MODELING ELECTRICITY PRICES IN BRAZIL: APPLICATION IN AN ELEPHANT GRASS BIOMASS POWER PLANT WITH SWITCH OPTION

Carlos Frederico Fontoura
ctfonto@gmail.com
IAG Business School, Pontifícia Universidade Católica do Rio de Janeiro

Luiz E. Brandão
brandao@iag.puc-rio.br
IAG Business School, Pontifícia Universidade Católica do Rio de Janeiro

Leonardo Lima Gomes
Pontifícia Universidade Católica do Rio de Janeiro
leonardolima@iag.puc-rio.br

Carlos Bastian-Pinto
Universidade do Grande Rio - Unigranrio
carbastian@gmail.com

Abstract

A major problem in the evaluation of power generation projects is the stochastic modeling of future prices. In Brazil, this problem is affected by the country's significant dependence on hydropower, which causes the price of short-term energy to behave differently from other markets. In this paper we propose a pricing model that incorporates both short-term uncertainties and long-run equilibrium through a mean-reverting model with jumps. We apply this model to an elephant grass power plant, which represents an alternative for the expansion of the Brazilian thermoelectric park since it helps to achieve the diversification of Brazil's energy matrix by means of a less polluting renewable source. Two scenarios were established. For the base case we adopt a power plant with no price uncertainty or operational flexibility. Next we assume the plant has the option to sell part of its energy in the uncertain short term electricity market or switch outputs and sell biomass in the form of energy briquettes through the installation of a briquetting unit. The results indicate that use of the proposed pricing model, associated with the insertion of the switch option increased the value of the project by 27.91% when compared to the base case. Considering that power generation project using biomass tends to gain an increasing importance especially with regard to the reuse of agricultural or industrial waste, the correct assessment of the value of these projects through an appropriate modeling of electricity prices in the short-term and the use of a methodology that incorporates the uncertainties of the market and managerial flexibilities, such as the real options theory, becomes essential to attract private sector investment.

Keywords: Electricity Price Models, Biomass, Elephant Grass, Switch Option

Introduction

The electricity market involves several agents, such as generators, consumers and regulators, whose respective generation capacity and energy demand decide the price of electricity in the market (Schweppe, Caraminis, Tabors, & Bohn, 1988). An important feature of this market is the need to estimate future demand and electricity price (Bunn, 2000).

According to Karakatsani & Bunn (2008), the electricity price time series of current markets have as main characteristics a high frequency, non-constant mean and variance, multi-seasonality, high volatility and a large number of atypical movements, such as price spikes with values much higher than average. All these characteristics are due to factors such as the difficulty of storage, the need for constant balance between generation and demand and the inelastic nature of short term demand, which also makes electricity a commodity different from others. The high volatility of these markets makes price forecasting tools essential to the survival of its agents (Birge, Cai, & Kou, 2010; Cartea & Figueroa, 2005; Deng, 2000; Hambly, Howison, & Kluge, 2009; Jong, 2006).

In Brazil, the operation of the power generation system is a responsibility of the National System Operator (ONS), whose aim is to guarantee the supply and minimize the total cost of operation over a planned horizon. The price of electricity in the short-term (known in Brazil as Differences Settlement Price, or PLD) is determined weekly by the Electric Energy Clearing Chamber (CCEE) through the use of an optimization model. This model, called Newave, consists of a coupling of a short-term and a long-term model that use the method of stochastic dual dynamic programming (Pereira & Pinto, 1991).

Due to the preponderance of hydroelectric generation in Brazil, the mathematical models used to calculate the PLD attempt to find the optimal balance solution between the present benefit of using water in the reservoirs and the future benefit of its storage, measured in terms of the expected fuel economy of thermal power plants. On the one hand, the maximum use of hydropower available in each period is the most economical option from the immediate point of view. This course of action minimizes fuel costs, but results in a higher risk of future deficits. On the other hand, the maximum supply reliability is obtained by maintaining reservoir levels as high as possible, using therefore, more thermal generation which results in greater operating costs (CCEE, 2010).

The need to maintain an optimal balance solution gives Brazil’s electricity market an important singularity when compared to other markets. Hydropower is associated with low marginal costs of production and the reservoirs can be considered as electricity stocks. Thus, the short-term price of electricity in Brazil hardly ever has an instantaneous large variation such as the spikes seen in markets where the energy matrix is essentially thermal such as the United States. Since the methodology used to determine the PLD causes its value to depend largely on the availability of water in reservoirs and the level of rainfall, this value rises gradually as the weather reports indicate a dry season longer than expected, which can take weeks. If the level of rainfall returns to normal, the value of the PLD gradually returns to normal as the risk of drought is reduced. However, if the long dry season actually happens, the PLD may remain at a considerable high value for weeks or even months. Thus, electricity price forecast models developed for other markets may not be fully appropriate for the characteristics of Brazil’s electricity market.

The strong dependence on hydropower has proved negative for Brazil in some situations, among which we can highlight the electricity rationing occurred between June 2001 and February 2002 due to low rainfall in previous years. As a result, in 2004 the Brazilian government created the Incentive Program for Alternative Sources of Energy (PROINFA) in order to increase the share of renewable energy sources like wind, biomass and small hydropower (SHP) in the energy matrix. PROINFA aimed at the installation of 144 power plants totaling 3,299 MW of installed capacity, with 685 MW from 27 plants based on biomass. The contracts were guaranteed for 20 years by the Brazilian Electric Power Company (Eletrobrás) and the projects could be financed by PROINFA’s Financial Support Program created by Brazil’s National Bank of Economic and Social Development (BNDES), which allowed the financing of up to 80% of the project and amortization in 12 years (MME, 2010b).

The Brazilian government also created a commercialization rule where power generation projects using fostered energy sources such as solar, wind and biomass with installed capacity up to 30MW had a discount of 50% or 100% in the transmission and distribution System usage fee. These actions promoted the adoption of these alternative sources and demonstrated Brazil’s potential in adopting a more sustainable energy matrix.

