Cumulative Leadership and New Market Dynamics

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Abstract

Two firms face market development uncertainty in a continuous-time investment model. They non-cooperatively choose when to invest in a lumpy capacity before competing in the market stage. The combined impact on equilibrium outcomes of the firms' relative ability to detect the new demand (or "alertness", Kirzner (1973)) and of a persistent first-mover advantage is characterized. With perfect alertness, equilibrium investments are always sequential. There is rent-equalization, with even more dissipation than without a first-mover advantage, which thus reduces both firms' ex-ante value. Limited alertness, as formalized by firm-specific investment trigger constraints, leads to qualitatively different outcomes that contrast with the known results in the literature. With nonzero probability simultaneous entry can occur, otherwise a firm maximizes value by investing late, though before its rival. A constraint level can always be defined that is so weak as to be slack in the benchmark scenario (perfect alertness and no first-mover advantage), and still result in more equilibrium value to the leader if it benefits from the market stage advantage. With more demand volatility, the impact of limited alertness on the entry sequence is less likely, and the leader-follower differential value decreases to the benefit of the less alert firm, although it enters even later.

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1 Introduction

Business circumstances in which rival firms contemplate entry into a new and growing market are prevalent. In most cases, the installation of production facilities is needed as a first step, before operations can start. Investments are typically lumpy and rarely recoverable. When demand is fluctuating and changes are uncertain, the timing of entry impacts the expected value of operations. An early investment is risky as demand may remain relatively low for a long time. On the other hand, a firm forgoes operating profits if it postpones entry to a distant period.

Firms' long-run investment choices are not made in isolation. There is competition for the lead position when it pays to start operations before others. This occurs if early entry results in monopoly profits for a while, before demand reaches a sufficiently high level that encourages new entrants. Firms also interact strategically in the short-run. Once more than one firm has entered, the formation of prices is a non-cooperative outcome. Each firm's ex-ante investment value thus depends on the nature of competition in the product market.

1.1. Connections to the theoretical literature

The continuous-time setup with demand uncertainty considered in this paper builds on several recent contributions to the theoretical literature. They all use a strategic real options methodology to study investment strategies when firms interact in a long-run investment game before competing in the market. More specifically, an extension to a stochastic environment of Fudenberg and Tirole (1985)'s analysis of technology adoption is introduced by Huisman and Kort (1999), where firms are already active on a market with uncertain demand. Smit and Trigeorgis (2004) discuss the impact of a marginal cost asymmetry in the new market case, where two firms must investing in productive assets to enter. In Kort and Pawlina (2006), the cost of investments is different across the two firms, which already compete before any investment occurs, and the effect of this difference on the nature of equilibrium is characterized. Mason and Weeds (2009) investigate the impact on the entry timing of a difference in flow profits to the benefit of the first entrant. In Boyer, Lasserre, and Moreaux (2011), in order to enter the firms must invest in lumpy capacities which can constrain quantity choices in the product market stage. In all these papers, among others, the firms are aware of the profit opportunities at all points in time.[1] For some parameter values a preemption equilibrium exists, in which investments occur sequentially with probability one. In the new market case, competition for being the first entrant always implies rent dissipation and equalization (the firms' expected value is the same in equilibrium), and the probability of simultaneous entry is zero.

[1]For recent surveys of game theoretic real options models, see Boyer, Gravel, and Lasserre (2010), Azevedo and Paxson (2011), and Chevalier-Roignant, Flath, Huchzermeier, and Trigeorgis (2011).

The objective of this paper is to compare the timing of investments and the value of firms across two scenarios that have been considered separately in the literature. In the first scenario, as soon as two firms hold finite production capacities they compete by choosing quantities simultaneously, so that their instantaneous profits are equal (as in Boyer, Lasserre, and Moreaux, 2011). In the second scenario, the firm which leads the entry process is also a first mover in the market subgame, and thereby earns higher instantaneous profits (as in Mason and Weeds, 2009). In both scenarios, firm-specific investment trigger constraints are introduced to formalize the limited ability of entrepreneurs to detect new profit opportunities in the early stage of the market development process.

The relative relevance of the Cournot and Stackelberg scenarios is discussed by Smit and Trigeorgis (2004) in the context of a continuous-time investment model. They see more merits in the former specification, on the grounds that, in the Stackelberg case, one of the two firms commits itself by moving earlier than its opponent. By doing so, the large quantity it chooses is not the same as with simultaneous moves, as it is not a best reply to the quantity chosen by the second mover. Then, if the market subgame is repeated, with firms making simultaneously quantity choices in all periods that follow the two initial sequential moves, the leader has an incentive to reduce its output. This reasoning holds when the leader cannot commit in some way to keep selling the large market share, and consumers are assumed to buy from one or the other supplier indifferently at all points in time, independently of the entry sequence of firms.[2] This however does not hold when brand loyalty, consumption habits, or network effects imply that clients are reluctant to switch to a new seller, or new consumers value an existing customer base. In that case, the consumers who started buying from the first entrant may keep doing so repeatedly, and a large share of new buyers can be more attracted by the leading firm than by a follower. In the present paper the Stackelberg specification refers to a large class of circumstances in which the first entrant benefits from such a *persistent* advantage.[3]

In the analytical framework that follows, the specification that firms invest in lumpy production capacities, as opposed to an abstract project, offers an explicit foundation to possible rankings of instantaneous profit levels, as earned in the market stage by each firm behaving as a monopolist, a Cournot duopolist, or a Stackelberg leader/follower. The rankings depend on the status of capacity constraints resulting from the firms' previous choices in the investment stage.

While several papers characterize the effect of introducing "time-to-build" *after* a firm has decided to invest in productive assets (Grenadier, 2000, and Pacheco-de-Almeida and Zemsky, 2003), this paper focuses on firm differences in the ability to detect new profit opportunities *before* an investment occurs. The fact that demand is taking off is not obvious at early stages, and entrepreneurs are likely

[2] For a discussion on time consistency and commitment issues in a Stackelberg model, see Fudenberg and Tirole (1991, pp. 75-77).

[3] See Cottrell and Sick (2002), and Kim and Lee (2011) for examples.

to overlook it until a relatively high level is reached. The ability to detect a new market opportunity relates to the "alertness" of investors, as coined by Kirzner (1973, 1979, 1997). He defines the role of an entrepreneur as essentially that of being "alert to opportunities created (...) by independently initiated changes" (1997, p. 5). Indeed there is no reason to assume that investors are equally able to detect a burgeoning demand. Barney, Wright, and Ketchen (2001) characterize the relative ability of firms to be "more alert to changes in their competitive environment" than competitors as a source of sustained competitive advantage (p. 631). Foss and Klein (2009, p. 1) see entrepreneurs as not all equally "alert to a new product" and in their propensity to respond to this opportunity "before others".

Therefore, an important specification of the model is that it includes firm-specific investment trigger constraints. They allow for a departure from the standard (and usually implicit) assumption that firms are perfectly alert, that is capable of seizing the opportunity to invest with no delay, and at any point in time, from the very beginning of the market development process. When constraints bind, they reflect real-world circumstances in which firm managers become aware of profitable market opportunities only if the level of demand is sufficiently high. When asymmetric, the constraints also play a role in the determination of the first investor, in the same way as a marginal cost asymmetry or a quality differential, as suggested by Smit and Trigeorgis (2004), or a difference in investment costs, as in Kort and Pawlina (2006).

1.2. The empirical evidence

To what extent does the first-mover advantage in the product market stage impact the entry sequence of firms? Does it preserve the rent equalization and dissipation properties, or can it generate more value to the leader? What is the consequence of an increase in growth or volatility with a first-mover advantage? These questions find only incomplete answers in the empirical literature that explicitly refers to a real options framework. In a research note, Folta and Miller (2002) conjecture that first-mover advantages accelerate entry. This is the outcome of analytical reasoning with no formal specification of firms' interactions in their investment choices. In a complementary paper, Folta and O'Brien (2004) use data from a broad array of industries to find support for their hypothesis that the choice by firms to enter a new activity is positively related to a measure of early-mover advantages. Using the same data, Folta, Johnson, and O'Brien (2006) examine the effects of irreversibility and uncertainty on the likelihood of entry into new activities. They find that greater uncertainty, measured at the industry level, decreases the likelihood of entry. Although they are consistent with the standard real options approach to investment, these findings cannot fully capture the strategic dimension (in the game theoretic sense) that characterizes the entry choice by several firms in the same market. More specifically, whether two firms enter almost simultaneously (the same year), or one after another over a longer period of time (in two different years), is not considered in the econometric model.

Therefore, it is not clear whether a first-mover advantage in the product market, as gained by an early entrant, impacts the timing of decision and the firms' values in a given industry. In another empirical study, Driver, Temple, and Urga (2008) investigate the connection between uncertainty and a firm's decision to invest more or less in the future. They find that a measure of irreversibility is a predictor of a negative effect of uncertainty on investment levels, while indicators of first-mover advantages contribute to a positive effect. However, the survey data they use are collected in a quarterly survey over two decades. Demand conditions are likely to change significantly over such a long time period. It is therefore not easy to disentangle the consequences of such changes from the role of industrywide characteristics on the timing of firms' investment choices. Moreover, none of these papers refers to firms differences in their ability to identify new market opportunities. They do not connect to another stream of empirical investigations which have evidenced that, depending on previous experience, or other organizational endowments, firms are not equally alert (Zaheer and Zaheer, 1997; Helfat and Lieberman, 2002; Nerkar and Roberts, 2004).

1.3. The main results

It is well known since Posner (1975) that rival firms take in value by competing for a lead position resulting in supernormal profit. The equilibrium analysis in this paper demonstrates that whether a persistent first-mover advantage *à la* Stackelberg reinforces this result or not in fact hinges on firms' relative alertness.

When firms are able to detect an emerging demand very early, and the first entrant benefits from a higher flow profit than the follower, investments are sequential in equilibrium. Each firm may enter first with the same probability, rent-equalization occurs, and there is more dissipation than without a first-mover advantage (as, for example, in Boyer, Lasserre, Moreaux, 2011). A different picture emerges when the perfect alertness assumption is relaxed. If a firm-specific trigger constraint binds, a possible outcome is simultaneous entry, which occurs with nonzero probability if and only if the binding constraints are symmetric and relatively mild. This contrasts with the usual conclusion in the literature that a simultaneous investment equilibrium does not occur in the new market case. Otherwise, when investments are sequential (preemption), a firm maximizes value by entering immediately before its rival, though later than with perfect alertness. In this equilibrium type, there is no rent equalization. The leader with a first-mover advantage is strictly better-off than in the unconstrained case, whereas the follower is at best indifferent. By dampening competition for the lead position, limited alertness transforms the market advantage into more value to the leader (a case of complementarity) in the investment stage, and less to the follower. There is cumulative leadership as with sequential investments the first entrant is the more alert firm, and also the one that earns higher flow profits. Another interesting result is that investment constraints can be so weak as to be slack in the benchmark scenario (Cournot players and perfect alertness), and still bind and imply more value to the leader when a first-mover advantage is introduced. Finally, limited alertness is less likely to impact the entry sequence, and to yield a large leader-follower differential value, in a highly uncertain market. In a constrained preemption equilibrium, more volatility in the market development process always benefits the less alert firm, and expands the endogenous distance between the two firms' investment triggers.

