $\mathcal{P}_i$ are expanded from the objective measure P_i by the set of density generators Θ_i, where $i=1$ and 2 denote the seller's cost and the buyer's revenue, respectively.

Then ambiguity in the seller's cost and the buyer's revenue are modeled by a set of probability distributions $\mathcal{P}_i=\{Q_i^{\theta_i} \mid \theta_i=(\theta_i(t))\in\Theta_i\}$, where $Q_i^{\theta_i}$ is derived from P_i (see the detailed definition in Nishimura and Ozaki (2007)). Moreover, we assume that $\forall\theta_i\in\Theta_i$ are restricted to the non-stochastic range $K_i=[-\kappa_i,\kappa_i]$, where κ_i ($\kappa_i\geq 0$) is the ambiguity/ignorance level in probability distributions, which is a constant given by some objective information and used to limit the scope of the density generators. This specification of ambiguity in continuous-time is called κ_i-ignorance by Chen and Epstein (2002) in a different context.

Since $dB_i^{\theta_i}=dB_i+\theta_i dt$ by Girsanov's theorem (Duffie, 2001, p.111, p. 337), we have for any $\theta_i\in\Theta_i$ the Ito process of S_1 and S_2 to the general set $\mathcal{P}_1$ and $\mathcal{P}_2$ yields under ambiguity:

$$dS_i(t)=(\mu_i-\sigma_i\theta_i)S_i(t)dt+\sigma_i S_i(t)dB_t^{\theta_i} \qquad \left(\forall t\geq 0,\forall\theta_i\in\Theta_i, i=1,2\right) \tag{1}$$

where $\mu_{iK}=\mu_i-\sigma_i\theta_i$, μ_{iK} represents the expected growth rate of S_i, σ_i is the volatility of S_i. Parameters μ_i and σ_i are assumed to be constant over time. $dB_t^{\theta_i}$ is a standard Brownian motion with respect to $Q_i^{\theta_i}$ by Girsanov's theorem, $dB_t^{\theta}=dB_t+\theta_t dt$, $E(dB_t^{\theta_i})=0$, $E[dB_1^{\theta_1},dB_2^{\theta_2}]=\varepsilon_{12}dt$. θ_i affects only the drift term, and not the volatility term.

The utilities of seller cost and buyer revenue is calculated by considering ambiguity preferences of negotiators and the level of ambiguity in probability distributions by applying the α-maxmin utility framework described in Marinacci (2002), Olszewski (2007) and Schroder (2011). The α-maxmin utility extends the multiple-priors utility in a min-max way by Gilboa and Schmeidler (1989) to include ambiguity loving/seeking features and separate the level of ambiguity in probabilities, and the specific ambiguity attitude captured by the individual parameter α_i. Then the expected present discounted value of $S_i(t)$ with respect to $Q_i^{\theta_i}$ from time t to time T is defined by the α-maxmin expected value $W_i(S_i(t))$:

$$W_i(S_i(t))=\alpha_i \sup_{Q_i^{\theta_i}\in P_i} E_t^{Q_i^{\theta_i}}[g_i(S_i)]+(1-\alpha_i)\inf_{Q_i^{\theta_i}\in P_i} E_t^{Q_i^{\theta_i}}[g_i(S_i)] \tag{2}$$

where $g_1(S_1)=\int_t^T S_1(\tau)e^{-r(\tau-t)}d\tau$, $g_2(S_2)=\int_t^T S_2(\tau)e^{-r(\tau-t)}d\tau$. For the seller, α_1 denotes pessimism with respect to seller cost reflecting the weight attributed to the worst case (the supremum of cost). When setting $\alpha_1=1$, α_1-expected value coincides with those under the maxmin preference of Gilboa and Schmeidler (1989) or the case of pure pessimism. For the buyer, α_2 is defined as the perceived optimism level for the buyer's revenue which reflects the weight attributed to the best case. $\alpha_2 = 1$ denotes the case of complete ambiguity loving or optimism (the supremum of revenue). The parameters α_1 and α_2 consider the tradeoff between pessimism and optimism and reflect the levels of ambiguity aversion and ambiguity loving of decision-makers.

Let the ambiguity level in probability distribution $K_i = [-\kappa_i, \kappa_i]$, The (multiple prior) α-maxmin expected seller cost and buyer revenue can be defined by

$$W_i(S_i(t)) = \lambda_i[1 - e^{-(r-\mu_i-\sigma_i\kappa_i)(T-t)}]S_i(t) \tag{3}$$

Where $\lambda_i = \dfrac{\alpha_i}{r-\mu_i-\sigma_i\kappa_i} + \dfrac{1-\alpha_i}{r-\mu_i+\sigma_i\kappa_i}$, $\kappa_i \geq 0$, λ_i is the ambiguity multiplier of $S_i(t)$, which connects the value of S_i at time t and expected present discounted value of $S_i(t)$ in the α-maxmin expectation framework $W_i(S_i(t))$ over some period of time, $\lambda_i \in R$. r is the discount rate, $\mu < r$, $\mu_i + \kappa_i\sigma_i < r$, otherwise if t is big enough, waiting longer is always a better strategy and the optimal solution does not exist.

We assume T approaches infinity to obtain close form solutions. Though this assumption has several limitations in practice, it can still be considered reasonable for long-term contracts or in situations where parties involved have exclusive and continuous relationships.

Consequently, under κ_i-ignorance, we obtain the α-maxmin expected value of $S_i(t)$:

$$W_i(S_i(t)) = \lambda_i S_i(t) \tag{4}$$

Equation (4) shows that the expected discounted values of $S_i(t)$ in the α-maxmin utility framework are determined, not only by the ambiguity multiplier of $S_i(t)$ that decision-makers themselves forecast and set, but also by the value of S_i at the starting time t of the supply contract.

For any $\theta_i \in \Theta_i$, since λ_i is constant over time, substituting $S_i(t) = W_i(S_i(t))/\lambda_i$, we have

$$dW_i(t) = (\mu_i - \sigma_i\theta_i)W_i(t)dt + \sigma_i W_i(t)dB_t^{\theta_i} \tag{5}$$

Where $i = 1$ and 2 denote the seller's cost and the buyer's revenue, respectively. We use $W_i(t)$ to denote $W_i(S_i(t))$ for convenience.

The optimal investment rule for a seller and a buyer is determined by solving the following stochastic optimal stopping problem, where the selling and buying opportunity values under Knightian uncertainty are denoted by $F_1(W_1)$ and $F_2(W_2)$.

$$F_1(W_1)=\max_t E_t^{Q_1^{\theta_1}}\left\{X-W_1(t)\right\} \tag{6}$$

$$F_2(W_2)=\max_t E_t^{Q_2^{\theta_2}}\left\{W_2(t)-X\right\} \tag{7}$$

Where X is the total contract price of the commodity (or service), which is negotiated based on the α-maxmin expected value of $S_1(t)$ and $S_2(t)$ under Knightian uncertainty.

Using Ito's Lemma (see Dixit and Pindyck, 1994, P80-82), we obtain

$$dF_i(W_i)=\frac{\partial F_i}{\partial t}dt+F_i'dW_i(t)+\frac{1}{2}F_i''\left(dW_i(t)\right)^2 \tag{8}$$

Where $F_i'=\partial F_i(W_i)/\partial W_i(t)$, $F_i''=\partial^2 F_i(W_i)/\partial W_i^2(t)$.

Since terms in $(dt)^2$ and $dB_t dt$ go to zero faster than dt, we have $\left(dW_i(t)\right)^2=\sigma_i^2\left(W_i(t)\right)^2 dt$. Since $\theta_i\in\Theta_i$ is restricted to the non-stochastic range $K_i=[-\kappa_i,\kappa_i]$, where κ_i ($\kappa_i\geq 0$). Then $E[\theta_i]=0$. Noting $E(dB_t)=0$, we obtain the expectation of $dF_i(W_i)$

$$E[dF_i(W_i)]=\mu_i F_i' W_i(t)dt+\frac{1}{2}F_i''\sigma_i^2\left(W_i(t)\right)^2 dt \tag{9}$$

Equation (9) expresses the equilibrium condition of the decision to postpone buying or selling, expressing seller and buyer willingness to hold their options.

In the waiting region the Bellman equation is (see e.g. Dixit and Pindyck, 1994, P140):

$$rF_i dt=E[dF_i(W_i)] \quad \text{for} \quad i=1,2 \tag{10}$$

Equation (10) says that over a time interval dt, the total expected return on the opportunity, $rF_i dt$, is equal to its expected rate of capital appreciation.

