Portfolios of Real Options and Capacity Expansion in Transmission Network Expansion Planning

Manuel V. Loureiro
INESC TEC (formerly INESC Porto)
Faculdade de Engenharia, Universidade do Porto
Faculdade de Economia, Universidade do Porto
mvml@inescporto.pt

Paulo J. Pereira
Faculdade de Economia, Universidade do Porto
pjpereira@fep.up.pt

João Claro
INESC TEC (formerly INESC Porto)
Faculdade de Engenharia, Universidade do Porto
jclaro@fe.up.pt

January 23, 2012

Abstract

In network investments, much of the real options value may reside "in" identifying the proper configuration to develop, in terms of the timing and sizing of its deployment. A longer-term perspective, with a treatment of uncertainty that moves beyond simply increasing deterministic specifications, has the potential to improve the design of networked infrastructures, such as power transmission networks. In this paper we present a new model for Transmission Network Expansion Planning, with uncertainty in demand, which considers investments in transmission lines as a portfolio of real options, and use this model to study the impact of uncertainty and demand correlation in basic network building blocks. Our analysis confirms the value of network sources of operational flexibility in a multi-stage setting. The results also show that uncertainty increases the network value in a non-monotonous way, due to the load curtailment costs and discrete capacity expansions, and higher demand correlations do not necessarily lead to reductions in the network value.

Keywords: Real options, Investment timing, Portfolios of real options, Transmission network expansion planning, Capacity expansion

1 Introduction

Transmission Network Expansion Planning (TNEP) is the problem of identifying the structure that an energy transmission network must have in order to allow a set of populations, located at its demand nodes, to fulfill their energy needs. This problem can be viewed as a particular case of a broader class of problems concerned with the design of networked engineering systems. Uncertainty is a key consideration in the design of these systems, as they are long lived, and interact with many and diverse contextual factors. However, these problems are usually addressed from a static perspective, neglecting the impact of uncertainty, and the evolution of uncertainty in time, in particular.

In this latter perspective, TNEP can be viewed as a problem of designing a physical system whose expansions are real options (Dixit and Pindyck 1994): TNEP investments are irreversible decisions, with significant costs associated, some leeway regarding their timing and dynamics, and made in the presence of significant uncertainties, for example regarding generation capacity and demand (Bustamante-Cedeño and Arora 2008).

Furthermore, TNEP is concerned with real options "in" projects (de Neufville and Scholtes 2011, Wang and de Neufville 2005): flexibility can be designed in the network, to address uncertainty in the future evolution of generation capacity or demand; with each new identified investment opportunity the complexity of the network increases significantly; path-dependency is crucial to reflect the impacts of capacity expansion in each possible scenario; the values of expansions are interdependent due to the existence of correlated demands and the fact that some expansions yield results similar to other expansions; the technical aspects of transmission networks can not be neglected.

TNEP can also be usefully conceptualized as a portfolio of real options, i.e., combinations of assets and real options associated with these assets that are subject to risk and constraints (Brosch 2008). Findings suggest that with real options value additivity does not hold (Trigeorgis 1993), thus a valuation of the whole portfolio of real options is required in order to determine its aggregate value. However, unlike the approach of Brosch (2008), we do not view decisions as switches between modes, but as options to expand the network, with new corridors that open up the possibility of investing in subsequent corridors, and increases in the number of lines in a corridor, allowing a higher power flow to meet demand. Moreover, this work includes a more complex treatment of the technical aspects of a physical system, going beyond economies of scale, to address aspects such as energy flows and energy losses.

Previous research in TNEP under uncertainty has considered two-stage models, with investment decisions under uncertainty, and network flow decisions made after uncertain parameters are revealed (Delgado and Claro 2011, Bustamante-Cedeño and Arora 2008). In this work, we extend that line of research, by introducing a multi-stage dimension (Claro and Sousa 2010, Ahmed et al. 2003), in order to understand the impact of leeway in the postponement of network configuration decisions. Our model is a mixed integer linear programming formulation of TNEP, based on the linearization suggested by Alguacil et al. (2003). Chow and Regan (2011) also address multi-period network design under uncertainty, but considering origin-destination demand, in the context of transportation networks, and using a heuristic approach to solve a lower bound model formulation.

This paper unfolds as follows: section 2 introduces the optimization model, section 3 describes a numerical study focused on a set of three basic network building blocks, and section 4 presents key conclusions and suggestions for further developments.

2 Model

2.1 Network Notation

To differentiate clearly between the network and the binomial tree, we will use the term *bus* for a network node, and the terms *corridor* and *line* for the links between buses. As such, a corridor can consist of any number of lines, depending on the investment decisions that maximize the total expanded net present value.