2 An Example: Investments in the Digital Music Industry

Many business situations illustrate a scenario of cumulative leadership in which the firm that detects in advance a new market opportunity preempts the lead position in the entry process, thereby benefitting from a first-mover advantage, which can be persistent.

The market for legal music downloads offers a clear illustrative example. A decade ago demand was nonexistent; now, it keeps growing at an unprecedented rate. Large firms compete by distributing goods (songs) available from very similar sources (catalogs of titles), and variable costs are negligible relative to capital investments (in technological diffusion capacities, advertising campaigns, or digital rights), which are – at least partly – sunk. In 2001, Apple first invested in this industry when the market was burgeoning with its iTunes Music Store (iTMS). It was an early move in that sales remained very limited for almost three years before accelerating sharply. According to the Recording Industry Association of America, in 2001, digital downloads represented only 0.2 percent of total sales. It rose to 0.5 percent in 2002, 1.3 percent in 2003, then fell to 0.9 percent, before jumping to 5.7% in 2005. It has been strongly increasing since then. The sales of digital music constituted 25% of the total market by value in 2007, 34% in 2008, 41% in 2009, 46% in 2010, and 50% in 2011.[4]

Firms in the digital music industry are not equally alert. In a press conference on the first anniversary of iTMS, Apple CEO Steve Jobs emphasized the fact that Apple was more alert than rivals to assess the sales potential of that market when it was only embryonic:[5] "Zero to 70 million in one year, you know, if a year ago anyone had predicted that iTunes would sell 70 million songs during its first year, they would have been laughed out of town." Steve Jobs was frequently portrayed as one who "has the phenomenal quality of figuring out where the next industry movement would be" (http://www.iipm.edu, Sept. 23, 2007).

This does not apply to the management of Microsoft, a more recent participant in the market for music downloads. It seems to be well accepted among business analysts that "[t]hroughout its history, Microsoft has been slow to grasp some of the computer industry's biggest technology shifts

[4]Source: the Recording Industry Association of America (http://www.riaa.com/keystatistics.php).

[5]See the transcript of the press conference from Steve Jobs concerning the first anniversary of the iTunes Music Store, dated April 29, 2004, available at: http://www.macobserver.com.

and business changes (...). It also was late coming to market with its own music player, and despite a push, remains far behind Apple" (http://online.wsj.com, July 30, 2007). In fact, Microsoft waited until late 2006 to launch its own player Zune and then MSN Music Store, when demand had reached a much higher level. At that time, the consensus among observers was that Apple's sales would not be strongly impacted by the new entrant. The president of Microsoft's entertainment division acknowledged that "analysts don't expect the early effort to make a serious dent in Apple's market share. (...). Apple's obviously still going to be the leader. I think that's fair. (...) While Microsoft is a great brand name it's, you know, not the first word that comes to your mind when someone says, Hey, music!" (http://www.businessweek.com, Nov. 13, 2006). Another expert was even more explicit, as it claimed that Apple "has something that the Zune certainly lacks: first-mover advantage. This advantage is primarily kept where people have bought their music via the iTunes store. The amount of effort to get your music in a format that is playable on the Zune (or any other player) is just too much" (http://www.zdnet.com, Sept. 20, 2006).

The first-mover advantage is likely to be a long-lasting phenomenon when it is supported by a combination of brand loyalty, use habits, and switching costs of all kinds, including network effects in connection to a technical device with a proprietary format. In that case, "first-movers benefit from building up an installed base early" (Koski and Kretschmer, 2004, p. 19). More specifically, intellectual property experts emphasize that "iPod owners derive benefits from the ease with which the iPod interconnects to a personal computer and iTMS," and certainly incur "high switching costs, one of which is learning alternative downloading methods" (Arewa and Sharpe, 2007, pp. 339 and 344). This occurs at the expense of new entrants. Microsoft's market share has never been more than a small portion of Apple's position over a long time period. A new consumer in the digital music market pays for the hardware (a portable media player) before getting access to an online music store. In addition, a customer gets used to routines (to search the catalog content, download files, pay) that render a shift to an alternative supplier costly. When satisfied with the first supplier, one is likely to keep purchasing from it. This represents many of the circumstances captured by the model in the next section.

3 The Model

■ *General specifications.* Two risk-neutral symmetric profit-maximizing firms, f and $-f$, contemplate entry in a new market to sell a non-differentiated good in quantities x_t^f and x_t^{-f}, respectively, at each point in time t. Production requires an investment in an asset, which allows a firm to supply up to a given finite output – a capacity – normalized to 1. The fixed cost of installing capacity is I in current value. Once installed, the asset has no resale value as it is firm-specific, and it does not depreciate.

The variable costs of production are negligible. At time $t \geq 0$, inverse demand is described by the function

$$P(X_t, Y_t) = Y_t D^{-1}(X_t), \tag{1}$$

where P is the final market price, $X_t = x_t^f + x_t^{-f} \geq 0$ is the total output, and $Y_t \geq 0$ is a random industrywide shock. The time-invariant function $D(.)$ is strictly decreasing, continuously differentiable, and integrable on R_+^1, with $D(0) = \lim_{P \downarrow 0} D(P) < \infty$; the mapping $X_t \mapsto X_t D^{-1}(X_t)$ is strictly concave on $(0, D(0))$. Aggregate shocks $(Y_t)_{t \geq 0}$ follow a geometric Brownian motion

$$dY_t = \alpha Y_t dt + \sigma Y_t dZ_t, \tag{2}$$

with $Y_0 > 0$, $\alpha > 0$ (growth), $\sigma > 0$ (volatility), and where $(Z_t)_{t \geq 0}$ is a standard Wiener process.[6] Demand is thus characterized by a deterministic positive trend and stochastic fluctuations that make future levels uncertain.

The timing of the game is as follows: 1) at any t, given the realization of Y_t, each firm chooses to invest or not in the productive asset; 2) given installed capacities, each firm selects an output level x_t which can be constrained; 3) given output levels, the market price is determined according to (1).

The solution concept is the Markov perfect equilibrium (MPE).[7] In a MPE a firm's investment and output decisions depend only on whether firms have already invested or not and on the current level of Y_t. It follows that, given installed capacities, firms cannot attempt to coordinate output decisions over time. At each date, they play the unique equilibrium of the market subgame. A MPE outcome is an ordered sequence of the firms' respective investment triggers with related quantities (x_t^f, x_t^{-f}). An investment trigger is denoted by y_{ij}, where $(i, j) \in \{0, 1\}^2$ refers to the firms' asset base immediately before Y_t reaches the threshold y_{ij} for the first time from below. Initially $i = j = 0$, then $i = 1$ if the firm has invested, and $j = 1$ when its competitor has invested.

■ *Market scenarios.* There are two versions of the model. The Cournot version of the model, in which the two firms choose quantities simultaneously, is the same as in Boyer, Lasserre, and Moreaux (2011) and is used as a benchmark. In the Stackelberg version, the first entrant is also a first mover in the market subgame, and thus benefits from a persistent advantage, as in Mason and Weeds (2009). In both scenarios, market outcomes depend on each firm's installed capacity:

(i) Cournot firms sell the same quantity x_C. Let $k_C = \lceil x_C \rceil$ be the minimum capital stock (an integer) required to produce x_C. The market subgame is assumed to admit a unique equilibrium

[6]The geometric brownian motion is derived from $Y_t = Y_0 \exp\left[\left(\alpha - \frac{1}{2}\sigma^2\right) t + \sigma Z_t\right]$ by using Itô's lemma. For the equation of motion to describe a market in expansion, it is assumed that $\alpha > \frac{\sigma^2}{2}$.

[7]See Appendix A1 for a formal definition.

(x_C, x_C), with $0 < x_C \leq 1$, so that $k_C = 1$. When both firms operate one capacity unit each, they cannot be constrained.

(ii) The Stackelberg leader benefits from a first-mover advantage over its rival. Let $k_L = \lceil x_L \rceil$ and $k_F = \lceil x_F \rceil$ be the minimum capital stocks required to produce x_L and x_F, as sold by the leader and the follower, respectively. For comparison with the previous benchmark scenario, it is assumed that the market subgame admits a unique equilibrium (x_L, x_F) with $0 < x_F \leq x_L \leq 1$, which is constant over time (persistency), so that $k_L = k_F = 1$. No firm is constrained when both have invested.

The specification that $k_C = k_L = k_F = 1$ formalizes the assumption that the capacity of the productive asset is "lumpy". This guarantees that investments stop as soon as *each* firm has invested in one capacity unit. However, in the two scenarios the monopoly output x_M can be greater than 1. In that case, and provided that only one firm has invested, production is capacity constrained until the other firm invests as well.[8]

■ *Profit rankings.* Given the installed asset base, from (1) the equilibrium of the market subgame does not depend on the multiplicative shock Y_t. A firm's instantaneous gross profit is $Y_t \pi_{ij}$, where π_{ij} depends on the asset base (i, j) and on the firm's rank in the investment sequence. Instantaneous profits are $\pi_{00} = \pi_{01} = 0$ for a non investor, and π_{10} in the monopoly case. When both firms have invested, π_{11} can take the values π_C (Cournot duopoly profits), π_F (follower's profits), or π_L (leader's profits), depending on the existence of a first-mover advantage. For simplicity we use the parameters $\phi \equiv \frac{\pi_F}{\pi_{10}}$, $\gamma \equiv \frac{\pi_C}{\pi_{10}}$, and $\lambda \equiv \frac{\pi_L}{\pi_{10}}$, with

$$0 < \phi \leq \gamma \leq \lambda \leq 1, \tag{3}$$

which is sufficiently general to capture a large class of circumstances.[9] In this quantity-setting duopoly model, the ranking holds whenever best-reply functions have a negative slope in the $\left(x_t^f, x_t^{-f}\right)$-plane.[10] The assumption that $\phi < (=) \lambda$ reflects the long lasting impact of brand loyalty, switching costs, or network effects (with an equality sign in the Cournot scenario).[11]

[8] This does *not* imply that the first investor would find it profitable to increase its capacity by investing in several productive assets. This is made formal in the next section as a comment of the value function $L(.)$ in (5).

[9] The possibility that the leader earns higher instantaneous profits than in monopoly, that is, $\pi_{10} < \pi_L$, is not considered. This case is precisely investigated by Mason and Weeds (2009, Proposition 5), where unusual comparative statics results characterize situations in which the follower's investment benefits the leader so much as to outweigh the effect of increased competition. However, we may have $\pi_F = \pi_C = \pi_L$, so that the Stackelberg market substage equilibrium coincides with the Cournot equilibrium (this occurs here if $x_C = 1 < x_M$).

[10] The ranking in (3) is rooted in the quantity-setting firms assumption. Indeed the same ranking cannot occur in a *price*-setting duopoly with standard specifications, where the slope of best-reply functions is positive, resulting in a *second*-mover advantage, as first demonstrated by Gal-Or (1985).