Then we have the Bellman equation (after dividing by dt)

$$\frac{1}{2}F_i''\sigma_i^2\left(W_i(t)\right)^2+\mu_i F_i' W_i(t)-rF_i(W_i)=0 \tag{11}$$

Equation (11) means that the real options value of delaying decision making should satisfy the condition: the expected future gain should be equal to the normal return $rF_i(W_i)$ to prevent any arbitrage profits from occurring.

2.2 The optimal selling time under Knightian uncertainty

The put option value under Knightian uncertainty $F_1(W_1)$ must satisfy the following boundary, value-matching and smooth-pasting conditions:

$$\lim_{W_1\to\infty}F_1(W_1)=0 \tag{12}$$

$$F_1(W_1^*)=X-W_1^* \tag{13}$$

$$F_1'(W_1^*) = -1 \tag{14}$$

Equation (12) reflects the fact that the put option value will be zero if the expectation of seller's cost W_1 is very large. Equation (13) is the value-matching condition such that at the moment the put option is exercised, its payoff is equal to the net present value of the selling decision. Equation (14) is the smooth-pasting condition such that the optimal selling trigger is the one that maximizes the value of the put option.

To find $F_1(W_1)$, we solve Equation (11) subject to the boundary conditions (12)-(14). Since $W_1(t) = \lambda_1 S_1(t)$, we have the put option value under Knightian uncertainty

$$F_1(W_1) = \begin{cases} A_1 \lambda_1^{\beta_1} S_1^{\beta_1} & if \quad S_1 > S_1^* \\ X - \lambda_1 S_1 & if \quad S_1 \le S_1^* \end{cases} \tag{15}$$

Where $A_1 = -\left(\lambda_1 S_{1K}^*\right)^{1-\beta_1} \big/ \beta_1$, $\lambda_1 = \dfrac{\alpha_1}{r-\mu_1-\sigma_1\kappa_1} + \dfrac{1-\alpha_1}{r-\mu_1+\sigma_1\kappa_1}$, $S_1^* = \dfrac{\beta_1}{(\beta_1 - 1)\lambda_1} X$, $\beta_1 = \dfrac{1}{2} - \dfrac{\mu_1}{\sigma_1^2} - \sqrt{\left(\dfrac{1}{2} - \dfrac{\mu_1}{\sigma_1^2}\right)^2 + \dfrac{2r}{\sigma_1^2}} < 0$. A_1, β_1 and λ_1 are constants. S_1^* is the trigger value of the product or service when the option to sell is exercised, and the commodity is sold.

When $\kappa_1 = 0$, we can obtain the option value under risk uncertainty $F_{1R}(W_1)$ comparable to Moon et al. (2011).

$$F_{1R}(W_1) = \begin{cases} A_{1R} (\phi_1 S_1)^{\beta_1} & if \quad S_1 > S_{1R}^* \\ X - \phi_1 S_1 & if \quad S_1 \le S_{1R}^* \end{cases} \tag{16}$$

Where the subscript R denotes risk uncertainty. $S_{1R}^* = \dfrac{\beta_1}{(\beta_1 - 1)\phi_1} X$, $A_{1R} = -\dfrac{(S_{1R}^*)^{1-\beta_1}}{\beta_1}$, $\phi_1 = \dfrac{1}{r-\mu_1}$, $r - \mu_1 > 0$, $\beta_{1R} = \beta_1 = \dfrac{1}{2} - \dfrac{\mu_1}{\sigma_1^2} - \sqrt{\left(\dfrac{1}{2} - \dfrac{\mu_1}{\sigma_1^2}\right)^2 + \dfrac{2r}{\sigma_1^2}} < 0$. $A_1 = (\lambda_1)^{1-\beta_1} A_{1R}$. $S_1^* = \dfrac{\phi_1 S_{1R}^*}{\lambda_1}$

The timing of selling a commodity is viewed as an investment opportunity. It is a standard optimal stopping problem. Indeed, the opportunity value is low when the seller's cost is large and more than S_1^* so it is best to wait and postpone selling. This defines the so-called "waiting region." The waiting region of S_1 given α_1=0 and S_1 given α_1=0 and κ_1=0.2 are shown in Figures 1 (a) and (b) respectively. Conversely, the opportunity value is relatively high when the seller's cost is small so it is optimal to exercise the put option immediately; this defines the "stopping region."

A decrease in the pessimism level of the seller cost (α_1) increases the affordable critical trigger value of S_1^* as shown in Figure 1(a). The critical trigger value under risk uncertainty S_{1R}^* is lower than critical trigger value S_1^* when $\alpha_1=0$ and is higher than S_1^* when $\alpha_1=1$.

Let us now compare in Figure 1 (b) the effect of an increase in ambiguity level in probability distributions (κ_1) on the critical trigger value of the seller's cost (S_1^*). An increase in κ_1 has opposite impacts on the choices of very pessimistic and optimistic decision-makers. As κ_1 increases, very pessimistic decision-makers ($\alpha_1=1$) need to lower the critical trigger value of cost (S_1^*) at the start time of the option and make deferral more likely in order to avoid losses under further uncertainty, while optimistic decision-makers accept a higher critical trigger value of cost (S_1^*).

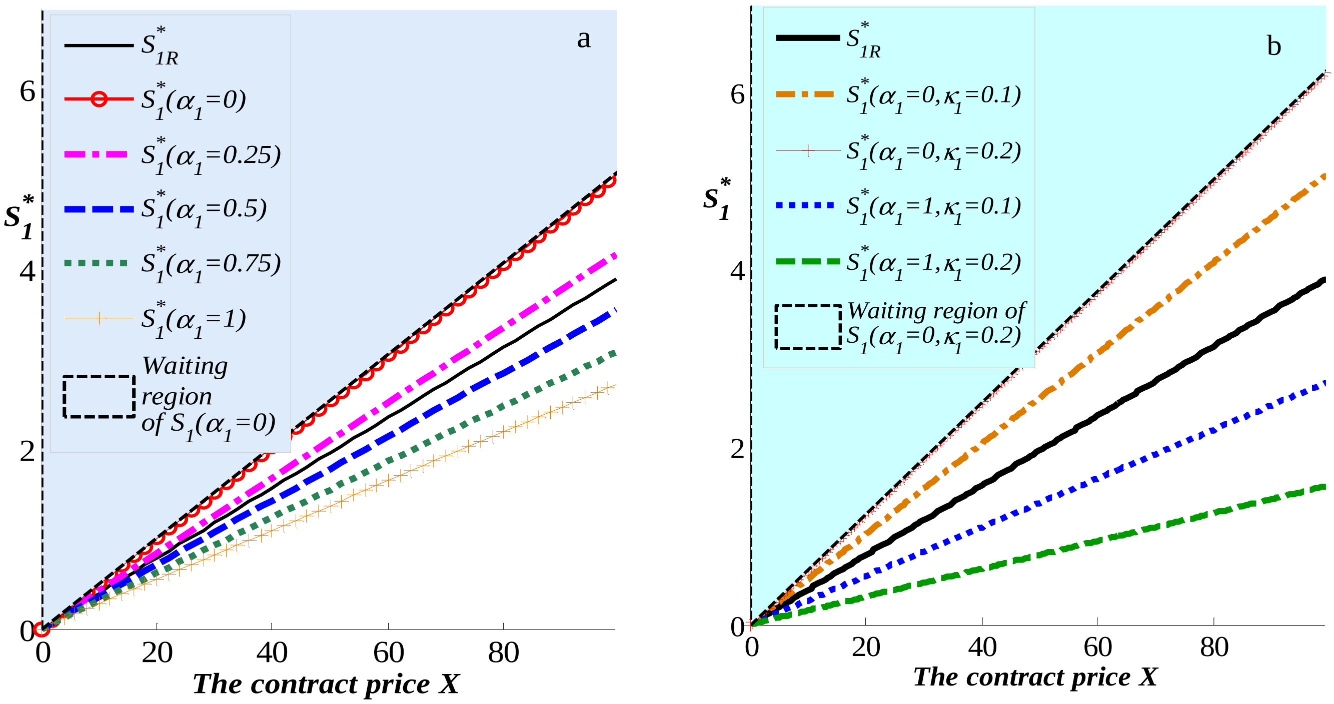

Figure 1 (a) the impacts of seller pessimism on the contract price and (b) the impacts of seller ambiguity level in probability distributions on the contract price. Here $r=0.08$, $\mu_1=0.03$, $\sigma_1=0.15$.