The network includes a set of buses $\mathcal{B}$, of which $\mathcal{B}_S$ are supply buses, $\mathcal{B}_D$ are demand buses, and eventually a third part are intermediate buses. Thus, the following conditions must hold:

$$\begin{aligned} \mathcal{B}_S &\subset \mathcal{B} \\ \mathcal{B}_D &\subset \mathcal{B} \\ \mathcal{B}_S \cup \mathcal{B}_D &\subseteq \mathcal{B} \\ \mathcal{B}_S \cap \mathcal{B}_D &= \emptyset \end{aligned}$$

Any bus can be either a part of subset $\mathcal{B}_S$ or $\mathcal{B}_D$, however no bus can represent both generation capacity and population demand.

The set of corridors $\mathcal{C}$ consists of all possible pairs of buses, excluding identical pairs, and can be defined as the Cartesian product of sets, with the pairs (i, i) removed. $\mathcal{C}_A$ is the subset of allowable connections, which allows the enforcement of different configurations. As direction is meaningless, i.e., energy can flow both ways, $\mathcal{C}_A$ must be enforced to include the corridor (j, i) if the corridor (i, j) is present:

$$\begin{aligned} \mathcal{C} &= (\mathcal{B} \times \mathcal{B}) \backslash \{(i, i) | i \in \mathcal{B}\} \\ \mathcal{C}_A &\subseteq \mathcal{C}, \forall (i, j) \in \mathcal{C}_A \exists (j, i) \in \mathcal{C}_A \end{aligned}$$

The set $\mathcal{K}$ is introduced to distinguish between lines in corridors, and a Cartesian product of sets is used to identify each line in each corridor, in a new set designated by $\mathcal{X}$. To simplify the formulation of constraints in our model, we will use x as a tuple that represents the starting bus i, the end bus j, and the line k; for the opposite direction, we will switch i with j and express it as y:

$$\begin{aligned} \mathcal{X} &= \mathcal{C}_A \times \mathcal{K} \\ x &= \{i, j, k\} \in \mathcal{X} \\ y &= \{j, i, k\} \in \mathcal{X}. \end{aligned}$$

2.2 Modeling the Porfolio of Real Options

Demand is, in general, considered as the key uncertain factor influencing the configuration of transmission networks. Other stochastic factors, such as selling prices or investment costs, were considered here as deterministic, to preserve simplicity in analysis.

The evolution of demand is modelled as a geometric Brownian motion (Marathe and Ryan 2005), with D_i designating the demand, and μ_i and σ_i, respectively, the trend and standard deviation of the evolution of demand, all corresponding to bus i:

$$dD_i = \mu_i D_i dt + \sigma_i D_i dW_i, \quad i \in \mathcal{B}_D.$$

A second key aspect in the description of the behavior of demand is the joint variation of population demands, that we characterize with the matrix of correlation coefficients ρ.

We compute the joint decisions of the portfolio of real options by numerical approximation using a binomial tree, following the approach of Cox et al. (1979), expanding it to the whole set of decisions being taken, and considering path-dependency (Figure 1).

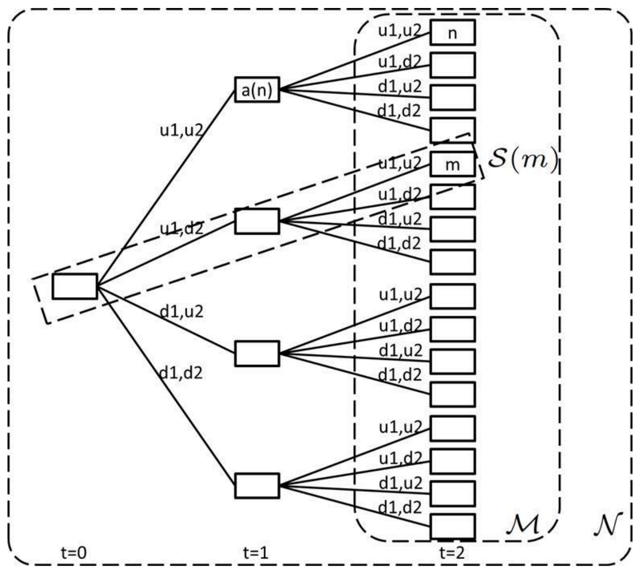

Figure 1: Path-dependent binomial tree for two underlying demands

For each uncertain demand, we have at each time stage an up move and a down move. However, a node which has as predecessors, for example, an up move followed by a down move, must be differentiated from one preceded by a down move followed by an up move. This way, instead of having $i+1$ new nodes for each new time stage, where i is the index of the time stage, we will have, at each time stage, 2^i new nodes. With n time stages, the total number of nodes will be