[11] In real-world circumstances, a first-mover advantage is likely to erode over time (see Cottrell and Sick (2005) for historical evidence). Therefore, the firms' equilibrium values, as derived in the present setting, should be seen as reference levels vis-à-vis more realistic situations in which the first investor's superiority is only temporary.

The specification that firms invest in a finite capacity – as opposed to an abstract project – offers a very natural set-up to generate all possible profit rankings that satisfy (3). For example, suppose that investing in one capacity unit is more than sufficient to supply the unconstrained Cournot output. Then, a linear demand $D(.)$ leads to a usual ranking: $x_C < 1 \Rightarrow \phi < \gamma < \lambda < 1$.[12] To compare, suppose that investing in one capacity unit is exactly sufficient to supply the unconstrained Cournot output but not the monopoly quantity. This gives a less usual ranking: $x_C = 1 < x_M \Rightarrow \phi = \gamma = \lambda < 1$.[13] As for the equality $\gamma = 1$, or $\lambda = 1$ with a first-mover advantage, it is a limit case that can only be approached here (demand is strictly decreasing).

■ *Trigger constraints.* Before any investment occurs, the firm-specific trigger constraints $y_{00} \geq y_f$ and $y_{00} \geq y_{-f}$, with $0 \leq y_f \leq y_{-f}$, formalize the assumption that at least one potential entrant is not able to seize profit opportunities until Y_t has reached a certain level. In case f detects the new demand before $-f$ and invests at $Y_t < y_{-f}$, by doing so it reveals the investment opportunity to its rival, which can thus choose to exert its option immediately after the first entrant. Firms are "perfectly alert" only if $y_f = y_{-f} = 0$. The thresholds y_f and y_{-f} cannot be adjusted in the short run. They capture organizational endowments and do not depend on the nature of competition in the market subgame.

4 Perfect Alertness

In this section, as standard in the literature, it is assumed that firms may invest with no delay at any point in time from $t = 0$ onward ($y_f = y_{-f} = 0$, so the trigger constraints are obviously slack). The MPE is characterized against the Cournot benchmark.

To do that, suppose first that a firm enters when $Y_t = y$, and that its competitor enters later, when Y_t reaches a higher level y_{01}. For all $y < y_{01}$, the value function of the latter firm is

$$F_{y_{01}}(y) = \left(\frac{y}{y_{01}}\right)^{\beta} \left(\frac{\pi_F}{r-\alpha} y_{01} - I\right), \tag{4}$$

where $\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \left[\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}\right]^{\frac{1}{2}}$, with the constant discount factor $r > \alpha$ implying that

[12] Still, the magnitude of profit differences depends on the status of other capacity constraints. More specifically, in this ranking π_{10} is relatively high if $x_M \leq 1$ (the slack case), and lower otherwise, though strictly above π_L, which is constant across the two cases.

[13] Absent capacity constraints, and without introducing an additional process of imitation, innovation, or externalities, Colombo and Labrecciosa (2008) establish that a very specific condition on the inverse demand function is needed for the Stackelberg market substage equilibrium to coincide with the Cournot equilibrium.

$\beta > 1$.[14,15] The maximum $F^*(y)$ with respect to y_{01} is obtained at $y_{01}^* = \frac{r-\alpha}{\pi_F} I \frac{\beta}{\beta-1}$, which is monotone decreasing in ϕ.[16] Note that $F^*\left(y_{01}^*\right) = \frac{I}{\beta-1}$, a constant which does not depend on instantaneous profits. For all $y \leq y_{01}^*$, the value function of the firm that invests immediately when $Y_t = y$, while its competitor remains out of the market as long as Y_t has not hit y_{01}^*, is

$$L(y) = \frac{\pi_{10}}{r-\alpha} y - I + \left(\frac{y}{y_{01}^*}\right)^\beta \frac{\pi_L - \pi_{10}}{r-\alpha} y_{01}^*. \tag{5}$$

If the two firms compete *à la* Cournot in the market subgame, define $y_C^* \equiv y_{01}^*|_{\phi=\lambda}$, which is used in $F_C^* \equiv F^*|_{\phi=\lambda}$ and $L_C \equiv L|_{\phi=\lambda}$. Note that $\phi = \gamma$ implies $y_C^* = y_{01}^*$ for all $\lambda \geq \phi$.

For all $y \geq Y_0$, if both firms invest simultaneously when $Y_t = y$, their common value is[17]

$$S(y) = \frac{\pi_C}{r-\alpha} y - I. \tag{6}$$

When a firm can commit, at $t = 0$, to lead the investment sequence, and chooses to postpone entry until Y_t reaches a higher level y_{00}, its value is $L_{y_{00}}(y) = \left(\frac{y}{y_{00}}\right)^\beta \left(\frac{\pi_{10}}{r-\alpha} y_{00} - I\right) + \left(\frac{y}{y_{01}^*}\right)^\beta \frac{\pi_L - \pi_{10}}{r-\alpha} y_{01}^*$, for all $y < y_{00}$. The maximum of $L_{y_{00}}(y)$ with respect to y_{00}, hereafter denoted by $L^*(y)$, is obtained at $y_{00}^L = \frac{r-\alpha}{\pi_{10}} I \frac{\beta}{\beta-1}$, which is strictly lower than (or equal to) y_{01}^* because $\pi_F < (=) \pi_{10}$.[18]

While the value $L^*(y_{00}^L)$ depends on π_F and π_L, the threshold y_{00}^L does not change with the level of profits earned in the market subgame. It remains the same with or without a first-mover advantage. This leads directly to a first claim:

Proposition 1 *If at $t = 0$ a firm may commit to lead the entry process, it chooses to enter when $Y_t = y_{00}^L$ independently of the nature of competition on the product market in the post-entry period, although $\phi < (=) \gamma < (=) \lambda$ implies $L(y_{00}^L) > (=) L_C(y_{00}^L)$ and $F^*(y_{00}^L) < (=) F_C^*(y_{00}^L)$.*

In the value analysis that follows, the threshold y_{00}^L, with which all other useful thresholds are compared, is used as a natural upper bound to the interval of Y_t that describes the early stage of the market development process.

[14]If $r \leq \alpha$ the firm's value is maximized by postponing investments forever. In (4) the term $\left(\frac{y}{y_{01}}\right)^\beta$ reads as the expected discounted value, measured when $Y_t = y$, of receiving one monetary unit at the first-hitting time $\tau_{01} = \inf\{t \geq 0 : Y_t \geq y_{01}\}$. If σ is formally set equal to zero (no uncertainty), we have $Y_t = Y_0 \exp(\alpha t)$ and $\beta = \frac{r}{\alpha}$ so that $\left(\frac{y}{y_{01}}\right)^\beta = e^{-r(\tau_{01}-t)}$, the usual discounting term.

[15]For a detailed exposition of the steps that lead to the expression of β, see Dixit and Pindyck (1994, pp. 140-144).

[16]$F_{y_{01}}(y)$ is concave in y_{01} if and only if $y_{01} < \left(1 + \frac{1}{\beta}\right) y_{01}^*$. This second-order condition is thus satisfied at $y_{01} = y_{01}^*$ for all $\beta > 1$.

[17]The expressions of the value functions in (4), (5), and (6), are standard in the real options literature. For an early introduction, see Dixit and Pindyck (1994, pp. 309-314). For a more technical presentation, see Huisman (2010, Ch. 7).

[18]In the next section, the property that $L_{y_{00}}(y)$ is concave in y_{00} if and only if $y_{00} < \left(1 + \frac{1}{\beta}\right) y_{00}^L$ will be useful to identify the maximizer when an investment trigger constraint binds.

A possible extension is to allow firms to invest in several assets. However, given the "lumpiness" assumption ($k_C = k_F = k_L = 1$) the second entrant never chooses to do so. As for the first entrant, a necessary (and not sufficient) condition for multiple investments is a constrained monopoly output ($1 < x_M$). In that case, once the firm has committed as a leader by investing first, it chooses the timing of any subsequent investment independently of the other firm. Then (5) generalizes to $L(y,k) = \frac{\pi_{10}}{r-\alpha} y - I + \sum_{i=1}^{k-1} \left(\frac{y}{y_{i0}^L}\right)^\beta \left(\frac{\pi_{(i+1)0} - \pi_{i0}}{r-\alpha} y_{i0}^L - I\right) + \left(\frac{y}{y_{0k}^*}\right)^\beta \frac{\pi_{k1} - \pi_{k0}}{r-\alpha} y_{0k}^*$, where $k \geq 1$ is the leader's number of investments, y_{i0}^L are maximizers (with $i = 1, \ldots, k-1$), and y_{0k}^* is the follower's choice. Then, $L(y, k+1) - L(y,k) > 0$ if and only if $\frac{\pi_{(k+1)0} - \pi_{k0}}{\pi_{11}} > \beta^{\frac{1}{\beta - 1}}$, so that the marginal profit must be sufficiently high above π_{11} for an additional investment to be profitable. This is ruled out in this paper as $i, j \in \{0,1\}$ WLG in the set of profit levels satisfying (3).

Suppose now that firms cannot commit vis-à-vis the investment sequence. Recalling that the functions F^* and L are both defined on $y < y_{01}^*$, we solve $F^*(y) - L(y) = 0$, to obtain:

Lemma 1 *There exists a unique positive* $y_{00}^p < y_{01}^*$ *such that* $L(y) > (=) F^*(y)$ *if and only if* $y > (=)\, y_{00}^p$.

Proof See Appendix A2. □

To compare MPE outcomes with and without a first-mover advantage, observe that, given π_{10}, one finds $\frac{d^2 F^*(y)}{d\phi dy} = \frac{d^2 L(y)}{d\lambda dy} = \frac{\beta^2}{\beta - 1} \left(\frac{y}{y_{01}^*}\right)^\beta \frac{I}{\phi y} > 0$, and $\frac{d^2 L(y)}{d\phi dy} = -\beta^2 (1-\lambda) \left(\frac{y}{y_{01}^*}\right)^\beta \frac{I}{\phi^2 y} < (=)\, 0$ for all $\lambda < (=)\, 1$. The slope of $F^*(y)$ is monotone increasing in ϕ, and the slope of $L(y)$ is monotone increasing in λ, and decreasing in ϕ. Therefore:

Remark 1 y_{00}^p *and* $F^*(y_{00}^p) = L(y_{00}^p)$ *are monotone decreasing in* λ/ϕ.

More specifically, for $y_C^p \equiv y_{00}^p|_{\phi=\lambda}$, we have $y_{00}^p < (=)\, y_C^p$ and $F^*(y_{00}^p) = L(y_{00}^p) < (=)\, F_C^*(y_C^p) = L_C(y_C^p)$, all $\phi < (=)\, \gamma < (=)\, \lambda$. This means that the two firms' respective values equalize at a lower level when there is a first-mover advantage in the market subgame. In the time dimension, $\tau_{00}^p = \inf\{t \geq 0 : Y_t \geq y_{00}^p\} < (=)\, \tau_C^p$.