2.3 The optimal buying time under Knightian uncertainty

The call option value under Knightian uncertainty $F_2(W_2)$ must satisfy the following boundary, value-matching and smooth-pasting conditions:

$$F_2(0)=0 \tag{17}$$

$$F_2(W_2^*)=W_2^*-X \tag{18}$$

$$F_2'(W_2^*)=1 \tag{19}$$

To find $F_2(W_2)$, we solve Equation (11) subject to the boundary conditions (17)-(19). Since

$W_2(t)=\lambda_2 S_2(t)$, we have

$$F_2(W_2)=\begin{cases}A_2\lambda_2^{\beta_2}S_2^{\beta_2} & if \quad S_2<S_2^* \\ \lambda_2 S_2-X & if \quad S_2\geq S_2^*\end{cases} \tag{20}$$

Where $W_2(t)=\lambda_2 S_2(t)$, $A_2=(\lambda_2 S_{2K}^*)^{1-\beta_2}/\beta_2$, $\lambda_2=\dfrac{\alpha_2}{r-\mu_2-\sigma_2\kappa_2}+\dfrac{1-\alpha_2}{r-\mu_2+\sigma_2\kappa_2}$, $S_2^*=\dfrac{\beta_2}{(\beta_2-1)\lambda_2}X=\dfrac{\phi_2 S_{2R}^*}{\lambda_2}$, $\beta_2=\dfrac{1}{2}-\dfrac{\mu_2}{\sigma_2^2}+\sqrt{\left(\dfrac{1}{2}-\dfrac{\mu_2}{\sigma_2^2}\right)^2+\dfrac{2r}{\sigma_2^2}}>1$.

The buyer will exercise the call option only when $S_2\geq S_2^*$. Otherwise, the buyer will delay buying the commodity until $S_2\geq S_2^*$.

When $\kappa_2=0$, the value of buying the commodity under risk uncertainty $F_{2R}(W_2)$ is given by

$$F_{2R}(W_2)=\begin{cases}A_{2R}(\phi_2 S_2)^{\beta_{2R}} & if \quad S_2<S_{2R}^* \\ \phi_2 S_2-X & if \quad S_2\geq S_{2R}^*\end{cases} \tag{21}$$

Where $S_{2R}^*=\dfrac{\beta_2}{(\beta_2-1)\phi_2}X$, $S_2^*=\dfrac{\phi_2 S_{2R}^*}{\lambda_2}$, $\phi_2=\dfrac{1}{r-\mu_2}$, $r-\mu_2>0$, $A_2=\dfrac{(S_{2R}^*)^{1-\beta_2}}{\beta_2}$, $A_2=(\lambda_2)^{1-\beta_2}A_{2R}$, $\beta_2=\beta_{2R}=\dfrac{1}{2}-\dfrac{\mu_2}{\sigma_2^2}+\sqrt{\left(\dfrac{1}{2}-\dfrac{\mu_2}{\sigma_2^2}\right)^2+\dfrac{2r}{\sigma_2^2}}>1$.

It is important to see the optimal and non-optimal exercise regions within the contract price domain. In Figures 2(a) and 2(b), S_2^* separates the optimal and non-optimal regions for the buyer. Waiting is optimal when buyer revenue (S_2) is less than its critical trigger value (S_2^*). Executing a call option is optimal when S_2 is greater than its critical trigger value S_2^*. The "waiting regions" of S_2 given α_2=1 and S_2 given α_2=1 and κ_2=0.2 are shown in Figures 2(a) and 2(b). Conversely, the buyer will exercise the call option if the product's or service's revenues reach the critical trigger value S_2^*. As the opportunity value will be relatively high when the revenue is large, it is optimal to buy immediately.

The impact of increased optimism with respect to buyer revenue (α_2) on the critical trigger value is depicted in Figure 2(a). An increase in α_2 decreases the critical trigger value of buyer revenue (S_2^*), which means more optimistic buyers will accept a lower revenue at the start time of the option. The critical trigger value under risk uncertainty S_{2R}^* is lower than critical trigger value (S_2^*) when α_2=0 and α_2=0.25 and is higher than S_2^* when α_2=0.5, α_2=0.75 and α_2=1.

The critical trigger value curves (S_2^*) for different ambiguity levels in probability distributions (κ_2) are shown in Figure 2 (b). If S_2 falls below the critical trigger value curves (S_2^*), the buyer will wait. The impact of an increase in κ_2 also depends on the pessimistic (or optimistic) levels of the negotiators. Increasing κ_2 from 0.1 to 0.2 increases (decreases) the critical trigger value S_2^* of very pessimistic (optimistic) decision-makers.

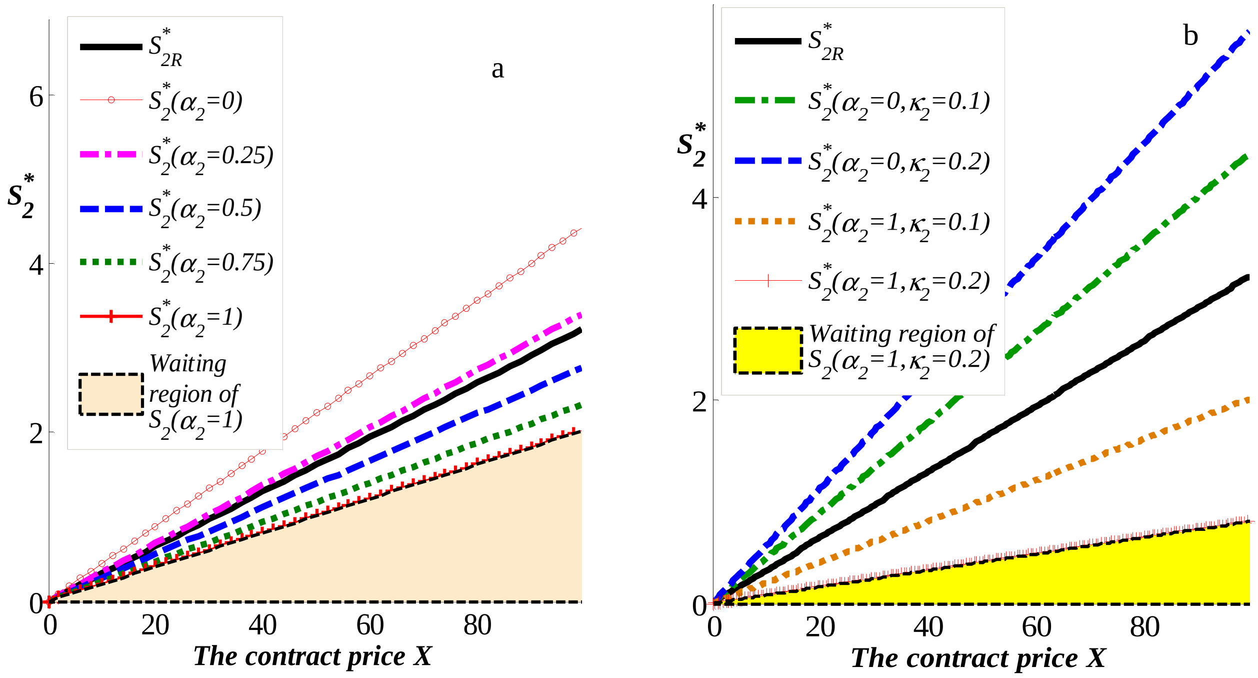

Figure 2 (a) the impacts of buyer optimism on the contract price and (b) the impacts of buyer ambiguity level in probability distributions on the contract price. Here $r=0.08$, $\mu_2=0.04$, $\sigma_2=0.15$.

3 The implicit zone of achievable agreement (IZOAA) under Knightian uncertainty

In this section we define the Implicit Zone of Achievable Agreement (IZOAA) under Knightian uncertainty and its existence condition.

A seller under Knightian uncertainty will supply when $S_1(t) \le S_1^* = \dfrac{X}{(1-1/\beta_1)\lambda_1}$ based on equation (15).

A buyer under Knightian uncertainty will purchase when $S_2(t) \ge S_2^* = \dfrac{X}{(1-1/\beta_2)\lambda_2}$ based on equation (20).

We refer to X_1 and X_2 as Implicit Reservation Prices for seller and buyer in negotiation under Knightian uncertainty. For given $S_1(t)$ and $S_2(t)$, we have

$$X_1 = (1-1/\beta_1)\lambda_1 S_1(t) \le X \quad \text{for a seller under Knightian uncertainty}$$

$$X_2 = (1-1/\beta_2)\lambda_2 S_2(t) \geq X \quad \text{for a buyer under Knightian uncertainty.}$$

The optimal buying and selling strategies are to sell when $X_1 \leq X$ and to buy when $X_2 \geq X$. Thus, we define the region $[X_1, X_2]$ determined by implicit reservation prices as an Implicit Zone of Achievable Agreement (IZOAA) under Knightian uncertainty.