Brazil has several comparative advantages regarding agricultural, agro-industrial and forestry products, in particular those dedicated to energy. Some of the most significant advantages are the vast expanse of agricultural areas available, intense solar radiation, the great availability of water, the climatic diversity and the interaction between the centers of agricultural research as the EMBRAPA and agribusiness (MME, 2007). Therefore, the use of biomass power plants represents an advantageous alternative for the expansion of the thermoelectric park since it helps achieve the diversification of Brazil’s energy matrix by means of a less polluting renewable source.

In this article we propose some modifications to a traditional electricity price model in order to adapt it to the particularities of the Brazilian electric market. We then apply this modified model to the case of an elephant grass power plant investment under conditions of electricity price uncertainty, considering that it has the flexibility to sell the generated electricity wholly or partly in the long or short-term markets. In addition, there is the possibility of setting up a briquetting unit so that the plant can sell electricity in short-term market or elephant grass in the form of energy briquettes.

This paper is organized as follows. After this introduction we analyze some of the electricity price forecast models present in the literature regarding Brazil and other international markets. In section 3 we analyze the use of biomass for power generation, and in particular the characteristics of elephant grass. In section 4 we present the proposed model to simulate the short-term electricity price and calculate the parameters for the Brazilian case. In section 5 we develop the model of a biomass power plant under different operating scenarios and then present the results and conclusions.

Modeling the short-term electricity price

According to Box, Jenkins & Reinsel (2008), an analysis of electricity prices in fourteen markets showed that the evolution of prices is different in each one, making it difficult to apply a single model for every market.

Two models widely used in the modeling of stochastic processes are the Geometric Brownian Motion (GBM) and Mean Reversion Movement (MRM). However, for the case of electricity prices is more common to use the MRM (Bastian, Jinxiang, Banunarayanan, &

Mukerji, 1999; Box et al., 2008; Contreras & Santos, 2006; Deb, Albert, Hsue, & Brown, 2000).

Besides the mean reversion characteristic and a strong seasonality, electricity prices also show significant but infrequent jumps. These jumps can multiply the value of the short-term electricity price several times in a period of up to one hour in markets with hourly variation and are due to unexpected fluctuations in generation or demand caused by severe weather conditions, technical problems or fuel supply. These jumps are usually of short duration and once the problem is solved, prices tend to return to their normal level. Several efforts were made to develop models that incorporate these two processes such as the ones presented in Baron, Rosenberg & Sidorenko (2002), Bunn (2000), Clewlow, Strickland & Kaminski (2000), Contreras & Santos (2006), Ethier & Dorris (1999) and Goldberg & Read (2000).

Eydeland & Geman (1999) suggested a similar model where the mean reversion component also appears in the jump component in order to characterize the fact that jumps are more severe during periods of high prices. Geman & Roncoroni (2003) proposed the use of downward jumps together with the process of mean reversion while Weron, Simonsen & Wilman (2003) suggested using upward jumps followed by downward jumps to capture the rapid price decline after the peak.

In Brazil, Amaral (2003) studied modeling strategies involving linear and nonlinear time series, Medeiros (2003) proposed the use of neuro-fuzzy systems for short-term price forecasting and Sousa (2003) suggested the use of structural models. Pemberton Jr. (2006) examined five models to describe the behavior of the short-term electricity price in Brazil which considered several factors such as mean reversion, regime switch and diffusion with Markovian jumps.

The model adopted in this paper as the basis for the short-term electricity price modeling is the mean reversion model with jumps proposed by Clewlow, Strickland & Kaminski (2000), modified to the particularities of the Brazilian market.

The Clewlow, Strickland & Kaminski (2000) modified model

The model proposed by Clewlow, Strickland & Kaminski (2000) (eq. (1)) can be described as the combination of a Schwartz (1997) model 1 with a Poisson process of jump diffusion.

$$dS = \eta(ln\bar{S} - lnS)Sdt + \sigma dz + kSdq \tag{1}$$

where $dS = \eta(ln\bar{S} - lnS)Sdt + \sigma dz$ is the Schwartz (1997) model 1, $kSdq$ is the Poisson process of jump diffusion and k is the proportional size of the jump which is random and determined by the natural logarithm of the proportional jumps being normally distributed:

$$ln(1-k) \sim N\left(ln\left(1+\bar{k}\right) - \tfrac{1}{2}\gamma^2, \gamma^2\right) \qquad (2)$$

where $\bar{k}$ is the average size and γ is the standard deviation of the proportional size of the jump.

The jump diffusion process is a discrete-time process where the jumps do not occur continuously, but in specific moments. Therefore, for typical frequencies of jumps, $dq = 0$ most of the time and only assumes value 1 when the random time of a jump occurs. So, when no jump is occurring the behavior of short-term electricity prices is identical to a simple movement of mean reversion.

In order to use the model to perform the simulation of the stochastic variable, you must first obtain its equation in discrete time. Clewlow, Strickland & Kaminski (2001) define $x = lnS$ and propose eq. (3) for price simulation:

$$\Delta S_i = \left\{\left[\eta(ln\bar{S} - S_i) - \tfrac{\sigma^2}{2}\right]\Delta t + \sigma\sqrt{\Delta t}\varepsilon_{1i} + \left(\bar{k} + \gamma\varepsilon_{2i}\right)(u_i < \phi\Delta t)\right\} \qquad (3)$$

where:

$\bar{k}$ and γ have been previously defined;

S is the stochastic variable;

$\bar{S}$ is the long-run equilibrium level of the stochastic variable;

η is the speed of reversion;

σ is the volatility;

ε_1 and ε_2 are independent random variables with standard normal distribution;

ϕ is the jump frequency;

u_i is a random number between 0 and 1 with uniform distribution.

Equation (3) can be split into two components where $\left[\eta(ln\bar{S} - x_i) - \frac{\sigma^2}{2}\right]\Delta t + \sigma\sqrt{\Delta t}\varepsilon_{1i}$ is the mean reversion component and $\left(\bar{k} + \gamma\varepsilon_{2i}\right)(u_i < \phi\Delta t)$ is the jump component.