We can now examine whether firms choose to be either the first or the second entrant, or to enter simultaneously. Let $Y_0 \leq y_{00}^p$ hereafter, for simplicity. There are three possible cases. Suppose first that $y < y_{00}^p$, which implies that $L(y) < F^*(y)$, hence no firm has an incentive to enter first. Suppose now that $y \geq y_{01}^*$ and that no firm has entered yet. In that case, extend beyond y_{01}^* the interval of y on which L and F^* are defined. As $\arg\max_{y_{01}} F_{y_{01}}(y) \equiv \{y_{01}^*\}$, the follower finds it profitable to invest immediately at current level y, that is at the same time as the leader (the dashed line in Figure 1), implying that $F^*(y) = L(y) = S(y)$. Eventually, in the intermediate interval $y_{00}^p \leq y < y_{01}^*$, it is valuable to take the lead because $F^*(y) < L(y)$. Therefore, each firm has an incentive to "undercut" its rival on the segment $[y_{00}^p, y_{00}^L]$ to preempt the lead position.

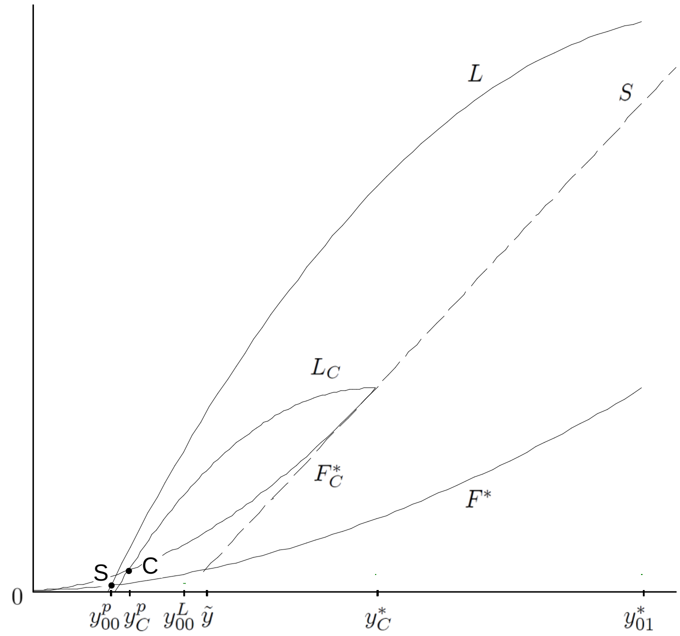

Figure 1: Firm values with $P(X, Y_t) = Y_t(1 - X)$, $r = 1/2$, $\alpha = 1/4$, $\sigma = 1/4$, and $I = 1/10$. If a firm can commit to lead, it enters at y_{00}^L. Without commitment, each firm may lead with probability 1/2. In the Cournot scenario (point C) the leader enters at y_C^p, and the follower at y_C^*, the firms' values are $L(y_C^p) = F^*(y_C^p)$. In the Stackelberg scenario (point S), the leader enters at y_{00}^p, the follower at y_{01}^*, and the firms' values are $L(y_{00}^p) = F^*(y_{00}^p)$.

The dynamics of competition for the lead position in the investment stage is qualitatively the same across the Cournot and Stackelberg scenarios. All properties of the preemption equilibrium, including rent dissipation and equalization, are robust to the introduction of a first-mover advantage. The only differences are that 1) the leader enters earlier, 2) the follower enters later, and 3) each firm's value is lower in equilibrium as a result of rent equalization. More formally:

Proposition 2 *Suppose $\phi < (=)\gamma < (=)\lambda$: (1) There exists a unique preemption equilibrium in which firm f invests with probability 1/2 at $y_{00}^p < (=) y_C^p$ at the first-hitting time $\tau_{00}^p < (=)\tau_C^p$, while firm $-f$ enters with probability 1/2 at $y_{01}^* > (=)\, y_C^*$; that is, it waits until $\tau_C^* > (=)\tau_{01}^*$; (2) The two firms' equilibrium value is $F^*(y_{00}^p) = L(y_{00}^p) < (=) F_C^*(y_C^p) = L_C(y_C^p)$.*[19]

Proof Equilibrium investment strategies for all levels of Y_t are derived in Appendix A1, and probabilities of leading/following are calculated in Appendix A4.[20] □

[19] Here $\tau_{01}^* = \inf\{t \geq 0 : Y_t \geq y_{01}^*\}$.

[20] Following Grenadier (1996), an alternative to the calculation of probabilities in Appendix A4 is to assume that, when firms choose the same point in time to enter, an exogenous random mechanism assigns the lead position to one of them by the flip of a fair coin.

In more intuitive terms, the preemption MPE is an outcome of exacerbated competition when each firm is interested not only in leading the investment process, but also in benefitting from a first-mover advantage in the market subgame. The possibility given by entering first, and being able to earn π_L (in lieu of π_C), is at first glance a bonus for the leader and a penalty for the follower. It makes the prize sweeter, and defeat bitter. However, in equilibrium, the first-mover advantage results in the leader entering earlier ($\phi < \lambda$ implies $y_{00}^p < y_C^p < y_{00}^L$), and the follower entering later ($y_{00}^L < y_C^p \leq y_{01}^*$). No rent results from the extended incumbency period because the rent equalization property holds. Actually, there is more dissipation in comparison to the reference value $L^*(y_{00}^L)$, as obtained when a firm can commit as a leader. The firms' equal investment option exercise values $L\left(y_{00}^p\right) = F^*\left(y_{00}^p\right)$ are lower than $L_C\left(y_C^p\right) = F_C^*\left(y_C^p\right)$, although the difference between L and F^* is higher than between L_C and F_C^* for all values of y. The first-mover bonus at the market level turns into more competition at the investment level, and *in fine* in a lower value for both entities.[21]

In Proposition 2, as the probabilities of leading or following are both equal to 1/2, the probability of a simultaneous investment at y_{00}^p (a "mistake" as coined by Fudenberg and Tirole (1985)) is nil. Firms may not escape that situation by cooperating to enter at the same time and later than when $Y_t = y_{00}^p$, at some stochastic date $\tau > \tau_{00}^p$. This would not be a self-enforcing deal (here, it is assumed such an agreement is not contractible, say for legal reasons). Supposing it takes place, the agreement would result in the same value $S_{y_{00}}(y) = \left(\frac{y}{y_{00}}\right)^\beta \left(\frac{\pi_C}{r-\alpha} y_{00} - I\right)$, as the firms do not invest at the current level $Y_t = y$, and choose to enter simultaneously when Y_t hits y_{00}. If $Y_t = y < y_{00}$ both firms are inactive, and if $Y_t = y \geq y_{00}$ their instantaneous flow profit is as in the Cournot scenario. Hence, $S_{y_{00}}(y)$ is identically equal to $F(y)$ with $\phi = \gamma$, and a maximum value $S^*(y)$ is obtained for $y_{00} = y_C^* < (=) y_{01}^*$ (for all $\phi < (=)\, \gamma$). When Y_t hits this threshold y_C^*, each firm's value is $S^*(y_C^*)$. If a firm f chooses to deviate while $-f$ sticks to the agreement, it may enter immediately before Y_t hits y_C^*, that is, at $y_C^* - \varepsilon$, with ε positive and arbitrarily small, for a value $L(y_C^* - \varepsilon)$. While in the case of simultaneous entry, each cooperating firm earns only π_C from $\tau_C^* \leq \tau_{01}^*$ on forever, in case of deviation f earns π_{10} from $\tau_C^* - \varepsilon$ to τ_{01}^*, and then π_L from τ_{01}^* on, again forever (it incurs the entry cost I at – almost – the same time in the two cases). Because $\pi_L \geq \pi_C$, the simultaneous entry agreement cannot be enforced for all $\pi_{10} \geq \pi_L \geq \pi_C > 0$.

Figure 1 illustrates Proposition 2. The distance between L and F^*, at y approaching y_{01}^*, can be measured by observing that $\lim_{y \uparrow y_{01}^*} L(y) - F^*(y_{01}^*) = I \frac{\beta}{\beta - 1}\left(\frac{\lambda}{\phi} - 1\right)$, which is proportional to $\frac{\lambda}{\phi}$. One also obtains a longer time period during which the first entrant is the unique supplier. Although it does not lead to blockaded entry, playing first in the short-run market subgame postpones the other

[21] The extension of the assumption $\pi_F < \pi_C < \pi_L$, and consequently of Proposition 2, to more than two firms is not straightforward. In a static hierarchical Stackelberg model where n firms choose outputs sequentially, Anderson and Engers (1992) provide a necessary and sufficient condition (on a demand parameter and on the number of firms) for the first mover to earn *lower* profits in Stackelberg than in Cournot if $n > 2$.

firm's investment for a while. This comes at some cost, since the leader enters earlier, at more risk, relative to the commitment case (investment trigger y_{00}^L). Finally, note that the difference between y_{00}^L and y_{01}^* depends on ϕ, but not on λ, while (4) and (5) are such that $y_{00}^p = y_C^p$ and $y_{01}^* = y_C^*$ if and only if ϕ and λ are equal. More precisely:

Remark 2 $y_{00}^p < (=)\, y_C^p < (=)\, y_{00}^L < (=)\, y_C^* < (=)\, y_{01}^*$ *if and only if* $\phi < (=)\, \gamma < (=)\, \lambda < (=)\, 1$.

When the profit π_{10} of a constrained monopolist (case $1 < x_M$) exactly matches the unconstrained Cournot profit π_C, the function $L_C(y)$ is linear and tangent to $F_C^*(y)$ at $y_C^p = y_{00}^L = y_C^*$. In the latter limit case only, the Cournot scenario collapses to the commitment situation in that the value of the two firms is exactly equal to the level of a firm protected from preemption.

There are situations in which we do not obtain either only equality signs or only strict inequality signs throughout the rankings in Remark 2. In particular, in the Cournot case $\phi = \gamma = \lambda < 1$ implies $y_{00}^p = y_C^p < y_{00}^L < y_C^* = y_{01}^*$.

5 Limited Alertness

From Proposition 2, we know that each firm may lead the investment schedule, or follow, with probability 1/2, independently of the first-mover advantage, if any. The indeterminacy of the identity of the leader/follower is a consequence of the assumption that firms are *ex-ante* symmetric. To avoid relying on a random selection of roles with no economic rationale, one may introduce a quality differential or a marginal cost asymmetry, as suggested by Smit and Trigeorgis (2004), or an investment cost asymmetry, as in Pawlina and Kort (2006). Then the firm with a lower fixed or marginal cost or with a higher demand is the one that preempts its less profitable competitor.

A complementary approach, adopted here, is to relax the assumption that firms are equally able to detect a new market or to make an investment decision at any point in time, with no delay, from $t = 0$ onward. Formally, a simple departure from this frictionless world is to introduce the investment trigger constraints $Y_t \geq y_f$ and $Y_t \geq y_{-f}$, with $0 \leq y_f \leq y_{-f}$. Because the analysis focuses on an infant industry, and the level y_{00}^L is not only lower than y_{01}^*, but also a reference point common to the Cournot and Stackelberg scenarios (see Proposition 1), in what follows it is specified that $y_f \leq y_{-f} \leq y_{00}^L$ (i.e., both firms are sufficiently "alert" to invest as soon as Y_t hits y_{00}^L).[22] In other words, beyond that threshold it is assumed that the market is so mature as to be discernible by both firms, so that investments can occur with no delay.