Setting an implicit reservation price for a seller or buyer guarantees the expected profit and considers the α-maxmin expected value of seller cost and buyer revenue. The contract price X is higher than implicit reservation prices for a seller (X_1), which makes the seller gain a surplus. Given X_1, a higher contract price X generates greater benefits for the seller. To reach negotiation agreement, the buyer's (X_2) should exceed the contract price X in order to gain consumer surplus. Setting implicit reservation prices also allows the seller or buyer to make informed decisions considering uncertainty preferences.

Theorem 1

The Implicit Zone of Achievable Agreement (IZOAA) under Knightian uncertainty on both buyer and supplier sides is a generalization of the risk uncertainty cases, including IZOAA under risk uncertainty on both sides and IZOAA under Knightian uncertainty on one side.

The ambiguity multiplier λ_i (defined in Equation (3)) is a generalization of ϕ_i (defined in Equations (16) and (21)) because when considering the case without ambiguity in probability distributions ($\kappa_i = 0$), λ_i reduces to $\phi_i = 1/(r-\mu_i)$. When T approaches infinity, we obtain $W_i(S_i(t)) = \phi_i S_i(t)$ which is identical to the standard expression for the expected present value under risk uncertainty (Dixit and Pindyck, P72). Therefore, we can justify Theorem 1. When $\kappa_i > 0$, the ambiguity multiplier λ_i incorporates the influence of α_i and κ_i and captures very optimistic and pessimistic cases. λ_i turns out to be positive since $\kappa_i \geq 0$ and $r - \mu_i - \sigma_i \kappa_i > 0$.

Thus, we define the region $[X_{1R}, X_{2R}]$ determined by implicit reservation prices as an Implicit Zone of Achievable Agreement (IZOAA) under risk uncertainty in the seller's cost and the buyer's revenue, where $X_{1R} = (1-1/\beta_1)\phi_1 S_1(t)$ for a seller under risk uncertainty, $X_{2R} = (1-1/\beta_2)\phi_2 S_2(t)$ for a buyer under risk uncertainty.

Theorem 2. (Condition for the existence of IZOAA under double-sided Knightian uncertainty)

Two parties involved in negotiation can reach an agreement with each other under Knightian uncertainty, when the following condition is satisfied

$$S_2(t)/S_1(t) \geq \delta_{KK} \tag{22}$$

where $\delta_{KK} = \dfrac{(1-1/\beta_1)\lambda_1}{(1-1/\beta_2)\lambda_2}$, δ_{KK} denotes the profit space threshold of the ratio of buyer revenue ($S_2(t)$) to seller cost ($S_1(t)$) when the negotiators exercise the put and call option at time t under Knightian uncertainty.

The threshold (δ_{KK}) reflects the profit space of signing the supply contract. A larger value of δ_{KK} means the cooperation between the seller and the buyer leads to more profits. While, a smaller value of δ_{KK} means the cooperation between the seller and the buyer results in less profits. It is necessary that δ_{KK} be greater than one for profits to be earned. The buyer's revenue should be larger than a multiple of the seller's cost and δ_{KK}.

When $\kappa_1 = 0$ and $\kappa_2 = 0$, the risk case of δ_{KK} becomes $\delta_{RR} = \dfrac{(1-1/\beta_1)\phi_1}{(1-1/\beta_2)\phi_2}$. Then $\dfrac{\delta_{KK}}{\delta_{RR}} = \dfrac{\lambda_1/\lambda_2}{\phi_1/\phi_2}$.

Theorem 3. (The impact of buyer and seller Knightian uncertainty on condition for the existence of IZOAA)

The relationship between the thresholds of the ratio of buyer revenue to seller cost under Knightian uncertainty δ_{KK} and under risk uncertainty δ_{RR} is:

$$\begin{cases} \delta_{KK} \geq \delta_{RR} \quad if \quad \dfrac{\lambda_1}{\lambda_2} \geq \dfrac{\phi_1}{\phi_2} \\ \delta_{KK} < \delta_{RR} \quad if \quad \dfrac{\lambda_1}{\lambda_2} < \dfrac{\phi_1}{\phi_2} \end{cases} \tag{23}$$

Equation (23) shows the relationship between λ_1/λ_2 and ϕ_1/ϕ_2 and determines the relationship between δ_{KK} and δ_{RR}.

The joint impacts of α_1 and α_2 on the profit space threshold δ_{KK} are illustrated in Figure 3. Since $\beta_1 < 0$ and $\beta_2 > 1$, we have $\partial\delta_{KK}/\partial\alpha_1 > 0, \partial\delta_{KK}/\partial\alpha_2 < 0$. A more pessimistic perception about seller costs (α_1) results in a higher profit space threshold. Increasing the optimistic perceptions related to buyer revenue (α_2) results in a lower profit space threshold. Given the ambiguity level in probability distributions ($\kappa_i, i = 1,2$), very optimistic negotiators ($\alpha_1 = 0$ and $\alpha_2 = 1$) accept a lower profit space threshold δ_{KK}. In contrast, very pessimistic negotiators ($\alpha_1 = 1$ and $\alpha_2 = 0$) look forward to the highest possible profit space threshold δ_{KK}. Increasing ambiguity level ($\kappa_i, i = 1,2$) induces a larger range for δ_{KK}. When the ambiguity level in probability distributions ($\kappa_i, i = 1,2$) equals to zero, ambiguity perceptions ($\alpha_i, i = 1,2$) do not affect δ_{KK}, which accords with the definition of the ambiguity multiplier λ_i in Equation (3).

The profit space threshold δ_{KK} as a function of the level of ambiguity in probability distributions ($\kappa_i, i=1,2$) for different values of ambiguity attitudes ($\alpha_i, i=1,2$) is depicted in Figure 4. δ_{KK} is a non-monotone function of κ_1. The monotonicity of δ_{KK} with repect to κ_1 depends on ψ_1, where $\psi_1 = \frac{1}{2} - \frac{\sigma_1\kappa_1(r-\mu_1)}{(r-\mu_1)^2+\sigma_1^2\kappa_1^2}$. If $\alpha_1 \geq \psi_1$, we have $\partial\delta_{KK}/\partial\kappa_1 \geq 0$. Else, $\partial\delta_{KK}/\partial\kappa_1 < 0$. The positive relationship between κ_1 and δ_{KK} occurs if the pessimism perception about seller cost α_1 is greater than ψ_1. If $\alpha_1 < \psi_1$, there is a negative relationship between κ_1 and δ_{KK}.

The effect of κ_2 on δ_{KK} exhibits non-monotonic behavior as shown in Figure 4. Whether δ_{KK} rises or falls with an increase in κ_2 depends on $\psi_2 = \frac{1}{2} - \frac{\sigma_2\kappa_2(r-\mu_2)}{(r-\mu_2)^2+\sigma_2^2\kappa_2^2}$. If $\alpha_2 \leq \psi_2$, we have $\partial\delta_{KK}/\partial\kappa_2 \geq 0$. Else, $\partial\delta_{KK}/\partial\kappa_2 < 0$. This relationship means a buyer with a very low optimism tends to need a higher profit space threshold. If the optimism perception of buyer revenue (α_2) is greater than ψ_2, the buyer views an increase in κ_2 as a good opportunity and thus decreases/lowers the profit space threshold (δ_{KK}).