In order to adapt the model to the Brazilian market, we replace the mean reversion component with the equation for Schwartz (1997) model 1 simulation proposed by Bastian-Pinto (2009), which provides an exact discretization, allowing the use of high values for Δt. Next we change the independent random variable ε_2 from a standard normal distribution to a standard log-normal distribution. This modification is necessary for two reasons: First, in this study we consider only upward jumps and the use of a variable with standard normal distribution would result in both upward and downward jumps. It is important to notice that given that the jump component acts directly on the price levels in our case, such downward jumps could result in negative values for the electricity prices, which is impossible to happen. This is due to the fact that while Clewlow, Strickland & Kaminski (2001) define the parameter values of the jump component for a series of proportional jump returns, we adopt a series of nominal values for the jumps. Considering the relationship between the log-normal and normal distributions shown in eq. (4), we find that the use of a log-normal distribution for the series of nominal values of jumps in this study is consistent with the use of a normal distribution for the series of proportional jump returns in the original model.

$$\text{If } X \sim N(\mu, \sigma^2) \text{ then } e^X \sim Log - N(\mu, \sigma^2) \tag{4}$$

Finally, we transform the model in a risk neutral process by subtracting the normalized risk premium $\left[\frac{(\mu - r)}{\eta}\ ou\ \frac{\pi}{\eta}\right]$ from the long-run equilibrium level, where μ is the risk adjusted discount rate, r is the risk free interest rate and π is the risk premium. Therefore, the risk-neutral modified equation used in this study is given by (5):

$$S_t = exp \left\{ \begin{matrix} ln[S_{t-1}]e^{-\eta \Delta t} + \left[ln(\bar{S}) - \frac{\sigma^2}{2\eta} - \frac{\pi}{\eta}\right](1 - e^{-\eta \Delta t}) + \\ \sigma \sqrt{\frac{1 - e^{-2\eta \Delta t}}{2\eta}} N(0,1) \end{matrix} \right\} + \left[\left(\bar{k} + \gamma Log - N(0,1)\right) \therefore (u_i < \phi \Delta t)\right] \tag{5}$$

As stated before, the jump diffusion process is a discrete-time process where jumps do not occur continuously, but at specific moments. In order to establish when these moments occur, we use the term $(u_i < \phi \Delta t)$ and define that its value is equal to 1 if the condition is true and 0 if false. If we consider the uniform distribution of the values of u_i and that $\phi \Delta t$ is the probability of a jump occurring, the probability of the value u_i being less than $\phi\Delta t$ is exactly the probability of a jump. Thus, it generates the jumps randomly in the correct average frequency in the limit where Δt tends to zero.

Parameters estimation

The parameters estimation of the mean reversion process (η, σ e $\bar{S}$) follows Bastian-Pinto (2009). Since the jumps can only be seen as part of a series that also includes the normal behavior of mean reversion, it is necessary to filter the jump diffusion process and determine its parameters ($\bar{k}$, γ e ϕ) before the estimation of the mean reversion parameters. Clewlow, Strickland & Kaminski (2001) adopt what they call a recursive filter. Although this methodology is suitable for markets where the electricity price is characterized by long periods of low values and instantaneous peaks (spikes), it has little precision in markets where the price rises or drops gradually, such as the Brazilian market. Given the inapplicability of the recursive filter we assume in this study that values above R$200.00 will be considered jumps.

To estimate the jump parameters we purged from the series the values below the limit of R$200.00 and determined its average ($\bar{k}$) and standard deviation (γ). Since the jump is a spurious occurrence, the estimation of an average value for this occurrence is not robust and, therefore, we will consider $\bar{k} = 0$.

Simulating the short-term electricity price

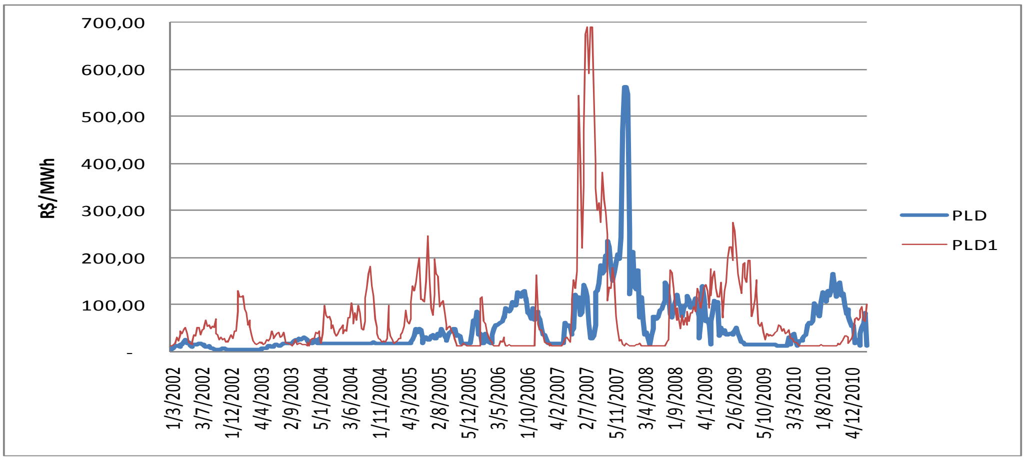
The short-term electricity prices time series can be obtained at CCEE's website. Although the series date back to 2001, through its start until early 2002 prices remained very high due to electricity rationing. The factors that led to this situation were subsequently mitigated through various actions such as the construction of new transmission lines and massive investments in power generation, and as a consequence, we assume that it is unlikely that this scenario will repeat itself in the future, at least at the previous level. Therefore, the time series used will skip this period and go from March 2002 to December 2010 on a weekly basis. Moreover, the series was deflated by the IGP-M index (FGV), since this is the index used to adjust electricity prices for inflation. Table 1 below shows the resulting values for the simulation parameters.