$$\sum_{i=0}^{n} 2^i.$$

Considering multiple different demands, with a multidimensionality of U, and an up and a down move for each, at each time stage and each node 2^U nodes will be added. The set of nodes in this path-dependent binomial tree is called $\mathcal{N}$, and its total number is

$$|\mathcal{N}| = \sum_{i=0}^{n} (2^U)^i.$$

In order to work with scenarios in the tree, we consider the subset $\mathcal{M} \in \mathcal{N}$ of the terminal nodes of the tree, a function $a(n)$ that returns the node that precedes node n, and a function $t(n)$ that returns the time stage for node n (the time stage for the root node is zero). A scenario is a path in the tree, from the root node (node 0) to a terminal node.

This problem can not be solved using backward induction alone. The investment decisions in any node, in general, influence the investment decisions in previous nodes in the same scenario, up to the root node. Although this is not the situation that is most commonly found in real options analysis, in this model, due to the additive nature of installed capacity, in general, all the investment decisions in the nodes of the subtree of a certain node will depend on the investment decisions in this node. As a consequence, in general, all the investment decisions in all nodes depend on all the other investment decisions in all the other nodes, which requires considering the binary tree as a whole for decision making.

Due to the increase in computational complexity caused by the interdependence between all decision nodes, and the fact that we are addressing a situation with multiple demands, we limit our analysis to instances with three time stages, and two population demands, which enables us to look for insights from the analysis of optimal solutions.

The probability of each scenario is computed by the joint probability of all nodes in its corresponding path. For each scenario, and the respective terminal node m, the probability is $Prob(m)$:

$$\sum_{m \in \mathcal{M}} Prob(m) = 1.$$

We compute the up and down factors, and respective probabilities, as Brosch (2008), for two underlying assets:

$$u_i = e^{\mu_i \Delta t + \sigma_i \sqrt{\Delta t}}$$
$$d_i = e^{\mu_i \Delta t - \sigma_i \sqrt{\Delta t}}$$

$$p1 = \frac{1}{4}\left[1 + \rho_{1,2}\right], \quad u_1, u_2$$
$$p2 = \frac{1}{4}\left[1 - \rho_{1,2}\right], \quad u_1, d_2$$
$$p3 = \frac{1}{4}\left[1 - \rho_{1,2}\right], \quad d_1, u_2$$
$$p4 = \frac{1}{4}\left[1 + \rho_{1,2}\right], \quad d_1, d_2.$$

Considering the subsets of nodes in each scenario $\mathcal{S}(m) \subset \mathcal{N}$, the expanded net present value is given by the sum of the present value of all net cash-flows of all nodes in a scenario, weighted by the scenario's probability:

$$\text{Expanded Net Present Value} = \sum_{m \in \mathcal{M}} Prob(m) \left[\sum_{n \in \mathcal{S}(m)} \text{Net Cash Flow}_n \right].$$

It should be noted that a node that is part of several scenarios will have its value weighted by the sum of the probabilities of all the scenarios that it is a part of. As a particular case, the root node, since it is a part of all scenarios, will have its full value reflected on the expanded net present value.

We assume that the net cash-flow of each node n is composed of an operational part CF_n and an investment I_n. For each scenario, i.e., each $m \in \mathcal{M}$ we also add a perpetuity component, using Gordon's model. Substituting, accounting for the time-value of money under risk-neutrality, and maximizing the expanded net present value, we get the objective function

$$max \sum_{m \in \mathcal{M}} Prob(m) \left[\sum_{n \in \mathcal{S}(m)} \left[\frac{CF_n - I_n}{(1+r)^{t(n)\Delta t}} \right] + \frac{CF_m(1+g)}{(r-g)(1+r)^{t(m)}} \right], \quad (1)$$

In equation 1 we express the fixed risk-free interest rate by r, the time interval between stages by Δt, and the growth rate of the perpetuity by g. Compound rates are not used to simplify the presentation of the perpetuity. As explained above, this modelling approach reflects the need to evaluate all the decisions in all the nodes of the binomial tree at the same time to account for their interdependencies.

