On the Technology Adoption with the Double Exponential Jump Diffusion Process

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Abstract

In this paper, we develop a model where the technological advancement evolves according to a mixed process. These processes are a combination between the geometrical Brownian motion (GBM) and the double exponential jump process. The combination of the arrival rate and the size of new technology (NT) confirms the idea that companies wait and adopt the NT when they are sufficiently advanced. We also incorporate absorptive capacity, so that the difference between the two optimum adoption levels of current and future technology becomes lower when the absorptive capacity (AC) of the firm increases.

Keywords: real option, technology adoption, the jump diffusion process

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I-Introduction to the model

Although the process of GBM can characterize well uncertainty on the technological market, it is unable to account the size of the NT, which the process of jump can defend, but by slackening the assumptions on the characteristics of the market. Zajdenweber (2009) highlighted the example of the pharmaceutical company Glaxo which adopted the new Zantac product which generated 50% of the sales. This profit continued until its Astra competitor adopted Losec which brought also 50% of the sales. Thus, the NT which appears has different sizes (in the sense that they do not bring the same profit) and can be continuous and loose but also discontinuous and prominent. It is Merton (1976) who first discovered this type of

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[2] Please note that this work is nearly a completed paper.

process. This model was unable to stand in the face of the success of Brownian geometrical which admits an analytical solution. Kou (2003) succeeded in solving this problem by supposing that the size of jump follows the double exponential law. It is this process which we use to characterize the evolution of technological advancement. We proceed according to the same logic of strategy of Grenadier (1997).

We note by S_t the state of the technological advancement which evolves according to the following stochastic process (under an equivalent martingale measure (EMM)):

$$dS_t = S_{t-}(\alpha - \lambda\xi)dt + S_{t-}\sigma dz + S_{t-}d(\sum_{i=1}^{Nt} V_i - 1) \quad (1)$$

With[3] :

N_t : following the Poisson process with the intensity rate $\lambda > 0$

V_i : is a sequence of independent identically distributed (i.i.d.) non-negative random variables such that $Y_i = LnV_i$ has an asymmetric double exponential distribution with the density:

$$f_Y(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y \geq 0\}} + p\eta_2 e^{\eta_2 y} 1_{\{y \leq 0\}} \text{ where } p,\ q \geq 0,\ p + q = 1$$

Here the condition $\eta_1 > 1$ is to ensure that the underlying variable has finite expectation. Note that the means of the two exponential distributions are $\frac{1}{\eta_1}$ and $\frac{1}{\eta_2}$ respectively. In the model, all sources of randomness $N(t),\ z(t),\ and\ Y$ are assumed to be independent.

$\lambda\xi$: represents the expectation of the realisation of jumps, where:

$$\xi = E\left[e^{Y_i}\right] - 1 = \frac{p\eta_1}{\eta_1 - \beta} + \frac{q\eta_2}{\eta_2 + \beta} - 1$$

If we apply the Ito lemma on (1), we obtain

$$S_t = S_0 e^{X_t}$$
$$with$$
$$X_t = (r - \frac{1}{2}\sigma^2 - \lambda\xi)t + \sigma z + \sum_{i=1}^{Nt} Y_i$$

We consider the moment generating function of X_t which is given by :
$E(e^{\beta X_1}) = e^{G(\beta)t}$, where

[3] see Kou and Wang (2003) and Kou, Petrella and Wang (2005) for more detail.

$$G(\beta)=\frac{\sigma^2}{2}\beta^2+(\alpha-\frac{\sigma^2}{2}-\lambda\xi)\beta+\lambda\left[\frac{p\eta_1}{\eta_1-\beta}+\frac{q\eta_2}{\eta_2+\beta}-1\right]$$

We can prove that for $G(\beta)=r$, with $r>0$, this equation admits four roots. We retain the two positive roots β_1 et β_2 while respecting the following condition:

$$0<\beta_1<\eta_1<\beta_2<\infty$$

II- The compulsive strategy

In this paper, we consider the strategy of technology adoption as a sequence of embedded options. In other words, we evaluate this strategy using real American sequential exchange option values. A decision to adopt a presently available version of a new technology will have an effect on the future options of the firm. The introduction of the new technology can either improve the process of production and increase productivity or enhance the product and increase the demand. In this paper, we suppose that technology increases the demand.