Table 1 – Simulation's parameters

Parameter	Value	Parameter	Value
Δt	1,0000	ϕ	0,0297
η	0,0503	$\bar{k}$	0
σ	0,3013	γ	151,2916
$\bar{S}$	77,8799	Π	0,0023

Finally, the initial value of the simulated series must be set. For this purpose, we used the PLD for the Southeast/Central-West submarket for the second week of March 2011 (12.08 R$/MWh). Applying the parameters shown in table 1 in eq. (5) and limiting the values to the minimum (12.08 R$/MWh) and maximum (689.18 R$/MWh) set by ANEEL for 2011 we obtain the simulation shown in Figure 1.

Figure 1 - Simulation of the short-term electricity price with limits



Biomass

The concept of biomass encompasses any renewable resource coming from organic matter. ANEEL defines the term biomass as:

> *Every renewable resource coming from organic matter (animal or plant) that can be used to produce energy. Just like hydropower and other renewable sources, biomass is an indirect form of solar energy. Solar energy is converted into chemical energy through photosynthesis, the basis of the biological processes of all living beings. (ANEEL, 2005 p. 77)*

This broad definition includes sources such as wood, agricultural, industrial or parks and garden wastes and, above all, dedicated energy crops that are forests or grass plantations designed specifically for this purpose (Mazzarella, 2007). Among the many advantages of using biomass for energy are the low cost of production in the case of wastes, and the zero balance of carbon dioxide emission. However, the diversity of sources also hinders government control over its origins and contributes to the association of biomass use with problems such as deforestation and desertification (ANEEL, 2008).

The growth of biomass' share in the Brazilian energy matrix has been steady in recent years as can be observed in the National Energy Balance 2010 report (EPE, 2010) that shows an increase of 17.5% in domestic supply of energy from biomass (from 23.3 TWh to 27.4 TWh). It is important to highlight that this growth occurred mainly in the form of co-generation systems in which it is possible to obtain thermal and electrical energy. Such power generation systems are usually installed on plants that weren't initially intended for the generation of electricity.

This fact can be inferred by observing the distribution of biomass power plants by type of input shown in table 2. The number of power generating units that have sugarcane bagasse as input is far superior to the others, as these units use the waste of sugar and ethanol plants to generate thermal energy and electricity for the productive process of the mill, with the surplus being exported to the Brazilian Electric System (SEB) grid.

Table 2 - biomass power plants in operation in 2008

Input	Thermal Power Plants	Total Installed Capacity
Sugarcane Bagasse	252	4000 MW
Black Liquor	13	944 MW
Wood	27	232 MW
Biogas	3	45 MW
Rice Husk	4	21 MW

Source: ANEEL (2008)

Even considering the recent growth, biomass still has a share of only 5.4% of the domestic supply of electricity in SEB. However, due to the importance given to investment in renewable sources of energy in the Energy Development Plan 2010-2019 report, it is expect an ever increasing growth of the installed capacity of biomass power generation. Nonetheless, this will not result in an increase in the biomass share of the total primary energy supply in Brazil due to the growth of other energy sources, as shown in Table 3.

Table 3 –Forecast of installed capacity of biomass power plants

Source	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
Biomass (MW)	5.380	6.083	6.321	6.671	7.071	7.421	7.621	7.771	8.121	8.521
Share	4,78%	5,14%	5,15%	5,10%	5,30%	5,27%	5,16%	5,11%	5,15%	5,10%

Source: MME (2010a)

Currently, the most explored source is the sugarcane bagasse, mainly due to the high productivity of sugarcane plantations, associated with the installation of cogeneration power unit in plants and distilleries producing sugar and ethanol. Other important advantages of this culture are the large amount of waste generated by the main activity (sugar and ethanol) that can be harnessed for power generation, the complementary timing of the harvest period with the dry period of Brazilian river basins that are the main source of energy in the SEB, and finally, the proximity to large consuming centers such as São Paulo and major capitals of the northeast (MME, 2007).

Elephant Grass

Elephant grass (*Pennisetum purpureum*) is a forage grass discovered in 1905 by Colonel Napier in tropical Africa. It was introduced in Brazil in 1920 and is now widespread throughout the country (Lopes, 2004). Being a perennial grass, it does not need to be replanted after each harvest and reaches 9 to 16 feet tall with 2 cm in diameter. Its use is closely associated with grazing, especially as dairy cattle feed, which is object of extensive research by Embrapa.

The first studies of the use of Elephant grass as a biomass source for energy date back from the mid-90s and regarded initially its use in the steel industry. Such interest is justified by its high biomass productivity due to its high photosynthetic efficiency. This productivity is apparent when we compare with other common biomass sources as shown in Table 4.

Table 4 – Comparison of energy yield per hectare annually

Energy Sources	Energy Yield	Energy Yield per Hectare Annually
	kcal/Kg	kcal/kg.ha per year
Elephant Grass Carajás (i.e. Paraíso)	4.200	189.000.000
Brachiaria brizantha grass	3.900	97.500.000
Eucalyptus grandis	4.641	92.820.000
Sugarcane Bagasse	3.700	29.600.000

Source: Vilela e Cerize (2010)

As can be seen in table 4, the energy yield per kilogram of elephant grass is only slightly higher than the energy yield of the *Brachiaria* grass and sugarcane bagasse. It was even lower than the amount yielded by eucalyptus, which is currently the main source of cellulose and charcoal. However, when we analyze the energy yield per hectare per year, we can see that it is far superior to the others. This is due to the fact that, while eucalyptus produces up to 20 tons of dry matter per hectare annually, elephant grass can produce between 20 and 60 tons. In addition, a forest of eucalyptus trees take up to seven years to grow to a level where it can use it as a source of biomass, while the first crop of elephant grass can be harvested after one hundred and eighty days (Vilela & Cerize, 2010). So, when compared with other sources of biomass such as sugarcane bagasse and eucalyptus the elephant grass is a source of higher energy yield potential that also adapts well to cultivation conditions existing in Brazil.