Note that the trigger constraints differ from the decision lags in Gilbert and Harris (1984, Section 3) where a firm is assumed to be able to invest strictly before its rival. In the latter reference paper,

[22] In stochastic time terms, $\tau_f \leq \tau_{-f} \leq \tau_{00}^L$.

the first investment can occur at any $t \geq 0$. The decisions lags, although arbitrarily close to zero, are always positive. To compare, in the present model, (i) we may have $y_f = y_{-f} (\geq 0)$, in which case the firms are fully symmetric; (ii) a firm has limited alertness only if $Y_t < y_{00}^L$, that is in the infant industry period; and (iii) no firm is *a priori* assumed to invest before its rival. This also differs from a real option model of competitive industry by Grenadier (2000), where a profit flow is received by any firm investing at t only from $t+d$ onward, with d defined as a positive constant. This time-to-build captures the lag between the initiation of an investment and completion. In Pacheco-de-Almeida and Zemsky (2003) there is also a parametrized lag, which formalizes a delay between firms' investment decisions and production. In both cases, the lag is not attached to the firms. It specifies the technological conditions of implementing a given decision to invest. In contrast, the trigger constraints here capture a pre-investment rigidity rooted in firm-specific organizational resources.[23]

The exact timing of entry and the firms' equilibrium values depend on the comparison of y_f and y_{-f} with y_{00}^p. When both firms are almost perfectly alert, in the sense that $Y_0 < y_{-f} \leq y_{00}^p$, we directly infer from Proposition 2 that a free MPE outcome is obtained in which only one firm enters at y_{00}^p, while the other firm waits until Y_t reaches y_{01}^*.[24] Any firm can be either the leader or the follower with equiprobability 1/2, and a well-known result is that the probability of a "mistake" – that is, a simultaneous investment – at y_{00}^p is exactly zero (see (25) in Appendix A4). In that case, the specification that y_f and y_{-f} may differ across firms plays no role.

Of more interest are all situations in which a constraint binds. Symmetric and asymmetric cases are considered in turn.

5.1 Symmetric Constraints

Whenever $y_{00}^p < y_f = y_{-f} = y_S$, the symmetric "floor" y_S prevents rent competition to fully dissipate and to equalize the monopoly rents. In the constrained preemption equilibrium, two distinct situations may occur that depend on the comparison of the common level y_S with a threshold $\tilde{y}$, which is implicitly defined as the unique solution to $S(y) - F^*(y) = 0$. We have:

Lemma 2 *There exists a unique $\tilde{y}$ such that $S(y) > (=) F^*(y)$ if and only if $y > (=) \tilde{y}$. The threshold $\tilde{y}$ is monotone increasing in ϕ, with $y_{00}^p < \lim_{\phi \downarrow 0} \tilde{y} = \frac{r-\alpha}{\pi_C} I < \lim_{\phi \uparrow 1} \tilde{y} = y_{01}^*$.*

Proof See Appendix A3. □

The threshold $\tilde{y}$ is thus bounded from below by $\frac{r-\alpha}{\pi_C} I$, which is lower than y_{00}^L whenever $\frac{1}{\gamma} < \frac{\beta}{\beta-1}$.

[23] See Azevedo and Paxson (2011) for a detailed discussion on alternative specifications of ex-ante asymmetry between firms in real-option game models.

[24] Here $y_{00}^p \leq y_C^p$ implies that, in the Cournot scenario, if $y_{-f} \leq y_{00}^p$, the leader enters at y_C^p (not at y_{00}^p) in a free premption equilibrium.

If $y_{00}^p < y_S < \tilde{y}$ (relatively weak trigger constraint), one of the two firms, say f, enters immediately at y_S, and $-f$ again enters at y_{01}^*. As formalized in the next proposition, the probability for each firm to lead or to follow is the same, though strictly less than 1/2 (a consequence of $y_{00}^p < y_S$). In that case, the probability of simultaneous entry at y_S is not zero anymore. If $y_{00}^p < \tilde{y} \leq y_S$ (sufficiently strong trigger constraint), the probability of simultaneous entry at y_S is equal to 1, and preemption never occurs. To summarize:

Proposition 3 (symmetric constraints) *Suppose that $y_f = y_{-f} = y_S$, with $y_{00}^p < (=)\, y_S \leq y_{00}^L$: firm f invests as a leader with probability $p_L^f(y_S)$ when Y_t reaches y_S for the first time, firm $-f$ enters as a follower with probability $p_F^{-f}(y_S)$ at y_{01}^*, or both firms invest simultaneously with probability $p_S(y_S)$. In the constrained equilibrium, investments are:*

(1) *either sequential (preemption) or simultaneous if $y_{00}^p < (=)\, y_S < \tilde{y}$, with*

$$0 < p_L^f(y_S) = p_F^f(y_S) = \frac{F^*(y_S) - S(y_S)}{L(y_S) + F^*(y_S) - 2S(y_S)} < (=)\, \frac{1}{2}; \tag{7a}$$

$$0 < (=)\, p_S(y_S) = \frac{L(y_S) - F^*(y_S)}{L(y_S) + F^*(y_S) - 2S(y_S)} < 1; \tag{7b}$$

(2) *simultaneous if $\tilde{y} \leq y_S \leq y_{00}^L$, with*

$$p_L^f(y_S) = p_F^{-f}(y_S) = 0 < p_S(y_S) = 1. \tag{8}$$

Proof See Appendix A4. □

As a particular case, in the Cournot scenario the absence of first-mover advantage implies that $S(y) < (=)\, F_C^*(y)$ for all $y < (=)\, y_C^*$. More formally:

Corollary 1 *$\tilde{y} = y_C^*$ if $\phi = \lambda$, implying that $y_S \leq y_{00}^L < \tilde{y}$. Then, $0 < p_L^f(y_S) = p_F^f(y_S) \leq \frac{1}{2}$ and $0 \leq p_S(y_S) < 1$.*

This means that, when trigger constraints are symmetric and there is no first-mover advantage, Proposition 3 simplifies to the situations described by (7), in which the probability that firms invest sequentially is non zero, and the probability of simultaneous entry is strictly less than 1. Moreover, while the probability of simultaneous entry at y_{00}^p is zero (no "mistake") for all parameter values, the likelihood that firms invest at the same time at $y_S > y_{00}^p$ can be positive, in contrast to the literature.[25] It depends on the flow profit reduction which penalizes the second investor:

Corollary 2 *Suppose that $\frac{r-\alpha}{\pi_C} I < y_S$. Then $p_S(y_S) = 1$ if ϕ is sufficiently low.*

[25] See Chevalier-Roignant and Trigeorgis (2011, Chapter 12) for a detailed analysis of the probability of simultaneous entry, in the absence of first-mover advantage, and without trigger constraints.

In other words, when the symmetric constraint parameter y_S is above the lower bound for $\tilde{y}$, which is monotone strictly increasing in ϕ (Lemma 2), the situation where both firms enter simultaneously with probability 1 is more likely when ϕ is low, all other things equal. In that case, the two firms' symmetric payoff is $S(y_S)$, with $F^*(y_S) < (=) S(y_S) < (=) L(y_S)$ for all $y_S > (=) \tilde{y}$ and $\gamma < (=) 1$.

5.2 Asymmetric Constraints

If $y_f < y_{-f}$, firm f is given the possibility *not* to invest immediately when Y_t hits y_f. It faces no threat of entry by the rival $-f$ whenever $Y_t < y_{-f}$, and thus may choose not to invest until Y_t reaches a higher level y_{00} in $[y_f, y_{-f}]$.[26] When positive, the difference in trigger constraints makes it impossible for firm $-f$ to contest the lead position for a while. Firm f may thus exploit the lack of market awareness of its rival to postpone entry and nevertheless to enter first, at less risk, and at some benefit. As opting for leadership in the entry process is a dominant strategy, and $y_{-f} \leq \arg\max_{y_{00}} L_{y_{00}}(y) \equiv \{y_{00}^L\}$ (see Proposition 1 and subsequent comment), firm f's optimal choice is to enter at $y_{-f} - \varepsilon$ with ε arbitrarily small (henceforth ε is set equal to zero for simplify).[27] We obtain:

Proposition 4 (asymmetric constraints) *Suppose that* $y_{00}^p < y_f < y_{-f} \leq y_{00}^L$: *the constrained equilibrium is sequential (preemption), firm* f *invests immediately with probability* 1 *when* Y_t *reaches* y_{-f} *for the first time, and firm* $-f$ *enters later with probability* 1 *at* y_{01}^*.

Proof Firm f, which is less constrained than $-f$, can choose to invest at any point in $[y_f, y_{-f})$ independently of firm $-f$. As $S(y) < (=) L^*(y)$ for all $y \leq y_{01}^*$, firm f has no incentive not to exploit the leadership possibility. As $L_{y_{00}}(y)$ (introduced in section 4) is concave for all $y_{00} \leq y_{00}^L$, with a maximum at y_{00}^L, and $y_{-f} \leq y_{00}^L$ (by assumption), firm f maximizes value by investing as late as possible, that is when Y_t reaches $y_{-f} \leq y_{00}^L$. □

This result establishes that asymmetric and quite stringent trigger constraints are sufficient for preemption to occur (the probability of a simultaneous investment is zero), and the identity of the first entrant is no longer indeterminate. Leadership is more cumulative than in the perfect alertness situation and in the symmetric trigger constraint case (here the more alert firm invests first with probability 1 and subsequently benefits from this timing in the market stage). In the constrained equilibrium of Proposition 4, an important property is that a first-mover advantage and limited alertness have a complementary effect on the two firms' value at the investment option exercise date. Formally:

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[26]In the time dimension, firm f may postpone entry to any point τ_{00} in $[\tau_f, \tau_{-f}]$ during which it is protected from preemption.

[27]The reasoning is similar to the choice of price in a Bertrand duopoly, with asymmetric marginal costs, when the low-cost supplier maximizes its profit by charging a price almost equal to the rival's marginal cost.

Proposition 5 (complementarity) *For all* $y_{-f} > Y_0$:

$$\frac{d^2 L(y_{-f})}{dy_{-f} d\lambda} = \frac{d^2 F^*(y_{-f})}{dy_{-f} d\phi} = \frac{\beta}{r-\alpha} \left(\frac{y_{-f}}{y_{01}^*}\right)^{\beta-1} \pi_{10} > 0.$$

Therefore, the marginal effect of an increase in the profit ratio λ (for a greater first-mover advantage) on the leader's constrained preemption equilibrium value increases in y_{-f}, which is the value-maximizing entry trigger. A reduction in ϕ (for a lower flow profit to the second-mover) results in the exact opposite effect on the follower's value.