The firm is confronted with a sequence of investment opportunities in technological innovation which is exogenous to the firm and follows a stochastic process with double exponential jump diffusion process as in equation (1). We know that the solution for this equation is:

$$S_t=S_0e^{X_t}$$

We note by F the value of the option to pass to the adoption of the future technology. The application of the integro-differential operator (since we have a diffusion process (differential equation) and jump (the integral)) gives us:

$$\frac{1}{2}\sigma^2F''+(\alpha-\frac{\sigma^2}{2}-\lambda\xi)F'+\lambda\int_{-\infty}^{+\infty}\left[F(X+Y)-F(X)\right]df_Y(y)=rF$$

Subject to the boundary conditions:

$$F(0)=0 \qquad (1)$$

$$F(X^*)=V(X^*)+\max(\Pi_1,\Pi_0) \qquad (2)$$

$$F'(X^*)=ce^{X^*} \qquad (3)$$

with

$$V(X^*)=ce^{X^*}$$

The first equation expresses the ideas according to which the option has no value if there is no technological progress. However, equation (2) (the value matching condition) shows that the option reaches its optimum when technological progress reaches a certain threshold X^* and the producer receives the maximum of profit. In fact, we suppose that the exercise of the option F at the threshold X^* brings two types of gain to the firm as follows:

- A direct profit in the form of monetary flows which will be represented by the difference between the generated profit of future technology Π_1 ($avec\ \Pi_1 = \delta\Pi_0$) and the profit generated by current technology Π_0.
- A technological profit or an indirect profit with each time the company adopts a NT, it accumulates more knowledge and increases its absorptive capacity (CA). The accumulation of this capacity generates additional profits indirectly. The company can reduce its costs of adoption, sell with a less expensive price, appropriate capacity to choose the least risky technologies, and have more flexibility when switching from one technology to another. This profit is represented by the term $V(X^*) = cX^*$ where the coefficient c represents the intensity of adoption (which can be approximated by the number of adoption of the NT of the company compared to the number of adoption on the market during a certain period).

The third constraint is a technical one (the smooth pasting condition) of the model (which is derived from equation (2)) to ensure the optimality of the threshold and its uniqueness.

The resolution of this program gives us the following solution:

$$F(X) = \begin{cases} Ae^{X\beta_1} + Be^{X\beta_2} & \text{si} \quad e^X < e^{X^*} \\ ce^X - \Pi_0(1-\delta) & \text{si} \quad e^X \geq e^{X^*} \end{cases}$$

avec

$$A = \frac{ce^{X^*}(1-\beta_2) + \beta_2\Pi_0(1-\delta)}{(\beta_1-\beta_2)e^{X^*\beta_1}}; B = \frac{ce^{X^*}(\beta_1-1) - \beta_1\Pi_0(1-\delta)}{(\beta_1-\beta_2)e^{X^*\beta_2}}; e^{X^*} = \frac{\Pi_0(1-\delta)}{\eta_1 c}\frac{\beta_1\beta_2(\eta_1-1)}{(1-\beta_1-\beta_2+\beta_1\beta_2)}$$

The following numerical example helps us to illustrate the relationship between the different parameters of the model and the optimal value for the adoption of new technology.

Fig1 :the relation between the profit0 and the optimal value for the adoption of NT

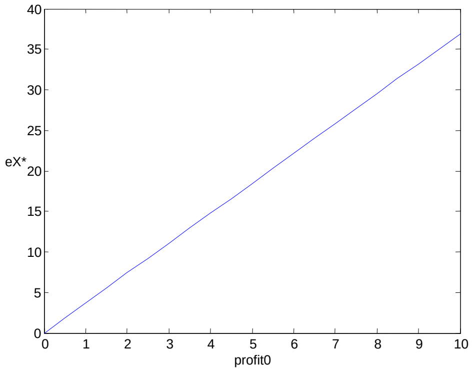


Fig2: the relation between eta1 and the optimal value for the adoption of NT

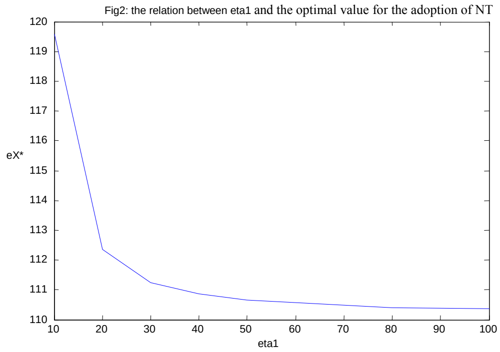


Fig3: the relation between absorption capacity (AC) and the optimum value for adoption