This higher energy yield is one of the main advantages of elephant grass both in economic, social and environmental terms, since the plant uses a smaller area of cultivation for the same installed capacity. For comparison purposes, a 30MW plant requires an elephant grass cultivation area with 6,094 ha. If the same plant was to use eucalyptus as its source of biomass, the cultivation area would need to be at least twice as large. Besides resulting in smaller acquisition and maintenance cost from the economic standpoint, elephant grass also optimizes the use of the land for energy purposes by occupying less agriculture land, which is one of the main arguments against energetic crops from the environmental and social point of view.

Elephant grass can also be used to produce briquettes. Briquettes are the result of a process of compaction of chopped biomass (less than 50 mm) in which the natural elasticity of the fiber is destroyed through the use of high pressure and/or high temperature. With the destruction of the elasticity, lignin acts as a binder for the biomass particles making the briquettes suitable for storage and transportation (Vilela, 2010). Briquettes have several advantages over uncompressed biomass such as homogenization of vegetable waste, drying and condensation of energy, particle size and shape more appropriate for the thermal process, lower dust content generating less ash, low risk of explosion and easier biomass transportation and storage.

Application in an elephant grass power plant with switch option

We consider a power plant with an installed capacity of 30 MW that has two possible scenarios for the commercialization of the electricity generated. In the first scenario, called the base case, the plant sells its entire capacity through a 20 year long-term supply contract at a pre-determined fixed price. In this scenario, there is no market uncertainty and the plant's future cash flows are known.

In the second scenario, called the case with switch option, the plant adopts a hybrid commercialization model where part of the installed capacity (25 MW) is sold through a 20 year long-term supply contract and the remainder (5MW) is sold in the short-term market, subject to the uncertainties of the electricity price. In order to mitigate the risk associated with this uncertainty, since these 5MW are equivalent to a certain amount of available biomass there is the possibility of setting up a briquetting unit. This unit will allow the conversion of the available biomass in a product with higher market value, thus creating the flexibility to switch outputs. Thus, in the case with the switch option, the plant may optimally choose on a weekly basis whether to sell electricity in the short-term market or briquettes.

Although other scenarios with different sales ratios in the long and short-term markets are technically feasible, we limited the amount sold in the short-term to 5MW in order not to impair the financing viability of the plant within the PROINFA program. Each case has different revenues, costs and tax rates.

Base Case

The net revenue from the sale of electricity through a long-term contract is given by eq. (6):

$$NR_{LT} = V_{LT}(P - TUST - VC_e - DT_e \times P) \tag{6}$$

Where:

V_{LT} Volume of electricity sold through long-term contracts;

P	Long-term electricity price;
$TUST$	Transmission System Usage Fee;
VC_e	Variable cost of electricity production;
DT_e	Direct taxes over electricity commercialization.

The net present value of the base case is given by eq. (7) $NPV_{BC} = \sum_{t=1}^{n} \frac{CF_{BC}(t)}{(1+\mu)^t} - I$, where

$$CF_{BC}(t) = EBIT(t) \times (1 - taxes) + Depreciation(t) - CAPEX - \Delta CGOL$$

and

$$EBIT(t) = NR_{LT}(t) - FC_e \times Cap_{plant} \times ER_{dollar} - FC_{Bio} \times A_{cultivation} - Depreciation$$

Where:

n	Lifetime of the project;
$CF_{CB}(t)$	Cash flow of the base case;
μ	Discount rate;
I	Investment in t=0;
$NR_{LT}(t)$	Net revenue from the sale of electricity through a long-term contract;
FC_e	Fixed cost of the plant;
Cap_{plant}	Capacity of the plant;
ER_{dollar}	Real x Dollar exchange rate (PTAX);
FC_{Bio}	Fixed cost of biomass production per hectare annually (cultivation area maintenance, biomass harvesting, chopping, drying, compaction and transportation);
$A_{cultivation}$	Area used for elephant grass cultivation;
Taxes	Taxes over EBIT;
Depreciation	Depreciation in the respective year;
CAPEX	Total investment in fixed assets;
$\Delta CGOL$	Net operating capital variation.

Since the project requires a biomass stock before it starts to generate electricity and the farming area needs to pass through a formation process before starting the cultivation of elephant grass, the necessary investment can be divided as follows:

- I_0 = 50% of the investment required for the construction of the plant (I_{ee}) + 100% of the cultivation area acquisition cost $\left(I_{aq}\right)$ + 100% of the formation process cost $\left(I_{cf}\right)$;
- $FC_{BC}(t = 1)$ = 50% of the investment required for the construction of the plant (I_{ee}) + 100% of $CF_{Bio} \times A_{cultivation}$ (first batch of biomass).

Case with switch option

When the short-term price of electricity falls, the company may choose not to generate electricity and use the available biomass to produce briquettes. The case with switch option is then characterized by a fixed annual cash flow from the sale of electricity through long-term contracts (equivalent to 25MW) added to the cash flow from the switch option of choosing between selling electricity in short-term market (equivalent to 5MW) or selling briquettes. Therefore, in order to calculate the total cash flow of this case, we need first to calculate the net revenue generated by the switch option.

To do that, we simulate the short-term electricity price for the next 20 years, which is the expected life of the project, on a weekly basis, since the PLD is determined by CCEE with this periodicity. Then we calculate the net revenue from the sale of energy in the short-term market with the following eq. (8):

$$NR_{spot} = V_{spot} \times [\left(P_{spot} + Premium\right)(1 - DT_e) - TUST - VC_e] \tag{8}$$

Where:

V_{spot}	Volume of electricity sold in the short-term market;
P_{spot}	Short-term electricity price;
$Premium$	Premium paid by the market for electricity from fostered energy sources;

Regarding the net revenue from the sale of briquettes, it will also be calculated on a weekly basis since its value must be compared to the net revenue from the sale of energy in the short-term market. The net revenue from the sale of briquettes is given by eq. (9):

$$NR_{briq} = V_{briq} \times [P_{briq} - \left(SC_{briq} + PC_{briq}\right) - DT_{briq} \times P_{briq}] \tag{9}$$

Where:

V_{briq}	Volume of briquettes sold;

P_{briq}	Price of a ton of briquettes in the wholesale market (CIF);
SC_{Briq}	Shipping cost to São Paulo;
PC_{Briq}	Packing cost;
DT_{briq}	Direct taxes over briquette commercialization.