Given this complementarity, for a given y_{-f} the leading firm does not necessarily enter at the same point in time in the Cournot and in the Stackelberg contexts. Therefore, the exact comparison of firm values across the two scenarios of market competition depends on the level of the trigger constraints. The analysis of extreme cases is straightforward. If $y_{-f} \leq y_{00}^p$, all (ϕ, λ), limited alertness has no impact on the equilibrium of the investment game. With a first-mover advantage, a more interesting situation occurs if $y_{00}^p < y_{-f} < y_C^p$ (i.e., the stronger trigger constraint is sufficiently mild as to bind only in the preemption equilibrium with no first-mover advantage). In that case, the leader enters earlier than in the unconstrained benchmark Cournot scenario (perfect alertness and symmetric flow profits), and still for a higher investment option exercise value, at the expense of the follower. This is made more formal in the following result:

Proposition 6 *Suppose that* $\phi < \lambda$*: (1) There exists a unique* $\hat{y}$ *in* $\left(y_{00}^p, y_C^p\right)$ *such that* $L(y_{-f}) > L_C(y_C^p)$ *if and only if* $y_{-f} > \hat{y}$*; (2) There exists a unique* $\breve{y} > \hat{y}$ *such that* $F^*(y_{-f}) < F_C^*(y_C^p)$ *if and only if* $y_{-f} < \breve{y}$*.*

Proof See Appendix A5. □

The comparison $\hat{y} < y_C^p$ indicates that the trigger constraint of the less alert firm needs *not* be strong for the short-run first-mover advantage to result in more equilibrium value. Whenever y_{-f} lies in $(\hat{y}, y_C^p)$, so that it cannot bind in the benchmark scenario, the leader's value, at y_{-f}, is higher with a first-mover advantage, although the firm still enters *before* Y_t reaches y_C^p, that is, earlier than in the free preemption equilibrium trigger of the Cournot case. On the other hand, the follower's equilibrium value is lower than (or the same if $\phi = \lambda$) in the Cournot benchmark with perfect alertness, provided that the leader enters at any y_{-f} below $\min\{y_{00}^L, \breve{y}\}$, where the latter threshold is defined above $\hat{y}$.

5.3 The Effect of Growth and Volatility

This section discusses the effect of a change in the parameters that appear in the geometric Brownian motion, namely α (growth) and σ (volatility), on the investment triggers and on the related value of firms. Toward this aim, define $\mathcal{G}(y) \equiv L(y) - F^*(y)$ for any y in the interval $(0, y_{00}^L]$. Recall that y_{00}^p, which is included in the latter interval, is implicitly defined by $\mathcal{G}(y_{00}^p) = 0$, and that $\mathcal{G}(y_{-f}) > 0$ for all y_{-f} in $(y_{00}^p, y_{00}^L]$.

■ *Growth* For all $y \leq y_{00}^L$, if $0 < \phi \leq \gamma \leq \lambda \leq 1$ one finds

$$\frac{d\mathcal{G}(y)}{d\alpha} = \underbrace{\frac{\partial \mathcal{G}(y)}{\partial \beta}}_{>0} \underbrace{\frac{d\beta}{d\alpha}}_{<0} + \underbrace{\frac{\partial \mathcal{G}(y)}{\partial \alpha}}_{\geq 0} \gtreqless 0. \tag{9}$$

Hence the effect of α on y_{00}^p and on firms' value is ambiguous.[28] A higher drift may decrease or increase the difference between $L(y)$ and $F^*(y)$ at any level of $y \leq y_{00}^L$, implying that y_{00}^p either increases or decreases, respectively. In the former case only, the continuity of value functions (hence of y_{00}^p) in α implies that more growth amounts to relaxing the trigger constraints, so that a free preemption equilibrium is more likely.

Example 1 *Suppose that $Y_0 < y_f \leq y_{-f}$. Let $r = 3/4$, $\alpha = 1/4$, and $\sigma = 1/2$, together with $\pi_F = 1/4$, $\pi_L = 1/2$. Solve $\mathcal{G}(y) = 0$ leads to $y_{00}^p|_{\alpha=1/4}$, which can be written as a function of π_{10} and I. Then $d\mathcal{G}(y)/d\alpha$, when evaluated at $y_{00}^p|_{\alpha=1/4}$, reduces to a simple expression that is positive (zero) if and only if π_{10} is less than (equal to)* 7/4.

Therefore, in a free preemption equilibrium the impact of a change in α on firms' value is also ambiguous. However, in a constrained equilibrium the payoffs are measured at y_{-f}, which does not depend on α. Then, firms' value can only increase with growth, both with simultaneous and sequential (preemption) investments.

■ *Volatility* For all $y \leq y_{00}^L$, we have

$$\frac{d\mathcal{G}(y)}{d\sigma} = \underbrace{\frac{\partial \mathcal{G}(y)}{\partial \beta}}_{>0} \underbrace{\frac{d\beta}{d\sigma}}_{<0} < 0. \tag{10}$$

The effect of σ on the difference $\mathcal{G}$ is thus univocal. Since (10) holds on the interval $(0, y_{00}^L]$, the impact of a change in σ on the level of Y_t at which entry occurs can be inferred from the negative sign of $d\mathcal{G}(y)/d\sigma$.

Proposition 7 *The higher the volatility σ, the higher the threshold y_{00}^p. Therefore, a free preemption equilibrium is more likely.*

In a free preemption MPE, the leader enters later as σ increases, at a higher y_{00}^p (a consequence of $L(y) \gtreqless F^*(y)$ if and only if $y \gtreqless y_{00}^p$).[29] The follower enters later also, at a higher y_{01}^* (this is

[28] All derivations in this section are straightforward. They are available from the author upon request.

[29] The negative sign of (10), and consequently the claim that y_{00}^p increases with σ, hold for $\pi_{10} \geq \pi_L$. When $\pi_{10} < \pi_L$, which may occur when the follower's investment is assumed to be highly beneficial to the leader (positive externality), Mason and Weeds (2009) find that more uncertainty can *lower* the leader's trigger point.

because $dy_{01}^*/d\sigma = (\partial y_{01}^*/\partial\beta)\,(d\beta/d\sigma) > 0)$.[30] At this level of generality, the sign of the change of the difference between y_{00}^p and y_{01}^*, and of the two firms' equal value $F^*(y_{00}^p) = L(y_{00}^p)$, is indeterminate.

In a constrained MPE, there are two cases. If the equilibrium investments are simultaneous (see case 2 in Proposition 3), the two firms' value is $S(y_S)$, which does not depend on σ. Otherwise, investments are sequential (preemption), the leader's entry date remains the same ($Y_t \geq y_{-f}$ binds, where y_{-f} does not depend on σ), and again the follower enters later, so that the difference between the two firms' respective investment triggers increases also.[31] Then (10) informs on the effect of a change in σ on $\mathcal{G}(y_{-f})$. At the firm level,

$$\frac{dF^*(y)}{d\sigma} = \underbrace{\frac{\partial F^*(y)}{\partial\beta}}_{<(=)0}\underbrace{\frac{d\beta}{d\sigma}}_{<0} > (=)0, \text{ all } y < (=)y_{00}^L, \tag{11}$$

and $\pi_L < (=)\pi_{10}$ implies

$$\frac{dL(y)}{d\sigma} = \underbrace{\frac{\partial L(y)}{\partial\beta}}_{>(=)0}\underbrace{\frac{d\beta}{d\sigma}}_{<0} < (=)0 \text{ iff } y < (=)y_{01}^* \exp(-1/\beta). \tag{12}$$

Consider the two derivatives at $y_{-f} \leq y_{00}^L$. When the derivative is negative (positive), more volatility penalizes (benefits) the leading firm f. As $y_{01}^* \exp(-1/\beta) < y_{00}^L$ if and only if $\phi > \exp(-1/\beta)$, firm f's equilibrium value increases with σ only when ϕ is sufficiently high for the entry threshold y_{-f} to be possibly in the interval $(y_{01}^* \exp(-1/\beta), y_{00}^L]$, in which case more volatility benefits firm f also.

Example 2 *Suppose that $Y_0 < y_f \leq y_{-f}$, $r = 3/4$, $\alpha = 1/4$, and $\sigma = 1/2$, together with $\pi_F = 1/2$, $\pi_{10} = 3/4$, $I = 1/2$. With these values $\phi > \exp(-1/\beta)$, so that $dL(y_{-f})/d\sigma$ is negative (zero) if $y_{-f} < (=)y_{01}^* \exp(-1/\beta)$, and positive otherwise. In particular $dL(y_{-f})/d\sigma|_{y_{-f}=y_{00}^p} < 0$, while $dL(y_{-f})/d\sigma|_{y_{-f}=y_{00}^L} > 0$.*

To summarize:

Proposition 8 *In a constrained sequential equilibrium (preemption), higher volatility reduces the positive rent differential, with the leading firm's value $L(y_{-f})$ decreasing if and only if $y_{-f} < y_{01}^* \exp(-1/\beta)$, and the follower's value $F^*(y_{-f})$ increasing for all $y_{-f} \leq y_{00}^L$. More uncertainty also increases the (stochastic) duration between the two firms' investment.*

[30] It is easy to check that $\partial y_{01}^*/\partial\beta < 0$.

[31] An increase in σ does not change the level of Y_t at which the leader enters until y_{00}^p (which does depend on σ) reaches y_{-f} from below, in which case one gets back to a free preemption equilibrium.

6 Final Remarks

The source of rent investigated in this paper is a limitation in firms' relative ability to detect a burgeoning demand. Absent this limitation, from an ex-ante viewpoint an entrepreneurial competitor cannot benefit from the market advantage it seeks by vying for the lead position. It only results in exacerbated competition in the investment process, hence in more rent dissipation. The perfect alertness assumption must be relaxed for a first-mover advantage to result in more value to the leader. This contradicts common wisdom, which sees only virtue in the ability to move fast. In this framework, a long duration between the entry dates of two competitors, as observed on a growing market, is no indication of a proportionally large gap in the managers' abilities to seize new opportunities. It follows that, in the illustrative case of the market for music downloads, where Apple clearly benefits from a persistent first-mover advantage, the late introduction of Zune by Microsoft should not be interpreted as blatant evidence of a timely inability to detect a new demand. It is consistent with a value-maximizing behavior when the difference in investors' "*vistas*" is only minor, and the market development process is highly volatile.

Provided that proxies for investment constraints can be measured in real-world circumstances, several theoretical outputs of this paper can be tested empirically. In particular, if in a nascent market the value of an early investor – as approximated ex post by the capital market value – can be observed to benefit a first mover, one may conjecture that the industry is not that prompt at detecting a new demand, or that uncertainty is reduced. If early entry does not result in large intra-industry profit differentials, it is likely that the perspective of a persistent first-mover advantage results in unleashed rent-seeking behavior, and that the level of future demand is highly uncertain. There is no *a priori* reason to assume, on purely analytical grounds, that the latter case is more frequent than the former.

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Appendix

A1. Strategies[32]

The information set for each firm is $h_t \equiv (Y_t, (i,j)_t)$. The state variables $Y_t \geq 0$ and $(i,j)_t \in \{0,1\}^2$ describe the industry-wide shock and the two firms' installed capacities, respectively.