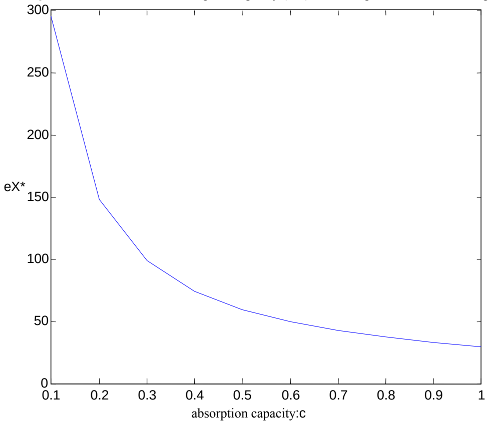


Fig4: the relation between delta and the optimum value for adoption

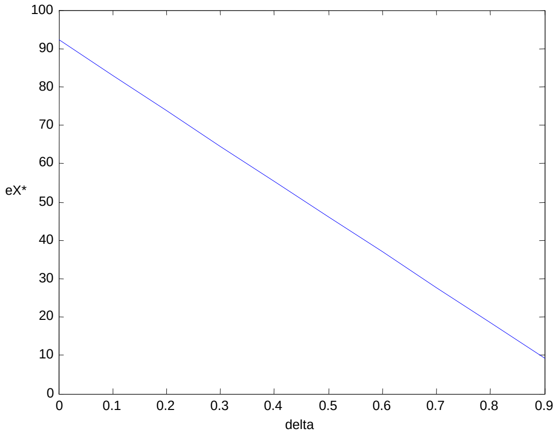
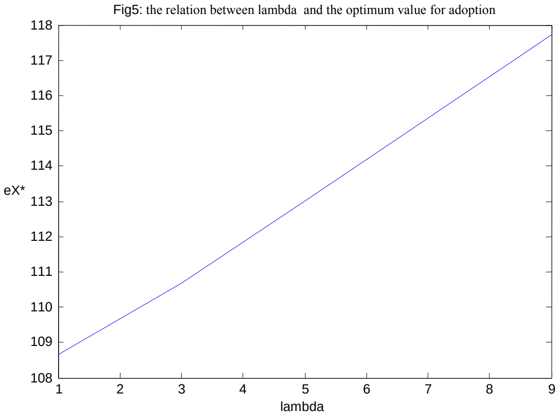


Fig5: the relation between lambda and the optimum value for adoption



The observation of these diagrams shows:

- A positive relation (Fig.1) between the current innovation profit and the optimum adoption level. Thus, the adoption of future technology will be postponed to a later date. Whereas, (Fig.4) shows that the adoption will be fast if the future benefit of NT will be important.
- Compared to the model of Farzin and Huissman (1998) our model verifies the intuition about the relationship between the size of the NT and its adoption. The

company will be encouraged to rapidly adopt the NT with a large size (diagram2). But we got the same result for the effect of arrival rate of NT (diagram5). This reinforces the idea that rapid technological progress does not necessarily mean more efficient technique. Rather, companies wait and adopt the NT when they are sufficiently advanced.

- The effect of absorption capacity (Fig.3) has the expected sign: the more capacity is important, the faster decision about adoption becomes.

III-The strategies of jumping and "buy and hold"

The G option consists in choosing between adopting current innovation, adopting the NT without the current or keeping the old technology. Thus, we suppose that there exists a level $e^{X^{**}} < e^{X^*}$ at which it is optimal to invest in current innovation. This choice is expressed by the following program:

$$\frac{1}{2}\sigma^2 G'' + (\alpha - \frac{\sigma^2}{2} - \lambda\xi)G' + \lambda\int_{-\infty}^{+\infty}[G(X+Y) - G(X)]df_Y(y) = rG$$

subject to the boundary conditions

$$G(e^{X^{**}}) = F(e^{X^{**}}) + \max(\Pi_0, \Pi_{-1}) \qquad (1)$$

$$G'(e^{X^{**}}) = F'(e^{X^{**}}) \qquad (2)$$

$$G(e^{X^*}) = ce^{X^*} + \max(\Pi_1, \Pi_{-1}) \qquad (3)$$

The first boundary is the value matching condition implies that the exercise of the option G allows to the firm: the option to move to the future NT and the difference between the profit of current technology (Π_0) and the old technology profit (Π_{-1}).