The switch option can be modeled as a sequence of European options since the option of selling one output in a week is totally independent from the choice made in any other week. The optimization of the choice process is given by eq. (10):

$$NR_{OP}(y) = max\left(NR_{spot}(y)\,, NR_{briq}(y)\right) \tag{10}$$

Where:

$NR_{OP}(y)$	Net revenue of the switch option in week y;
$NR_{spot}(y)$	Net revenue from the sale of energy in the short-term market in week y;
$NR_{briq}(y)$	Net revenue from the sale of briquettes in week y.

Since the net revenue for the case with switch option results from adding the net revenue from the sale of electricity through long-term contracts to the one from the switch option, this cases net present value can be obtained through eq. (11) $NPV_{CO} = \sum_{t=1}^{n} \frac{CF_{CO}(t)}{(1+\mu)^{t}} - I$, where:

$$CF_{CO}(t) = EBIT(t) \times (1 - Taxes) + Depreciation(t) - CAPEX - \Delta CGOL$$

and

$$EBIT(t) = NR_{LT}(t) + \sum_{1}^{52} NR_{OP}(t) - FC_{e} \times Cap_{plant} \times ER_{dollar} \text{ - } FC_{briq} \times CapBriq_{unit} \text{ - } FC_{Bio} \times A_{cultivation}$$

Where:

$CF_{CO}(t)$	Cash flow of the case with switch option;
$\sum_{1}^{52} NR_{OP}(t)$	Net revenue of the switch option in year t calculated trough the addition of the weekly net revenues for the respective year;
FC_{briq}	Fixed cost of the briquetting unit;
$CapBriq_{unit}$	Annual production capacity of the briquetting unit.

The net revenues of the switch option are weekly, but they were consolidated in an annual basis in order to facilitate the calculation of the net present value of the case. Since the biomass will only be available at the end of the second year, the investment in the briquetting unit can be made integrally in the second year. Therefore, the necessary investment can be divided as follows:

- I_0 = 50% of the investment required for the construction of the plant (I_{ee}) + 100% of the cultivation area acquisition cost $\left(I_{aq}\right)$ + 100% of the formation process cost $\left(I_{cf}\right)$;
- $FC_{CO}(t = 1)$ = 50% of the investment required for the construction of the plant (I_{ee}) + 100% of $CF_{Bio} \times A_{cultivation}$ (first batch of biomass) + 100% briquetting unit cost (I_{Briq}).

Assumptions, used data and results

The main variables estimated for this study can be divided into three groups: variables related to investment and fixed costs of the power plant, to the net revenue from the sale of briquettes and to the net revenue from the sale of electricity in the short and long-term markets. We adopted a risk adjusted discount rate of 11.5%, based on the discount rates used in the Brazilian National Energy Plan 2030 (8, 10 and 12%). In addition, there will be an analysis of sensitivity to the discount rate using these same rates.

Table 5 – Variables related to investment and fixed costs of the power plant

Variable	Value	Unit	Source
ER_{dollar}	1.6669	BRL/USD	PTAX of February 18th, 2011
Cap_{plant}	30	MW	Defined for the project
$CapBriq_{unit}$	24,546	t/year	Defined for the project
n	20	years	Defined for the project
I_{ee}	1,500	USD/kW	MME (2007)
I_{Briq}	2,630,299.10	BRL	Adapted from (Silva, Felfli, Pérez, Rocha, & Simões, 2006)
I_{aq}	3,565.49	BRL/ha	Adapted from IEA (2010)
I_{cf}	1,515.21	BRL/ha	Mazzarella (2007)
$Elephant\ grass\ productivity$	37.5	t.ms/ha/year	(Andrade et al., 2000a); (Andrade et al., 2000b); (Embrapa, 2009)
FC_e	55	USD/kW	MME (2007)
FC_{briq}	10.33	BRL/t	Adapted from (Silva et al., 2006)
FC_{Bio}	1,449.19	BRL/ha	Adapted from Mazzarella (2007)

$A_{cultivation}$	6,094	ha	$Area = \frac{instaled\ capacity}{efficiency\ \times productivity}$
$Taxes$	34	%	Defined for the project
$Depreciation$	10	%/year	Defined for the project

Table 6 – Variables related to the net revenue from the sale of briquettes

Variable	Value	Unit	Source
P_{briq}	Triangular Distribution (120;270;400)	BRL/t	Adapted from Gentil (2008)
V_{briq}	755	t	Adapted from (Silva et al., 2006)
SC_{briq}	33.65	BRL/t	Adapted from Gentil (2008)
PC_{briq}	22.50	BRL/t	(Silva et al., 2006)
DT_{briq}	21.65	%	(Silva et al., 2006)

Table 7 – Variables related to the net revenue from the sale of electricity in the short and long-term markets.

Variable	Value	Unit	Source
P	170	BRL/MWh	(Energia, 2010; EnergiaDireta, 2011)
VC_e	6	USD/MWh	MME (2007)
$TUST$	2.5	BRL/MWh	MME (2007)
DT_e	9.75	%	MME (2007)

For the base case and the case with switch option, this study adopted the values shown in tables 8 and 9 respectively.

Table 8 – Value for the variables of the base case

Variable	Value	Unit	Source
V_{LT}	262,800	MWh/year	Defined for the project

Table 9 – Value for the variables of the case with switch option

Variable	Value	Unit	Source
V_{LT}	219,000	MWh/year	Defined for the project
V_{spot}	4,200	MWh/week	Defined for the project
$Premium$	15.46	BRL/MWh	15,46 BRL/MWh – 50% TUSD Eletropaulo 04/06/2011 (fostered energy).

Based on the variables used and respective data, the mean net present value for the base case was R$ 16,811 thousand and for the case with the switch option was R$ 21,504 thousand. Therefore, adding the switch option raised the value of the project by R$ 4,693 thousand, or 27.91%.