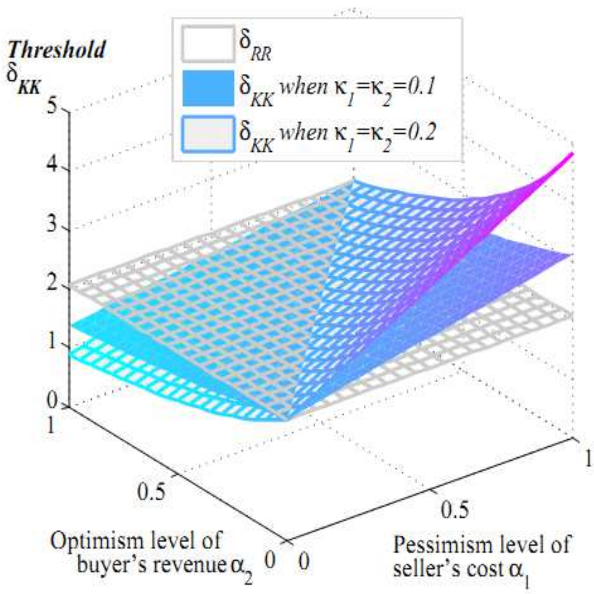

Figure 3 the impacts of α_1 and α_2 on δ_{KK}

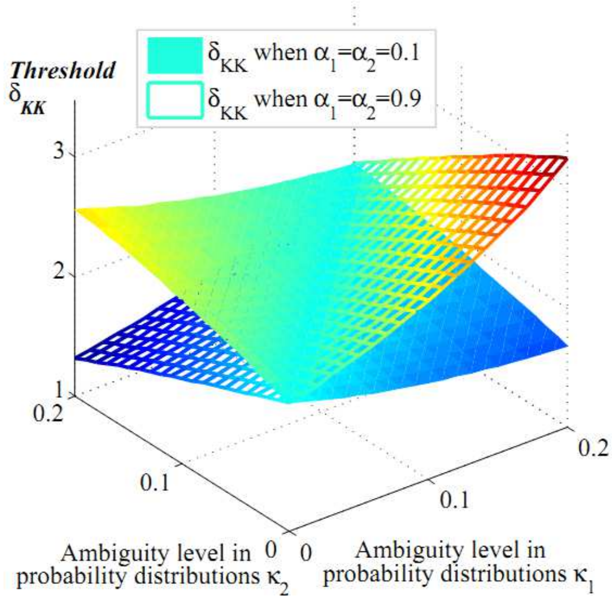

Figure 4 the impacts of κ_1 and κ_2 on δ_{KK}

Here $r = 0.08$, $\mu_i = 0.03$, $\sigma_i = 0.1$.

To understand how implicit reservation prices change when attitudes towards ambiguity (α_i) and ambiguity level in probability (κ_i) change, we have Proposition 1 below.

Proposition 1. (Comparative statics for implicit reservation prices under Knightian uncertainty)

For a given valuation at time t, $S_i(t)$ for $i=1,2$ the implicit reservation price of a seller (a buyer) increases (decreases) monotonically as pessimism about seller costs (optimism about buyer revenues) increases. The implicit reservation price of a seller (a buyer) is a non-monotone function of the ambiguity level in probability distributions. There is a critical value for monotonicity.

$$\frac{\partial X_i}{\partial \alpha_i} \geq 0, \quad \begin{cases} \partial X_i / \partial \kappa_i \geq 0 & if \quad \alpha_i \geq \psi_i \\ \partial X_i / \partial \kappa_i < 0 & if \quad \alpha_i < \psi_i \end{cases}$$

Where $\psi_i = \frac{1}{2} - \frac{\sigma_i \kappa_i (r-\mu_i)}{(r-\mu_i)^2 + \sigma_i^2 \kappa_i^2}$.

Proposition 1 shows that the seller asks for a higher price if he/she is more pessimistic about production costs ($\partial X_1 / \partial \alpha_1 \geq 0$). The buyer provides a higher price if he is more optimistic about revenues ($\partial X_2 / \partial \alpha_2 \geq 0$). If the pessimism level about seller cost (α_1) is greater than or equal to ψ_1, this seller asks for a higher price because of an increase in the ambiguity surrounding probability distributions (κ_1) ($\partial X_1 / \partial \kappa_1 \geq 0$). If α_1 is less than ψ_i, there is a negative relationship between κ_1 and X_1. If the optimism surrounding buyer revenue (α_2) is greater than or equal to ψ_2, this buyer offers a higher price ($\partial X_2 / \partial \kappa_2 \geq 0$). If α_2 is less than ψ_2, there is a negative relationship between κ_2 and X_2.

4 Negotiation agreement probability under Knightian uncertainty

In negotiations, parties are often influenced consciously or unconsciously by their assessments of possible alternatives and the probabilities of mutual agreement induced from each of these alternatives. It is almost always helpful to compare possible probability outcomes before making a decision during negotiation. Since the seller cares about costs and the buyer cares about revenues, the negotiation agreement probability is affected by the seller's cost and buyer's revenue. This section helps understand how negotiation agreement probability can be determined and studies how changes in the ambiguity surrounding probability distributions affect the likelihood of negotiation agreement for very optimistic and pessimistic decision makers.

$S_1(t)$ and $S_2(t)$ follow a lognormal distribution and their two-dimension probability distribution density function is:

$$h(S_1,S_2)=\frac{\exp\left\{-\frac{1}{2\left(1-\rho_{12}^2\right)}\left[\left(\frac{\log S_1-\mu_{1N}}{\sigma_{1N}}\right)^2-2\rho_{12}\frac{(\log S_1-\mu_{1N})(\log S_2-\mu_{2N})}{\sigma_{1N}\sigma_{2N}}+\left(\frac{\log S_2-\mu_{2N}}{\sigma_{2N}}\right)^2\right]\right\}}{2\pi\sigma_{1N}\sigma_{2N}\sqrt{1-\rho_{12}^2}S_1S_2} \quad (24)$$

Where μ_{iN} and σ_{iN} are the expected value and standard variance of $\log S_i$ under Knightian uncertainty. $\mu_{iN}=(\mu_i-\sigma_i\theta_i-\frac{1}{2}\sigma_i^2)t$, $\sigma_{iN}=\sigma_i\sqrt{t}$, for $i=1,2$, $\forall t\geq 0, \forall\theta_i\in\Theta_i$. The subscript N implies $\log S_i$ follows the normal distribution. ρ_{12} is the correlation parameter between S_1 and S_2.

We find the process followed by S_1S_2 (e.g. Dixit and Pindyck (1994), P82)

$$d(S_1S_2)=(\mu_1-\sigma_1\theta_1+\mu_2-\sigma_2\theta_2+\varepsilon_{KK}\sigma_1\sigma_2)S_1S_2dt+(\sigma_1 dB_t^{\theta_1}+\sigma_2 dB_t^{\theta_2})S_1S_2 \quad (25)$$

Since the expected value and standard variance of S_i under Knightian uncertainty are given by $E(S_i)=S_i(0)\exp\left[(\mu_i-\sigma_i\theta_i-\frac{1}{2}\sigma_i^2)t\right]$, $std(S_i)=S_i(0)\sqrt{\exp[2(\mu_i-\sigma_i\theta_i-\frac{1}{2}\sigma_i^2 t)]\left[\exp(\sigma_i^2 t)-1\right]}$, we have

$$E(S_1S_2)=S_1(0)S_2(0)\exp\left[(\mu_1-\sigma_1\theta_1+\mu_2-\sigma_2\theta_2+\varepsilon_{12}\sigma_1\sigma_2)t\right] \quad (26)$$

We need to calculate the factor ρ_{12} to obtain the probability distribution density function.

$$\rho_{12}=\frac{E(S_1S_2)-E(S_1)E(S_2)}{std(S_1)std(S_2)} \quad (27)$$

Substituting $E(S_1S_2)$ into ρ_{12}, we have

$$\rho_{12}=\frac{\exp\left[(\mu_1-\sigma_1\theta_1+\mu_2-\sigma_2\theta_2+\varepsilon_{12}\sigma_1\sigma_2)t\right]-\exp\left[(\mu_1-\sigma_1\theta_1)t+(\mu_2-\sigma_2\theta_2)t\right]}{\sqrt{\exp(2(\mu_1-\sigma_1\theta_1)t)\left[\exp(\sigma_1^2t)-1\right]}\sqrt{\exp\left[2(\mu_2-\sigma_2\theta_2)t\right](\exp(\sigma_2^2t)-1)}} \quad (28)$$

Where $E[dB_1^{\theta_1}, dB_2^{\theta_2}]=\varepsilon_{12}dt$.

Thus, we obtain the two-dimension probability distribution density function for $\log S_1$ and $\log S_2$. Then we derive the negotiation agreement probability under Knightian uncertainty.

Theorem 4 (negotiation agreement probability under Knightian uncertainty)

The negotiation agreement probability under Knightian uncertainty is a generalization of the risk uncertainty case.