Given a history h_t of state variables, denote by $A^f(h_t)$ the set of firm f's pure actions available at t. As in our model a firm faces only two possible actions in the investment stage (it may either invest only once or not), so that the investment process stops as soon as each firm holds one capacity unit, we have $A^f(h_t) = \{\text{invest}, \text{wait}\}$ if $(i,j)_t < (1,1)$, and $A^f(h_t) = \{\text{wait}\}$ otherwise. Then, denote by $(\alpha^f(h_t), 1-\alpha^f(h_t)) \in [0,1]^2$ a probability distribution over the elements of $A^f(h_t)$, with $\alpha^f(h_t)$ defined as the probability of investing at time t. (With symmetric notations for firm $-f$.)

At any t, firm f chooses a behavioral investment strategy:

Definition 1. *A behavioral investment strategy of firm f at time t is a map which assigns to each history h_t of state variables a probability distribution $(\alpha^f(h_t), 1-\alpha^f(h_t))$ over the elements of $A^f(h_t)$.*[33]

[32] The first formal treatment of premption appears in Fudenberg and Tirole (1985) in a deterministic framework. Huisman and Kort (1999) extend it to a stochastic environment. Here we adapt the recent presentation by Chevalier-Roignant, Huchzermeier, and Trigeorgis (2010).

[33] If $(i,j)_t \geq (1,1)$, implying that $A^f(h_t) = \{\text{wait}\}$, the distribution degenerates to $(0,1)$.

For simplicity, in what follows we slightly abuse terminology by referring to $\alpha_t^f \equiv \alpha^f(h_t)$ as firm f's behavioral investment strategy at time t. To characterize the chosen strategies, suppose that no firm has invested yet at t, and consider a 2×2 game Γ in which each firm can choose either to invest or to wait. In strategic form:

		$-f$	
		invest $\left(\alpha_t^{-f}\right)$	wait $\left(1-\alpha_t^{-f}\right)$
f	invest $\left(\alpha_t^f\right)$	$S(y), S(y)$	$L(y), F^*(y)$
	wait $\left(1-\alpha_t^f\right)$	$F^*(y), L(y)$	$V(\alpha_t^f, \alpha_t^{-f}), V(\alpha_t^f, \alpha_t^{-f})$

Figure 2: The 2×2 game Γ, for all $t \geq 0$ (first payoff for f, second payoff for $-f$).

In the strategic-form representation of Γ, the function $V^f : [0,1]^2 \to R_+^1$ describes firm f's present expected value of replaying the same game as a function of the two firms' respective probabilities of investing, α_t^f and α_t^{-f}. In that case, firm f's expected value of replaying the game verifies the recursive expression

$$V^f(\alpha_t^f, \alpha_t^{-f}) = \\ \alpha_t^f \alpha_t^{-f} S(y) + \alpha_t^f (1-\alpha_t^{-f}) L(y) + (1-\alpha_t^f)\alpha_t^{-f} F^*(y) + (1-\alpha_t^f)(1-\alpha_t^{-f}) V^f(\alpha_t^f, \alpha_t^{-f}),$$

where $F^*(y)$, $L(y)$, and $S(y)$ are defined in (4), (5) and (6). Here the displayed expression of $V^f(\alpha_t^f, \alpha_t^{-f})$ is firm f's value for a history h_t, with a symmetric expression for $-f$. Moreover, with a Markov refinement, only the *current* state values are used, for any information set, in the definition of the value functions $F^*(y)$, $L(y)$, and $S(y)$. This leads to the following solution concept:

Definition 2. *At any $t \geq 0$, given h_t, a pair of behavioral strategies $(\hat{\alpha}_t^f, \hat{\alpha}_t^{-f})$ is a Markov Nash equilibrium of Γ if $V^f(\hat{\alpha}_t^f, \hat{\alpha}_t^{-f}) \geq V^f(\alpha_t^f, \hat{\alpha}_t^{-f})$ and $V^{-f}(\hat{\alpha}_t^f, \hat{\alpha}_t^{-f}) \geq V^{-f}(\hat{\alpha}_t^f, \alpha_t^{-f})$, all $\alpha_t^f, \alpha_t^{-f}$.*

For subgame perfection, the condition must hold at each point in time.

Definition 3. *A collection of behavioral strategies $\{(\hat{\alpha}_t^f, \hat{\alpha}_t^{-f})\}_{t\geq 0}$ is a subgame perfect Markov Nash equilibrium if for every $t \in [0, \infty)$ the pair of behavioral strategies $(\hat{\alpha}_t^f, \hat{\alpha}_t^{-f})$ is a Markov Nash equilibrium of Γ.*

The next step is to compute equilibrium strategies. For all $(\alpha_t^f, \alpha_t^{-f}) \neq (0,0)$, a reorganization of terms gives

$$V^f(\alpha_t^f, \alpha_t^{-f}) = \frac{\alpha_t^f \alpha_t^{-f} S(y) + \alpha_t^f (1-\alpha_t^{-f}) L(y) + (1-\alpha_t^f)\alpha_t^{-f} F^*(y)}{\alpha_t^f + \alpha_t^{-f} - \alpha_t^f \alpha_t^{-f}}, \tag{14}$$

which describes firm f's objective for a given h_t as a function of its own strategy α_t^f and of the rival's strategy α_t^{-f}. The first-order derivative of V^f in α_t^f is

$$\frac{\partial V^f(\alpha_t^f,\alpha_t^{-f})}{\partial \alpha_t^f}=\alpha_t^{-f}\frac{\alpha_t^{-f}\left(S(y)-L(y)\right)+L(y)-F^*\left(y\right)}{\left(\alpha_t^f+\alpha_t^{-f}-\alpha_t^f\alpha_t^{-f}\right)^2}. \tag{15}$$

with a symmetric expression for $-f$. The sign of the derivative in (15) is the same as the sign of the numerator, which depends on the comparison of $S(y)$ with $L(y)$, and of $L(y)$ with $F^*(y)$. If the numerator is zero, the value $V^f(\alpha_t^f,\alpha_t^{-f})$ does not depend on α_t^f, and there is no cost to specify that firm f plays the same strategy as what is obtained when the numerator is either negative or positive.

We already know from Lemma 2 that $S(y)<(=)F^*(y)$ if and only $y<(=)\tilde{y}\in(y_{00}^p,y_{01}^*)$, and from the definitions in (5-6) that $S(y)<(=)L(y)$ for all $y<(\geq)y_{01}^*$ whenever $\pi_C<(=)\pi_{10}$ (the instantaneous market profit to the leader is $\pi_{10}\geq\pi_C$ when $y<y_{01}^*$ and is followed by $\pi_L\geq\pi_C$ when $y\geq y_{01}^*$, while the profit is π_C all along in case of simultaneous investment). It remains to consider the partition of the support of y in three intervals, as follows.

Case 1: $y<\bar{y}_{00}^p$. We know from above that $S(y)<L(y)$, and from (5) and (6) that $L\left(y\right)<F^*\left(y\right)$. It follows from (15) that $\frac{\partial V^f(\alpha_t^f,\alpha_t^{-f})}{\partial\alpha_t^f}<0$ and $\frac{\partial V^{-f}(\alpha_t^f,\alpha_t^{-f})}{\partial\alpha_t^f}<0$, for all $(\alpha_t^f,\alpha_t^{-f})$, implying that f and $-f$ maximize V^f and V^{-f}, respectively, by choosing the lowest possible probability of investing. Therefore, the unique equilibrium is $\hat{\alpha}_t^f=\hat{\alpha}_t^{-f}=0$.

Case 2: $\bar{y}_{00}^p\leq y<\bar{y}_{01}^*$. Again $S(y)<(=)L(y)$ for all $y\leq y_{01}^*$, and we know from (5-6) that $L\left(y\right)>(=)\,F^*\left(y\right)$ if $y_{00}^p<(=)\,y<y_{01}^*$. Therefore, the sign of the derivative in (15) is *a priori* indeterminate. There are now two situations that depend on the level of y relative to $\tilde{y}$.

First, if $y<(=)\,\tilde{y}$, we have $S\left(y\right)<L\left(y\right)$ and $S\left(y\right)<F^*\left(y\right)$, which leads to three subcases that depend on the level of α_t^{-f}:

(i) Suppose that $0\leq\alpha_t^{-f}<\frac{L(y)-F^*(y)}{L(y)-S(y)}$, implying that $\frac{\partial V^f(\alpha_t^f,\alpha_t^{-f})}{\partial\alpha_t^f}\geq0$, for all α_t^f. Then firm f maximizes V^f by choosing $\alpha_t^f=1$. Hence $\frac{\partial V^{-f}(1,\alpha_t^{-f})}{\partial\alpha_t^{-f}}=S(y)-F^*\left(y\right)<(=)\,0$ for all $y<(=)\,\tilde{y}$, so that $-f$ maximizes V^{-f} with $\alpha_t^{-f}=0$, and the solution is asymmetric.

(ii) Suppose that $\frac{L(y)-F^*(y)}{L(y)-S(y)}<\alpha_t^{-f}\leq1$, implying that $\frac{\partial V^f(\alpha_t^f,\alpha_t^{-f})}{\partial\alpha_t^f}<0$, for all α_t^f. Then firm f maximizes V^f by choosing $\alpha_t^f=0$. This implies in turn that $\frac{\partial V^{-f}(0,\alpha_t^{-f})}{\partial\alpha_t^{-f}}=0$, for all $\alpha_t^{-f}>0$ sufficiently high to satisfy the initial supposition, leading to an asymmetric solution.

(iii) The last possibility is $\alpha_t^{-f} = \frac{L(y)-F^*(y)}{L(y)-S(y)}$, implying that $\frac{\partial V^f(\alpha_t^f,\alpha_t^{-f})}{\partial \alpha_t^f} = 0$, all $\alpha_t^f \in [0,1]$. The latter continuum renders admissible the unique symmetric strategy

$$\hat{\alpha}_t^f = \hat{\alpha}_t^{-f} = \frac{L(y) - F^*(y)}{L(y) - S(y)}, \tag{16}$$

so that firm values are $V^f(\hat{\alpha}_t^f, \hat{\alpha}_t^{-f}) = V^{-f}(\hat{\alpha}_t^f, \hat{\alpha}_t^{-f}) = L(y) = F^*(y) > S(y)$.

Only in (iii) we have symmetry. From here on we focus on this case. Then (16) implies that $\hat{\alpha}_t^f = \hat{\alpha}_t^{-f}$ describes a non zero probability of simultaneous entry for all $y \in (y_{00}^p, y_{01}^*)$, with $\lim_{y \to y_{00}^p} \hat{\alpha}_t^f = \lim_{y \to y_{00}^p} \hat{\alpha}_t^{-f} = 0$, and $\hat{\alpha}_t^f\Big|_{y=\tilde{y}} = \hat{\alpha}_t^{-f}\Big|_{y=\tilde{y}} = 1$.