Sensitivity analysis

The definition of the risk adjusted discount rate may vary and it is very common to see different investor applying different discount rates to the same project. In order to see if the proposed model is robust against this variation we conducted a sensitivity analysis using discount rates of 8, 10, 11.5 and 12%. We also created a test case where there was no briquetting unit and the plant would always sell part of its capacity in the short-term market in order to verify what portion of the option value derived from the uncertainties of the short-term electricity price and how much actually came from the switch option itself. The results are shown in table 10.

Table 10 – Sensitivity test results (R$ 1,000,00)

Discount rate	8%	10%	11,50%	12%
Base case	54,074	30,901	16,812	12,629
Case with switch option	60,905	36,400	21,501	17,079
Option value (BRL) (1)	6,831	5,499	4,689	4,449
Option Value (%)	12.63%	17.80%	27.89%	35.23%
Testing case	57,171	32,319	17,324	12,893
Base case - testing case (2)	3,097	1,418	512	263
(2) / (1)	45.33%	25.79%	10.93%	5.92%

As we can observed in table 10, the value of the switch option decreases percentage wise rapidly as we decrease the value of the adjusted discount rate. With a discount rate of 8%, the option value falls to 12.63%. Also, by dividing the increase in value obtained through the testing case by the one obtained through the case with the switch option, we can infer that as we lower the discount rate the influence of the uncertainties of the short-term electricity price grows. With a discount rate of 8%, the value that comes from these uncertainties represents 45.33% of the option value.

However the opposite remains true and for higher values of discount rate we have both high option value e low influence of those uncertainties. Comparing the results shown in table

10, we can infer that the proposed model has viable results for discount rates of 10% or higher.

Conclusions and Recommendations

In this study we proposed an electricity pricing model that incorporates both short-term uncertainties and the long-run equilibrium through a mean-reverting model with jumps adapted to the characteristics of the Brazilian market. We also analyzed the value of inserting a switch output option in a project of power generation with biomass as source. An elephant grass power plant was chosen as the object of study for three reasons: it represents an advantageous alternative of expanding the Brazilian thermoelectric park since it helps achieve the diversification of Brazil's energy matrix with a less polluting renewable source; allows the use of flexible forms of commercialization and/or operation there are more profitable; and elephant grass has one of the highest biomass productivities demanding less area to produce the same amount of energy.

We defined that the elephant grass power plant would have an installed capacity of 30 MW and two possible scenarios for the commercialization of its generated electricity. In the first scenario with no market uncertainty, called the base case, the plant sold its entire capacity through a long-term supply contract with a pre-determined fixed value. In the second scenario, called the case with switch option, the plant set up a briquetting unit and adopted a hybrid commercialization model where part of the installed capacity (25 MW) was sold through a long-term supply contract and the remainder (5MW or the biomass equivalent) was sold either in the short-term market subject to the uncertainties of the electricity price or in the form of briquettes, whichever generated a higher net revenue.

In order to forecast the short-term electricity price, this study proposed a modified version of the Clewlow, Strickland & Kaminski (2000) model that is adapted to the characteristics of the Brazilian market. The modifications can be summarizes as: replacing the discretization of its mean reversion component with the equation for Schwartz (1997) model 1 simulation proposed by Bastian-Pinto (2009); and changing the independent random variable ε_2 from a standard normal distribution for a standard log-normal distribution in order to prevent downward jumps and the possibility of obtaining negative values for the short-term electricity price. The result was a new equation in discrete time for the respective model that may prove useful in future researches related to the theme.

The results indicate that the briquetting switch output option increases the project NPV by 27.91% compared to the base case. We also conducted a sensitivity analysis where we show that the proposed model has viable results for discount rates of 10% or higher.

Considering the need of an ever clean, renewable and diversified energy matrix, biomass powered projects tend to gain an increasing importance, especially regarding the reuse of agricultural or industrial waste. In this context, the use of options that reduce the risk and increase the value of these projects and the use of appropriate models to forecast the short-term electricity price is essential to attract private sector investment.

References

Amaral, L. F. M. d. 2003. ***Modelos lineares e não lineares na modelagem do preço spot de energia elétrica do Brasil***. Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro.

Andrade, A. C., Fonseca, D. M. d., Gomide, J. A., Alvarez V., V. H., Martins, C. E., & Souza, D. P. H. d. 2000a. Produtividade e valor nutritivo do capim-elefante cv. Napier sob doses crescentes de nitrogênio e potássio. ***Revista Brasileira de Zootecnia***, 29: 1589-1595.

Andrade, J. B. d., Ferrari Junior, E., & Beisman, D. A. 2000b. ***Avaliação do capim-elefante (pennisetum purpureum schum.) visando o carvoejamento***. Paper presented at the 3º Encontro de Energia no Meio Rural, Campinas.

ANEEL. 2005. Atlas de energia elétrica do Brasil - 2ª Edição. Brasilia: Agência Nacional de Energia Elétrica.

ANEEL. 2008. Atlas de energia elétrica do Brasil - 3ª Edição. Brasília: Agência Nacional de Energia Elétrica.

Baron, M., Rosenberg, M., & Sidorenko, N. 2002. Divide and conquer: forecasting power via automative price regime separation. ***Energy and Power Risk Management***(Março): 70-73.

Bastian-Pinto, C. L. 2009. ***Modelagem de Opções Reais com Processos de Reversão à Média em Tempo Discreto: Uma Aplicação na Indústria Brasileira de Etanol***. Pontifícia Universidade Católica Rio de Janeiro.

Bastian, J., Jinxiang, Z., Banunarayanan, V., & Mukerji, R. 1999. Forecasting energy prices in a competitive market. ***Computer Applications in Power, IEEE***, 12(3): 40-45.

Birge, J. R., Cai, N., & Kou, S. G. 2010. A Two-Factor Model for Electricity Spot and Futures Prices, ***Texas Quantitative Finance Festival***. Austin.

Box, G. E. P., Jenkins, G. M., & Reinsel, G. C. 2008. ***Time Series Analysis: Forecasting and Control 4th Edition***.