The negotiation agreement probability under Knightian uncertainty P_{KK} is defined by

$$P_{KK} = P(X \geq X_1 \quad and \quad X \leq X_2; \quad t) = P(S_1 \leq S_1^* \quad and \quad S_2 \geq S_2^*; \quad t)$$
$$= \int_{-\infty}^{Y_1^*}\int_{Y_2^*}^{+\infty} \frac{\exp\left\{-\frac{1}{2\left(1-\rho_{12}^2\right)}\left[\left(\frac{Y_1-\mu_{1N}}{\sigma_{1N}}\right)^2 - 2\rho_{12}\frac{(Y_1-\mu_{1N})(Y_2-\mu_{2N})}{\sigma_{1N}\sigma_{2N}} + \left(\frac{Y_2-\mu_{2N}}{\sigma_{2N}}\right)^2\right]\right\}}{2\pi\sigma_{1N}\sigma_{2N}\sqrt{1-\rho_{12}^2}} dY_1 dY_2 \tag{29}$$

Where $\mu_{1N} = (\mu_1 - \sigma_1\theta_1 - \frac{1}{2}\sigma_1^2)t$, $\sigma_{1N} = \sigma_1\sqrt{t}$, $\mu_{2N} = (\mu_2 - \sigma_2\theta_2 - \frac{1}{2}\sigma_2^2)t$, $\sigma_{2N} = \sigma_2\sqrt{t}$, $Y_1 = \ln\left(\frac{S_1(t)}{S_1(0)}\right)$,

$Y_2 = \ln\left(\frac{S_2(t)}{S_2(0)}\right)$, $Y_1^* = \ln\left(\frac{S_1^*}{S_1(0)}\right) = \ln\left(\frac{X}{(1-1/\beta_1)\lambda_1 S_1(0)}\right)$ and $Y_2^* = \ln\left(\frac{S_2^*}{S_2(0)}\right) = \ln\left(\frac{X}{(1-1/\beta_2)\lambda_2 S_2(0)}\right)$.

$S_1^* = \frac{X}{(1-1/\beta_1)\lambda_1}$, $S_2^* = \frac{X}{(1-1/\beta_2)\lambda_2}$.

Let $\kappa_1 = 0$ and $\kappa_2 = 0$, we obtain the expectation and variance of S_i and $\log S_i$ under risk uncertainty, $\mu_{iNR} = (\mu_i - \frac{1}{2}\sigma_i^2)t$, $\sigma_{iNR} = \sigma_i\sqrt{t}$, $E\left(S_{iR}\right) = S_i(0)\exp[(\mu_i - \frac{1}{2}\sigma_i^2)t]$,

$std(S_{iR}) = S_i(0)\sqrt{\exp[2(\mu_i - \frac{1}{2}\sigma_i^2)t(\exp(\sigma_i^2 t)-1)}$, $i = 1, 2$. The subscript R denotes risk uncertainty.

Using the same logic as above, we have the negotiation agreement probability under risk uncertainty P_{RR}:

$$P_{RR} = P(X_{1R} \leq X \quad and \quad X_{2R} \geq X; \quad t) = \int_{-\infty}^{\frac{X}{(1-1/\beta_1)\phi_1 S_1(0)}} \int_{\frac{X}{(1-1/\beta_2)\phi_2 S_2(0)}}^{\infty} h(S_1, S_2) dS_1 dS_2 \tag{30}$$

The probability that negotiation agreement in Equations (29) and (30) will be reached depends on the value of S_i at time 0, the ambiguity multiplier λ_i, the social discount rate r, the parameters of Geometric Brownian motion followed by S_i as defined in Equation (1).

The negotiation agreement probabilities for very optimistic negotiators ($\alpha_1 = 0$ and $\alpha_2 = 1$), very pessimistic negotiators ($\alpha_1 = 1$ and $\alpha_2 = 0$) and risk neutral parties ($\kappa_1 = \kappa_2 = 0$) are illustrated in Figure 5. The very optimistic (pessimistic) scenario corresponds to a situation in which the pessimism surrounding seller cost is at the lowest (highest) level and the optimism about buyer revenue is at the highest (lowest) level. The very optimistic cases have higher agreement probabilities than the very pessimistic cases. In the case of very pessimistic decision-makers, the negotiation agreement probability decreases when ambiguity ($\kappa_i, i=1,2$) increases, which reflects a conservative attitude towards great uncertainty. In the contrast, very optimistic decision-makers increase the negotiation agreement probabilities as ambiguity in probability distributions ($\kappa_i, i=1,2$) increases.

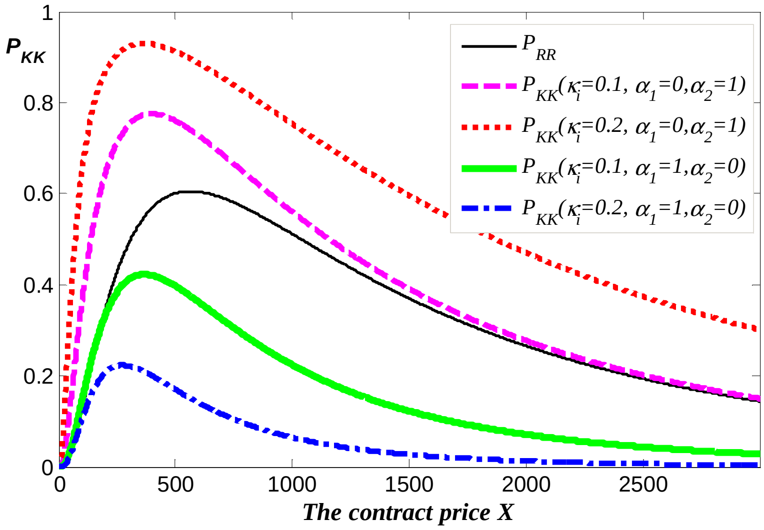

Figure 5 Negotiation agreement probability for the very optimistic ($\alpha_1 = 0$ and $\alpha_2 = 1$), very pessimistic case ($\alpha_1 = 1$ and $\alpha_2 = 0$) when $\kappa_i = 0, 0.1, 0.2$, $i = 1, 2$

Here t=30; $S_1(0)$=6.23; $S_2(0)$ =48.03; r=0.08; μ_1=0.03; μ_2=0.04; $\sigma_1 = \sigma_2 = 0.15$.

When the revenue and the cost functions are independent of each other, the probability of negotiation agreement can have the following closed form solution.

$$P_{KK} = P(S_1 \le S_1^*;t)P(S_2 \ge S_2^*;t) = \Phi\left(\frac{Y_1^* - (\mu_1 - \sigma_1\theta_1 - \frac{1}{2}\sigma_1^2)t}{\sigma_1\sqrt{t}}\right)\left(1 - \Phi\left(\frac{Y_2^* - (\mu_2 - \sigma_2\theta_2 - \frac{1}{2}\sigma_2^2)t}{\sigma_2\sqrt{t}}\right)\right) \tag{31}$$

Where $P(S_2 \ge S_2^*;t) = \int_{Y_2^*}^{+\infty} \frac{1}{\left(2\pi\sigma_2^2 t\right)^{1/2}} \exp\left\{-\frac{1}{2}\left[\left(\frac{Y_2 - (\mu_2 - \sigma_2\theta_2 - \frac{1}{2}\sigma_2^2)t}{\sigma_2\sqrt{t}}\right)^2\right]\right\} dY_2,$

$$P(S_1 \le S_1^*;t) = \int_{-\infty}^{Y_1^*} \frac{1}{\left(2\pi\sigma_1^2 t\right)^{1/2}} \exp\left\{-\frac{1}{2}\left[\left(\frac{Y_1 - (\mu_1 - \sigma_1\theta_1 - \frac{1}{2}\sigma_1^2)t}{\sigma_1\sqrt{t}}\right)^2\right]\right\} dY_1.$$

Note that the closed form solution defined in Equation (31) is also a generalization of the risk uncertainty cases. This includes negotiation agreement probability under risk uncertainty on both buying/selling sides and negotiation agreement probability under Knightian uncertainty on one side.

5 Negotiation power under Knightian uncertainty

Repeated offers and counteroffers constitute a sequence of bargaining games over time. This section discusses the effects of buyer and seller influence or negotiation power in the supply chain on the real options dynamics of negotiation under Knightian uncertainty examined above.

In our case, the two parties (i.e., seller and buyer) are to negotiate a contract that would distribute a surplus (e.g., profit) generated from mutual efforts. Let us assume that the seller's negotiation power is $\gamma \in [0,1]$ and then the buyer's negotiation power is $(1-\gamma)$. From the generalized Nash Bargaining game (Gurnani and Shi, 2006; Nagarajan and Sosic, 2008; Moon et al., 2011), a contract price is determined as follows:

$$X = (1-\gamma)W_1 + \gamma W_2 \tag{32}$$

Equation (32) shows that the contract price is equal to the weighted sum of the expected present discounted value of $S_i(t)$ in the α-maxmin expected value framework.

The buyer's payoff function with negotiation power under Knightian uncertainty is defined by

$$U_2(W_1,W_2) = \max_t E_t^{Q_2^{\theta_2}}\{S_2(t) - X\} = (1-\gamma)\max_t E_t^{Q_2^{\theta_2}}\{[S_2(t) - S_1(t)]\} = (1-\gamma)\max_t\{V(t)\} \tag{33}$$

Where $V(t) = W_2(t) - W_1(t)$, $V(t)$ is the total discounted surplus of the supply contract, $W_i(t) = \lambda_i S_i(t)$, $i = 1,2$.