Second, if $y > \tilde{y}$ then $\alpha_t^{-f}(S(y) - L(y)) + L(y) - F^*(y) > 0$, so that $\frac{\partial V^f(\alpha_t^f,\alpha_t^{-f})}{\partial \alpha_t^f} \geq 0$ from (15), all $\alpha_t^{-f} \geq 0$. Then firm f maximizes V^f with $\alpha_t^f = 1$. Hence $\frac{\partial V^{-f}(1,\alpha_t^{-f})}{\partial \alpha_t^{-f}} = S(y) - F^*(y) > 0$, so that $-f$ maximizes V^{-f} with $\alpha_t^{-f} = 1$, leading to a symmetric solution:

$$\hat{\alpha}_t^f = \hat{\alpha}_t^{-f} = 1. \tag{17}$$

Case 3: $y_{01}^* \leq y$. By definition $F^*(y) = L(y) = S(y)$, implying from (14) that $V^f(\alpha_t^f, \alpha_t^{-f}) = V^{-f}(\alpha_t^f, \alpha_t^{-f}) = S(y)$ also, with both firms investing immediately, hence $\hat{\alpha}_t^f = \hat{\alpha}_t^{-f} = 1$.

A2. Proof of Lemma 1

The value function $F^*(y)$, which is strictly increasing with y, is also strictly convex as

$$\frac{d^2F^*(y)}{dy^2} = \frac{I\beta}{y^2}\left(\frac{y}{y_{01}^*}\right)^\beta > 0, \tag{18}$$

all $y > 0$, while $L(y)$ is strictly (weakly) concave if and only if $\pi_L < (=)\pi_{10}$ because

$$\frac{d^2L(y)}{dy^2} = \frac{\beta(\beta-1)}{y^2}\left(\frac{y}{y_{01}^*}\right)^\beta \frac{\pi_L - \pi_{10}}{r - \alpha} y_{01}^* < (=) 0, \tag{19}$$

all $y > 0$. It follows that $F^*(y) - L(y) = 0$ may admit up to two roots. Consider first the benchmark case $\phi = \lambda$ (Cournot), implying that $y_C^* \equiv y_{01}^*|_{\phi=\lambda}$ is a root. Then $F_C^*(0) = 0 > L_C(0) = -I$, together with

$$\left.\frac{dF_C^*(y)}{dy}\right|_{y=y_C^*} = \frac{\pi_C}{r-\alpha} > \left.\frac{dL_C(y)}{dy}\right|_{y=y_C^*} = \frac{\pi_C}{r-\alpha} - (\beta - 1)\frac{\pi_{10} - \pi_C}{r - \alpha}, \tag{20}$$

are sufficient to conclude that there exists another positive root $y_C^p < (=) y_C^*$ if $\pi_C < (=)\pi_{10}$. Then, consider the more general case $\phi \leq \gamma \leq \lambda$. Define $\mathcal{F} \equiv F^* - F_C^*$ and $\mathcal{L} \equiv L - L_C$, to compare the slopes of $F^*(y)$ and $L(y)$ with the slopes of $F^*_C(y)$ and $L_C(y)$, respectively. Recalling that $\beta > 1$, we find

$$\frac{d\mathcal{F}(y)}{dy} = \frac{I}{y}\frac{\beta}{\beta-1}\left[\left(\frac{y}{y^*_C}\right)^{\beta} - \left(\frac{y}{y^*_{01}}\right)^{\beta}\right] < 0 \tag{21}$$

if $\pi_F < \pi_C$, and

$$\frac{d\mathcal{L}(y)}{dy} = \frac{\beta}{r-\alpha}\left[\left(\frac{y}{y^*_C}\right)^{\beta-1}(\pi_{10} - \pi_C) - \left(\frac{y}{y^*_{01}}\right)^{\beta-1}(\pi_{10} - \pi_L)\right] > 0 \tag{22}$$

if $\pi_C < \pi_L$, all $y > 0$. In the Stackelberg scenario the slope of the second entrant's value function is strictly lower than in Cournot, whereas the slope of the first entrant's value function is higher. Together with $F^*(0) = F^*_C(0) = 0$ and $L(0) = L_C(0) = -I$, this leads to the conclusion that F^* and L intersect at a lower y than F^*_C and L_C. This establishes that $y^p_{00} < y^p_C$. In addition, it is sufficient to check that $F^*(y^*_{01}) < \lim_{y\uparrow y^*_{01}} L(y)$ when $\phi < \lambda$ to conclude that F^* and L cannot intersect for any $y > y^p_{00}$ on which the value functions are defined. □

A3. Proof of Lemma 2

Set $\phi = 1$. From (4) and (6), the slope of $F^*(y)$ is (weakly) lower than the slope of $S(y)$ for all $y < (=)\, y^*_{01}$. Then $F^*(y) = 0 > S(y) = -I$ and $\lim_{y\uparrow y^*_{01}} F^*(y) = S(y^*_{01})$ imply that $F^*(y) > (=)\, S(y^*_{01})$ for all $y < (=)\, y^*_{01}$. Moreover, it is immediate to check that the slope of $F^*(y)$ is monotone decreasing when ϕ departs from 1 toward 0, while $S(y)$ is unchanged, for all $y \leq y^*_{01}$. It follows that there exists a unique $\tilde{y}$ in $(0, y^*_{01})$ such that $F^*(y) > (=)\, S(y)$ if and only if $y < (=)\, \tilde{y}$, with $\tilde{y}$ strictly monotone increasing in ϕ, and $\lim_{\phi\uparrow 1} \tilde{y} = y^*_{01}$.

Next, observe from (5) that $L^*(y)$ is strictly concave, while $S(y)$ is linear, for all $y < (=)\, y^*_{01}$, so that $L^*(y) - S(y) = 0$ has at most two roots. We know that $L^*(0) = S(0)$, and for $\phi = \lambda$ that $L^*(y^*_{01}) = S(y^*_{01})$. As $L(y)$ gets steeper when λ/ϕ rises (Remark 1), while $S(y)$ in (6) is unchanged, we have $L^*(y) > S(y)$, all $y \in (0, y^*_{01})$, including $\tilde{y}$, hence $L^*(\tilde{y}) > S(\tilde{y}) = F^*(\tilde{y})$. Then it is sufficient to recall that $L^*(y) > F^*(y)$ if and only if $y > y^p_{00}$ to obtain $\tilde{y} > y^p_{00}$. Finally, as $\lim_{\phi\downarrow 0} F^*(y) = 0$, while $S(y)$ is unchanged, when π_F approaches 0 the root $\tilde{y}$ converges to the solution to $S(y) = 0$, that is $\frac{r-\alpha}{\pi_C} I$. □

A4. Proof of Proposition 3

The probability that only one firm, say f, invests as a leader, while $-f$ waits and invest as a follower, is given by $p^f_L = \alpha^f_t\left(1 - \alpha^{-f}_t\right) + \left(1 - \alpha^f_t\right)\left(1 - \alpha^{-f}_t\right) p^f_L$. For all $(\alpha^f_t, \alpha^{-f}_t) \neq (0,0)$, a reorganization of terms gives

$$p^f_L = \alpha^f_t \frac{1 - \alpha^{-f}_t}{\alpha^f_t + \alpha^{-f}_t - \alpha^f_t \alpha^{-f}_t}. \tag{23}$$

Next, the probability of simultaneous investments is $p_S = \alpha_t^f \alpha_t^{-f} + \left(1 - \alpha_t^f\right)\left(1 - \alpha_t^{-f}\right) p_S$. For all $(\alpha_t^f, \alpha_t^{-f}) \neq (0,0)$, a reorganization of terms gives

$$p_S = \frac{\alpha_t^f \alpha_t^{-f}}{\alpha_t^f + \alpha_t^{-f} - \alpha_t^f \alpha_t^{-f}}. \tag{24}$$

In a symmetric equilibrium $\alpha_t = \alpha_t^f = \alpha_t^{-f}$, (23) and (24) simplify to $p_L^f = \frac{1-\alpha_t}{2-\alpha_t}$ and $p_S = \frac{\alpha_t}{2-\alpha_t}$. It remains to plug in the latter expressions the equilibrium strategies $\left(\hat{\alpha}_t^f, \hat{\alpha}_t^{-f}\right)$ derived in Appendix A1. Again there are two cases that depend on the level of y relative to $\tilde{y}$:

► If $y_{00}^p \leq y < \tilde{y}$, the symmetric equilibrium strategies $\hat{\alpha}_t^f = \hat{\alpha}_t^{-f} = \frac{L(y)-F^*(y)}{L(y)-S(y)}$ in (16) give

$$p_L^f(y) = \frac{F^*(y) - S(y)}{L(y) + F^*(y) - 2S(y)} \qquad \text{and} \qquad p_S(y) = \frac{L(y) - F^*(y)}{L(y) + F^*(y) - 2S(y)}. \tag{25}$$

Then $L(y_{00}^p) = F^*(y_{00}^p) \Rightarrow p_L^f(y_{00}^p) = \frac{1}{2} > p_S(y_{00}^p) = 0$, and $L(y) > F^*(y) \Rightarrow p_S(y) > 0$ for all $y \in (y_{00}^p, \tilde{y})$.

► If $\tilde{y} \leq y < y_{01}^*$, the symmetric equilibrium strategies $\hat{\alpha}_t^f = \hat{\alpha}_t^{-f} = 1$ in (17) lead to

$$p_L^f(y) = p_F^{-f}(y) = 0 \qquad \text{and} \qquad p_S(y) = 1. \tag{26}$$

In Proposition 3, (7a) and (7b) result directly from (25), and (8) results from (26). □

A5. Proof of Proposition 6

(1) From Proposition 2 we know that $\phi < (=)\lambda$ implies $L\left(y_{00}^p\right) < (=)L_C\left(y_C^p\right)$, with $y_{00}^p < (=)y_C^p$. Moreover $L_C(y) < (=)L(y)$ for all $y > 0$ (from (22)), implying that $L_C\left(y_C^p\right) < (=)L\left(y_C^p\right)$, hence $L\left(y_{00}^p\right) < (=)L_C\left(y_C^p\right) < (=)L\left(y_C^p\right)$ by transitivity. As L is monotone strictly increasing in $y \geq 0$, when $\phi < \lambda$ there exists a unique $\hat{y}$ in (y_{00}^p, y_C^p) such that $L(y) < L_C(y_C^p)$ iff $y < \hat{y}$.

(2) We know from Remark 2 that $\gamma < (=)1$ implies $y_C^p < (=)y_C^*$, and from (4) that F_C^* is monotone strictly increasing in $y \geq 0$, implying in turn that, for all $y_C^p > 0$, we have $0 < F_C^*(y_C^p) < (=)F_C^*(y_C^*)$. The latter inequality, together with $F_C^*(y_C^*) = F^*(y_{01}^*) = I/(\beta - 1)$, leads to $0 < F_C^*(y_C^p) < (=)F^*(y_{01}^*)$ by transitivity. Recalling that $F^*(0) = 0$ and that F^* is also monotone increasing in $y \geq 0$, there exists a unique $\check{y}$ in $(0, y_{01}^*]$ such that $F^*(y) < (=)F_C^*(y_C^p)$ iff $y < (=)\check{y}$. As a consequence, recalling that $\phi < (=)\gamma$ implies $F^*(y) < (=)F_C^*(y)$ for all $y > 0$ (from (21)), we have $y_C^p < (=)\check{y}$. Then $\hat{y} < (=)y_C^p$ implies $\hat{y} < (=)\check{y}$. □