Bunn, D. W. 2000. Forecasting loads and prices in competitive power markets. ***Proceedings of the IEEE***, 88(2): 163-169.

Cartea, Á. & Figueroa, M. G. 2005. Pricing in Electricity Markets: A Mean Reverting Jump Diffusion Model with Seasonality. ***Applied Mathematical Finance***, 12(4): 313 - 335.

CCEE. 2010. Visão Geral das Operações na CCEE: versão 2010: Câmara de Comercialização de Energia Elétrica.

Clewlow, L., Strickland, C., & Kaminski, V. 2000. Jumping the gaps. ***Energy and Power Risk Management Magazine***(Dezembro): 26-27.

Clewlow, L., Strickland, C., & Kaminski, V. 2001. Extending mean-reversion jump diffusion. ***Energy and Power Risk Management Magazine***(Fevereiro): 30-31.

Contreras, J. & Santos, J. R. 2006. ***Short-term demand and energy price forecasting***. Paper presented at the Electrotechnical Conference, 2006. MELECON 2006. IEEE Mediterranean.

Deb, R., Albert, R., Hsue, L.-L., & Brown, N. 2000. How to Incorporate Volatility and Risk in Electricity Price Forecasting. ***The Electricity Journal***, 13(4): 65-75.

Deng, S. 2000. Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes, ***Program on Workable Energy Regulation (POWER)***. Berkeley: University of California Energy Institute.

Embrapa. 2009. Potencial produtivo e eficiência da fixação biológica de nitrogênio em diferentes genótipos de capim-elefante (*Pennisetum purpureum* Schumach) para uso como fonte alternativa de energia, ***Boletim de Pesquisa & Desenvolvimento 41***. Seropédica: Embrapa Agrobiologia.

Energia, J. d.; Preço do leilão torna energia eólica viável no mercado livre; http://www.abeeolica.org.br/zpublisher/materias/Noticias.asp?id=19572; 03 de Março, 2011.

EnergiaDireta; Cotações; https://www.energiadireta.com.br/index.php?sub=lista_cota.php; 17 de Janeiro, 2011.

EPE. 2010. Balanço Energético Nacional 2010 - ano base 2009. In E. d. P. Energética (Ed.). Rio de Janeiro.

Ethier, R. & Dorris, G. 1999. Do not Ignore the Spikes. ***Energy and Power Risk Management***(Julho/Agosto): 31-33.

Eydeland, A. & Geman, H. 1999. Fundamentals of Electricity Derivatives. In R. Books (Ed.), ***Energy Modelling and the Management of Uncertainty***.

Geman, H. & Roncoroni, A. 2003. A Class of Marked Point Processes for Modelling Electricity Prices: ESSEC Research Center, ESSEC Business School.

Gentil, L. V. B. 2008. ***Tecnologia e Economia do Briquete de Madeira***. Universidade de Brasília, Brasília.

Goldberg, R. & Read, J. 2000. Dealing with a Price-Spike World. ***Energy and Power Risk Management***(Julho/Agosto): 39-41.

Hambly, B., Howison, S., & Kluge, T. 2009. Modelling spikes and pricing swing options in electricity markets. ***Quantitative Finance***, 9(8): 937-949.

IEA. 2010. Valor de Terra Nua: Instituto de Economia Agrícola.

Jong, C. D. 2006. The Nature of Power Spikes: A Regime-Switch Approach. ***Studies in Nonlinear Dynamics & Econometrics***, 10(3).

Jr., T. O. P. 2006. ***Modelando o Preço Spot de Energia Elétrica no Brasil: um modelo estocástico com reversão à média, mudança de regime Markoviano e difusão com saltos***. IBMEC São Paulo, São Paulo.

Karakatsani, N. V. & Bunn, D. W. 2008. Forecasting electricity prices: The impact of fundamentals and time-varying coefficients. ***International Journal of Forecasting***, 24(4): 764-785.

Lopes, B. A. 2004. O Capim Elefante. Viçosa: Universidade Federal de Viçosa.

Mazzarella, V. 2007. Capim Elefante como fonte de energia no Brasil: realidade atual e expectativas, ***INEE - Instituto Nacional de Eficiência Energética***.

Medeiros, L. d. 2003. ***Previsão do preço spot no mercado de energia elétrica***. Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro.

MME. 2007. PNE 2030 - Geração Termelétrica (Biomassa). In E. d. P. E. EPE (Ed.). Brasília.

MME. 2010a. Plano Decenal de Expansão de Energia 2019. In M. d. M. e. Energia (Ed.). Brasília.

MME; PROINFA - PROGRAMA DE INCENTIVO AS FONTES ALTERNATIVAS DE ENERGIA ELÉTRICA; http://www.mme.gov.br/programas/proinfa/; 03/07/2010, 2010.

Pereira, M. V. F. & Pinto, L. M. V. G. 1991. Multi-stage stochastic optimization applied to energy planning. ***Math. Program.***, 52(2): 359-375.

Schwartz, E. S. 1997. The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. ***The Journal of Finance***, 52(3): 923-973.

Schweppe, F. C., Caraminis, M. C., Tabors, R. D., & Bohn, R. E. 1988. ***Spot pricing of electricity***: Kluwer Academic Publishers.

Silva, C. A. d., Felfli, F. F., Pérez, J. M. M., Rocha, J. D., & Simões, A. F. 2006. ***Estudo da viabilidade técnico: econômica de uma fabrica de briquetes para fins de geração energética***. Paper presented at the 6º ENCONTRO DE ENERGIA NO MEIO RURAL, Campinas.

Sousa, R. L. d. 2003. ***Modelagem estrutural aplicada a previsão do preço spot de energia elétrica do Brasil***. Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro.

Vilela, H. 2010. Produção de Briquete de Capim Elefante, ***Agronomia***.

Vilela, H. & Cerize, D. 2010. Capim Elefante Paraíso na geração de energia, ***Agronomia***.

Weron, R., Simonsen, I., & Wilman, P. 2003. Modeling highly volatile and seasonal markets: evidence from the Nord Pool electricity market: EconWPA.