The seller's put option value with negotiation power under Knightian uncertainty is

$$U_1(W_1,W_2) = \max_t E_t^{Q_1^{\theta_1}}\{X - S_1(t)\} = \gamma \max_t\{V(t)\} \tag{34}$$

The seller and buyer have the same objective of maximizing the total option value $V(t)$. Their option value is a function of their negotiation power. For tractability, we define

$$W_2(t)/W_1(t) = z \tag{35}$$

Where z reflects the ratio of the α-maxmin expected value, $W_1(t)$ that is the α-maxmin expected value of $S_1(t)$, and $W_2(t)$ the α-maxmin expected value of $S_2(t)$.

According to equation (35), we have

$$V(t) = (z-1)W_1(t) \tag{36}$$

Equation (36) shows the total discounted surplus of the supply contract. $V(t)$ depends on z and $W_1(t)$.

For given negotiation powers for each party, the buyer's call option value and the corresponding optimal time to purchase under Knightian uncertainty can be derived as follows:

$$U_2(W_1,W_2)=\begin{cases} d_2(\lambda_2 S_2)^{b_2}(\lambda_1 S_1)^{1-b_2} & if \quad z<z_2^* \\ (1-\gamma)\lambda_2 S_2-(1-\gamma)\lambda_1 S_1 & if \quad z\geq z_2^* \end{cases} \tag{37}$$

Where $d_2=\dfrac{1-\gamma}{b_2}\left(z_2^*\right)^{1-b_2}$, $z_2^*=\dfrac{b_2}{b_2-1}=\dfrac{1}{1-1/b_2}$,

$$b_2=\frac{1}{2}-\frac{\left(\mu_2-\sigma_2\kappa_2-(\mu_1-\sigma_1\kappa_1)+\frac{1}{2}\sigma_1^2\right)}{\sigma_1^2-\varepsilon_{12}\sigma_1\sigma_2+\sigma_2^2}+\sqrt{\left(\frac{1}{2}-\frac{\mu_2-\sigma_2\kappa_2-(\mu_1-\sigma_1\kappa_1)+\frac{1}{2}\sigma_1^2}{\sigma_1^2-\varepsilon_{12}\sigma_1\sigma_2+\sigma_2^2}\right)^2+\frac{2r}{\sigma_1^2-\varepsilon_{12}\sigma_1\sigma_2+\sigma_2^2}}>1.$$

The corresponding put option value for the seller is

$$U_1(W_1,W_2)=\begin{cases} d_1(\lambda_2 S_2)^{b_1}(\lambda_1 S_1)^{1-b_1} & if \quad z<z_1^* \\ \gamma\lambda_2 S_2-\gamma\lambda_1 S_1 & if \quad z\geq z_1^* \end{cases} \tag{38}$$

Where $z_1^*=z_2^*=\dfrac{b_2}{b_2-1}$, $d_1=\dfrac{\gamma}{b_2}\left(z_1^*\right)^{1-b_2}$.

z_2^* (that is equal to z_1^*) denotes the exercise boundary for the call and put options and reflects the space profit of the ratio of the α-maxmin expected value of buyer revenue to the α-maxmin expected value of seller cost and becomes the exercise boundary for the call or put option. When z is less than the ratio threshold z_2^*, the total option value is too low for cooperation or mutual agreement. Then neither the buyer nor the seller will sign the supply contract. When z is larger than the ratio threshold z_2^* (that is equal to z_1^*), it is worth cooperating.

Theorem 5. (IZOAA with negotiation powers and its existence condition under Knightian uncertainty)

The Implicit Zone of Achievable Agreement (IZOAA) with negotiation powers is a generalization of the risk uncertainty case, including IZOAA with negotiation powers under risk uncertainty on both sides and IZOAA with negotiation powers under Knightian uncertainty on one side.

If we consider the case without ambiguity in probability distributions $\kappa_i=0$, λ_i reduces to $\phi_i=1/(r-\mu_i)$ (as defined in Equations (3), (16) and (21)). The proof of this theorem is in line with Theorem 1.

The buyer's call option value and the corresponding optimal time to purchase under risk uncertainty in both seller costs and the buyer revenues can be derived as follows:

$$U_2(W_{1R},W_{2R})=\begin{cases} d_{2R}(\phi_2 S_2)^{b_{2R}}(\phi_1 S_1)^{1-b_{2R}} & if \quad z<z_{2R}^* \\ (1-\gamma)\phi_2 S_2-(1-\gamma)\phi_1 S_1 & if \quad z\geq z_{2R}^* \end{cases} \tag{39}$$

Where $d_{2R} = \frac{1-\gamma}{b_{2R}} \left(z_{2R}^{*} \right)^{1-b_{2R}}$, $\phi_1 = \frac{1}{r-\mu_1}$, $\phi_2 = \frac{1}{r-\mu_2}$, $z_{2R}^{*} = \frac{b_{2R}}{b_{2R}-1}$,

$$b_{2R} = \frac{1}{2} - \frac{\left(\mu_2 - \mu_1 + \frac{1}{2}\sigma_1^2 \right)}{\sigma_1^2 - \varepsilon_{12}\sigma_1\sigma_2 + \sigma_2^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu_2 - \mu_1 + \frac{1}{2}\sigma_1^2}{\sigma_1^2 - \varepsilon_{12}\sigma_1\sigma_2 + \sigma_2^2} \right)^2 + \frac{2r}{\sigma_1^2 - \varepsilon_{12}\sigma_1\sigma_2 + \sigma_2^2}}.$$

The corresponding put option value under risk uncertainty is

$$U_1(W_{1R}, W_{2R}) = \begin{cases} d_{1R}(\phi_2 S_2)^{b_{1R}} (\phi_1 S_1)^{1-b_{1R}} & if \quad z < z_{1R}^{*} \\ \gamma\phi_2 S_2 - \gamma\phi_1 S_1 & if \quad z \geq z_{1R}^{*} \end{cases} \tag{40}$$

Where $b_{1R} = b_{2R}$, $z_{1R}^{*} = z_{2R}^{R*} = \frac{b_{2R}}{b_{2R}-1}$, $d_{1R} = \frac{\gamma}{b_{1R}} \left(z_{1R}^{*} \right)^{1-b_{1R}}$.

Figure 6 depicts the impact of the level of ambiguity in probability distributions (κ_1 or κ_2**)** on the exercise boundary for the call and put options, (z_2^{*}).

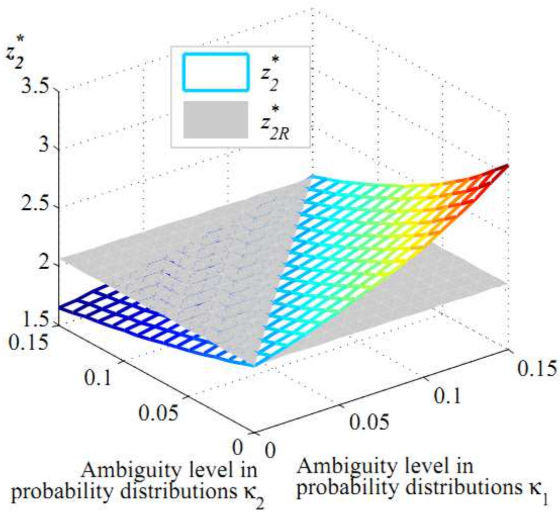

Figure 6 the impacts of κ_1 and κ_2 on the ratio of W_2 to W_1

Here $r = 0.08$, $\mu_1 = 0.03$, $\mu_2 = 0.04$, $\sigma_i = 0.15$, ε_{12}=0.1, $i = 1, 2$.

According to the optimal discounted space profit z_1^{*} (that is equal to z_2^{*}) and combining Equations (32) and (37), the seller can obtain a relationship between S_1 and the seller's implicit reserve price with negotiation power X_1^{np} (see Moon et al., 2011).

$$X_1^{np} = \left(\frac{b_2 - (1-\gamma)}{b_2 - 1} \right) \lambda_1 S_1 \tag{41}$$

Similarly, the relationship between S_2 and the buyer's implicit reserve price with negotiation power X_2^{np}

$$X_2^{np} = \left(\frac{b_2 - (1-\gamma)}{b_2}\right)\lambda_2 S_2 \tag{42}$$

Similar to Section 3, we define the Implicit Reservation Prices for seller and buyer but this time with negotiation power. Thus, we define the region $[X_1^{np}, X_2^{np}]$ determined by implicit reservation prices as an Implicit Zone of Achievable Agreement (IZOAA) with negotiation power under Knightian uncertainty in the seller's cost and the buyer's revenue.