The second boundary is the smooth pasting condition. In the third boundary the firm hasn't adopted the current innovation. Consequently, the firm must choose either adopting the future NT at the moment of its arrival (jumping the current technology) or keeping the old technology until the arrival of the future technology. At this point the firm adopts the current technology.

The resolution of this program gives the following solution:

$$G(X)=\begin{cases} Ce^{X\beta_1}+De^{X\beta_2} & \text{si} \quad e^X < e^{X^{**}} \\ \mathrm{F}(e^{X^{**}})+\Pi_0-\Pi_{-1} & \text{si} \quad e^{X^{**}} \le e^X < e^{X^*} \\ cX^*+\Pi_1-\Pi_{-1} & \text{si} \quad e^X \ge e^{X^*} \end{cases}$$

With[4]

$$D = B - \frac{\beta_1}{(\beta_2-\beta_1)}\Pi_0(1-\delta)e^{-\beta_2 X^{**}}$$

$$C = A + \frac{\beta_2}{(\beta_2-\beta_1)}\Pi_0(1-\delta)e^{-\beta_1 X^{**}}$$

$$e^{X^{**}} = \left(\frac{f}{g}\right)^{-\frac{1}{\eta_1}} e^{X^*}$$

$$f = \frac{\Pi_0(1-\delta)\beta_2}{(\beta_2-\beta_1)(\beta_1-\eta_1)} - \frac{\Pi_0(1-\delta)\beta_1}{(\beta_2-\beta_1)(\beta_2-\eta_1)} + \frac{\Pi_0-\Pi_{-1}}{\eta_1}$$

$$g = \frac{ce^{X^*}}{(1-\eta_1)} - \frac{Ae^{\beta_1 X^*}}{(\beta_1-\eta_1)} - \frac{Be^{\beta_2 X^*}}{(\beta_2-\eta_1)} + \frac{\Pi_0(1-\delta)}{\eta_1}$$

This option high lights three important remarks:

- As shown in Table 1: the difference between the tow optimum adoption levels of current and future technology becomes lower when the absorptive capacity (AC) of the firm increases.

Table1: relation between AC, e^{X^*} and $e^{X^{**}}$

CA	**e^{X^*}**	**$e^{X^{**}}$**	**The difference : e^{X^*}- $e^{X^{**}}$**
0.3	218.9169	109.3169	109.6
0.5	131.3502	65.5901	65.7601
0.6	109.4585	54.6585	54.8
0.7	93.8215	64.8501	28.9714
1	65.6751	32.7951	32.88

[4] A, B and e^{X^*} are given by option F.

- However, table 2 shows that this gap widens when the profit of the current innovation is very significant. Adopting the future technology is postponed to a later date.

Tableau 2 : relation between Π_0, $\mathbf{e}^{X^*}$ and $\mathbf{e}^{X^{**}}$

profit0	e^{X^*}	$e^{X^{**}}$	The difference : $e^{X^*} - e^{X^{**}}$
10	131.3502	65.5901	65.7601
20	262.7003	132.4189	130.2814
50	656.7508	332.5271	324.2237
80	1058	536.0446	521.9554

- The increase in volatility on the market (σ) leads to a negative effect on the adoption decision. The higher this uncertainly on the market is, the more the firm postpones the current innovation. Consequently the speed of adoption of new technologies is slowing down.

Table 3: relation between σ, $\mathbf{e}^{X^*}$and $\mathbf{e}^{X^{**}}$

σ	e^{X^*}	$e^{X^{**}}$	The difference : $e^{X^*} - e^{X^{**}}$
0.1	152.3097	0	
0.2	202.72	80.8122	121.9078
0.3	271.6953	110.5912	161.1041
0.5	467.3039	186.516	280.7879
0.8	916.4041	380.5845	535.8196

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