Two parties incorporating negotiation powers under the two-sided Knightian uncertainty can reach agreement, when the following condition is satisfied: $X_1^{np} \le X \le X_2^{np}$. Then to achieve mutual agreement, X_2^{np} should be greater than X_1^{np}.

Theorem 6. (Condition for the existence of IZOAA with negotiation power under Knightian uncertainty)

Two parties incorporating negotiation powers under the Knightian uncertainty can achieve mutual agreement, when the following condition is satisfied:

$$S_2/S_1 \ge \delta_{KK}^{np} \tag{43}$$

Where $\delta_{KK}^{np} = \dfrac{\lambda_1}{(1-1/b_2)\lambda_2}$,. δ_{KK}^{np} denotes the profit space threshold with negotiation power of the ratio of buyer revenue $S_2(t)$ to seller cost $S_1(t)$ at time t under Knightian uncertainty.

Theorem 7. (The impact of two-sided Knightian uncertainty with negotiation power and the existence condition of the IZOAA)

The relationship between the threshold with negotiation power under Knightian uncertainty (δ_{KK}^{np}) and the threshold with negotiation power under risk (δ_{RR}^{np}) uncertainty is

$$\begin{cases} \delta_{KK}^{np} \ge \delta_{RR}^{np} \quad if \quad \dfrac{\lambda_1}{(1-1/b_2)\lambda_2} \ge \dfrac{\phi_1}{(1-1/b_{2R})\phi_2} \\ \delta_{KK}^{np} < \delta_{RR}^{np} \quad if \quad \dfrac{\lambda_1}{(1-1/b_2)\lambda_2} < \dfrac{\phi_1}{(1-1/b_{2R})\phi_2} \end{cases} \tag{44}$$

Equation (44) shows the relationship between δ_{KK}^{np} and δ_{RR}^{np} is determined by λ_1, λ_1, ϕ_1, ϕ_2, b_2 and b_{2R}.

Figure 7 shows how δ_{KK}^{np} depends on α_1 and α_2, which is in line with the dynamics described in Figure 3. More pessimism about seller costs (α_1) results in a higher profit space threshold. Increasing the optimistic level surrounding buyer revenues (α_2) results in a lower profit space threshold. Larger values of κ_1 and κ_2 produce a wider range for δ_{KK}^{np} with respect to α_1 and α_2. When $\kappa_1=\kappa_2$=0, δ_{KK}^{np} does note depend on α_1 and α_2. Very pessimistic (optimistic) negotiators expect the highest (lowest) possible profit space threshold δ_{KK}.

The profit space threshold δ_{KK}^{np} as a function of the level of ambiguity in probability distributions ($\kappa_i, i=1,2$) for different ambiguity attitudes ($\alpha_i, i=1,2$) is shown in Figure 8. Results are comparable to those in Figure 4. When $\alpha_1 \geq \psi_1$ or $\alpha_2 \leq \psi_2$, an increase in κ_i undermines agreement potential, where $\psi_i = \frac{1}{2} - \frac{\sigma_i\kappa_i(r-\mu_i)}{(r-\mu_i)^2+\sigma_i^2\kappa_i^2}$, $i=1,2$. When $\alpha_1 < \psi_1$ or $\alpha_2 > \psi_2$, an increase in κ_i encourages negotiation agreement.

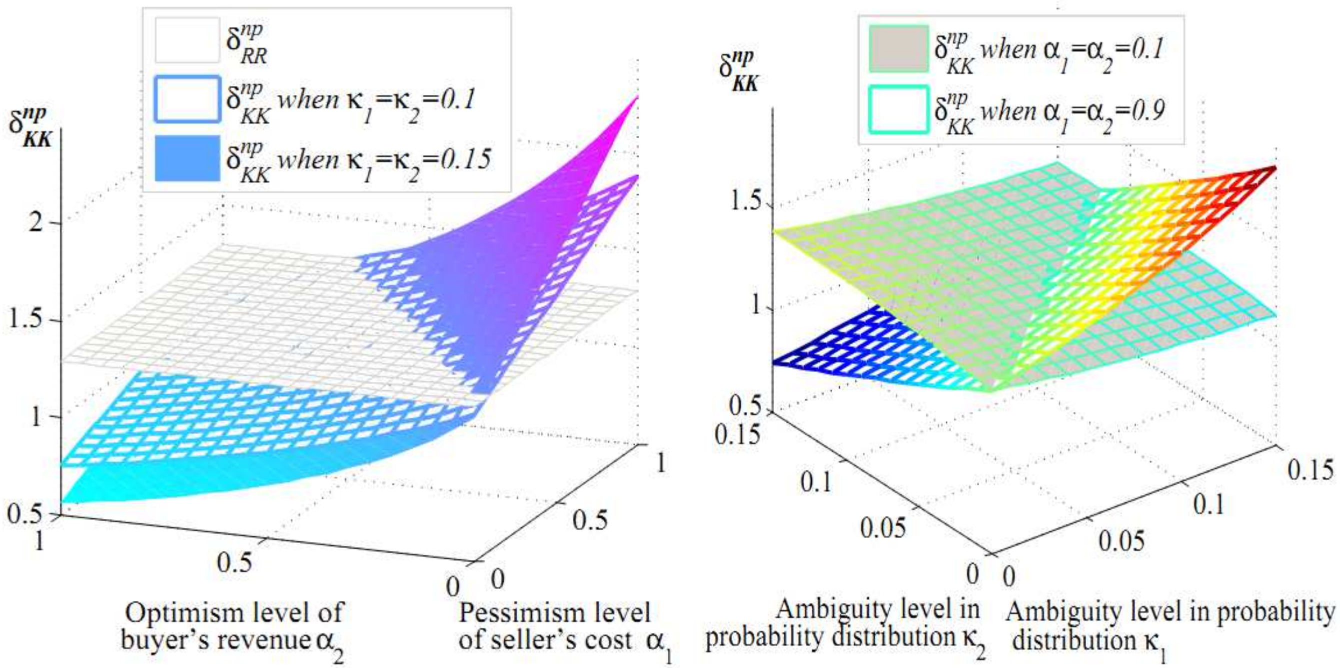

Figure 7 the impacts of α_1 and α_2 on δ_{KK}^{np} Figure 8 the impacts of κ_1 and κ_2 on δ_{KK}

Here $r=0.08$, $\mu_1=0.03$, $\mu_2=0.04$, $\sigma_i=0.1$, ε_{12}=0.1, $i=1,2$.

Proposition 2. (Comparative statics for implicit reservation prices with negotiation power under Knightian uncertainty)

There is a positive (negative) relationship between the seller's (buyer's) negotiation power and his implicit reservation price. The derivatives of implicit reservation prices for a seller and a buyer with negotiation power (X_1^{KK} and X_2^{KK}) with respect to α_i and κ_i are akin to the results without

negotiation power of Proposition 1.

$$\frac{\partial X_1^{np}}{\partial \gamma} > 0 \, , \frac{\partial X_2^{np}}{\partial (1-\gamma)} < 0 \, ,$$

$$\frac{\partial X_i^{np}}{\partial \alpha_i} \geq 0 \, , \quad \begin{cases} \partial X_i^{np} / \partial \kappa_i \geq 0 & if \quad \alpha_i \geq \psi_i^{np} \\ \partial X_i^{np} / \partial \kappa_i < 0 & if \quad \alpha_i < \psi_i^{np} \end{cases}$$

where $i = 1,2$, $\psi_i^{np} = \psi_i = \frac{1}{2} - \frac{\sigma_i \kappa_i (r - \mu_i)}{(r-\mu_i)^2 + \sigma_i^2 \kappa_i^2}$, γ and $1-\gamma$ stand for the negotiation power of seller and buyer, respectively.

From the derivatives of X_1^{np} with respect to seller negotiation power γ, if the seller has more negotiation power, he/she tends to trade at a higher contract price which generates higher benefits. Similarly, increasing the negotiation power $(1-\gamma)$ results in a lower implicit reservation price for the buyer who also aims for higher revenues.

6 Conclusion

In this contribution, we examined the real options dynamics of bilateral price negotiation under Knightian uncertainty using multiple-priors. Besides generalizing risk uncertainty results found in previous research, our findings highlighted the moderating effect of negotiators' perceived ambiguity (i.e., pessimism and optimism) on the process of negotiation and its related outcomes and provided insights into the formulation of robust optimal (buying/selling) strategies for negotiation under scenarios of deep uncertainty. Our models also helped us identify conditions under which mutual agreement is warranted with and without negotiation power.

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