The operational cash-flow is given by

$$CF_n = \sum_{i \in \mathcal{B}_D} [(D_{i,n} - \Gamma_{i,n})\delta_i - \Gamma_{i,n}\gamma_i] - \sum_{i \in \mathcal{B}_S} E_{i,n}c_i \quad , n \in \mathcal{N}. \quad (2)$$

Its computation requires knowing the amount of demand we are able to satisfy at a price δ_i, at all demand buses in each node, which is given by the difference between the demand $D_{i,n}$ and the amount of unfulfilled demand, i.e., the load curtailment $\Gamma_{i,n}$. We also consider an opportunity cost for unfulfilled demand, γ_i, and the cost of energy consumption $E_{i,n}$, i.e., the demand fulfilled and the power losses in the network, with a unit cost of c_i. Prices and costs are assumed fixed through time.

In any node, the current network configuration is given by the sum of previous investments Ω and current investments ω:

$$\Omega_{x,n} = \Omega_{x,a(n)} + \omega_{x,n} \quad , \forall n > 0 \in \mathcal{N}, x \in \mathcal{X} \quad (3)$$

$$\Omega_{x,0} = \omega_{x,0} \quad , \forall x \in \mathcal{X} \quad (4)$$

$$\Omega_{x,n} = \{0,1\} \quad , \forall n \in \mathcal{N}, x \in \mathcal{X} \quad (5)$$

$$\omega_{x,n} = \{0,1\} \quad , \forall n \in \mathcal{N}, x \in \mathcal{X}. \quad (6)$$

It should be noted that we assume no lead time for the investments to be operational, as we are considering a time step sufficiently large to allow for construction and use. This assumption could later be relaxed either by reducing the contribution of ω or by postponing its effects. Ω and ω are binary variables that have a value of 1 if line x has been built or 0 if it has not (5 and 6).

The total investment cost in any given node n, I_n, is then simply computed by adding up all the costs C of the new lines:

$$I_n = \sum_{x \in \mathcal{X}} C_x \omega_{x,n} \quad , n \in \mathcal{N}. \quad (7)$$

2.3 Adding the Transmission Network Expansion Planning Constraints

In this subsection we present all the technical constraints that are included in the model, with a brief description and pointing to appropriate references for further detail.

2.3.1 Network Flow

The constraints that model the network flows are the following:

$$E_{i,n} - (D_{i,n} - \Gamma_{i,n}) - \sum_{x \in \mathcal{X}} \left[f_{x,n} + \frac{1}{2} h_{x,n} \right] = 0 \quad , \forall n \in \mathcal{N}, \quad i \in \mathcal{B} \tag{8}$$

$$f_{x,n} = -f_{y,n} \quad , \forall n \in \mathcal{N}, \quad x, y \in \mathcal{X} \tag{9}$$

$$h_{x,n} = h_{y,n} \quad , \forall n \in \mathcal{N}, \quad x, y \in \mathcal{X} \tag{10}$$

$$0 \le E_{i,n} \le E_i^{max} \quad , \forall n \in \mathcal{N}, \quad i \in \mathcal{B}_S \tag{11}$$

$$-F_x^{max} \Omega_{x,n} \le f_{x,n} \le F_x^{max} \Omega_{x,n} \quad , \forall n \in \mathcal{N}, \quad x \in \mathcal{X} \tag{12}$$

$$0 \le h_{x,n} \le F_x^{max} \Omega_{x,n} \quad , \forall n \in \mathcal{N}, \quad x \in \mathcal{X} \tag{13}$$

$$f_{x,n} + \frac{1}{2} h_{x,n} \le F_x^{max} \Omega_{x,n} \quad , \forall n \in \mathcal{N}, \quad x \in \mathcal{X} \tag{14}$$

$$-f_{x,n} + \frac{1}{2} h_{x,n} \le F_x^{max} \Omega_{x,n} \quad , \forall n \in \mathcal{N}, \quad x \in \mathcal{X} \tag{15}$$

Equation 8 states that a bus can not store energy. All energy supplied E either serves demand, or flows to another bus, f, with losses h. Equations 9 and 10 relate the flow in any line between buses (i, j) to its symmetric in (j, i), holding the sign for losses. Non-negativity of supplied energy is enforced through equation 11, with a maximum value possibly defined using E^{max}. Equations 12, 13, 14 and 15 define limits F^{max} for the flow and losses in each line, individually and jointly. Further detail can be obtained, e.g, in Bustamante-Cedeño and Arora (2008).

2.3.2 Linearized Flow and Losses

The linearization of power flows and losses is achieved with the following constraints:

$$-(1 - \Omega_{x,n}) M^* \le \frac{f_{x,n}}{B_x} + (\theta^+_{i,j,n} - \theta^-_{i,j,n}) \le (1 - \Omega_{x,n}) M^* \quad , \forall n \in \mathcal{N}, \quad i, j \in \mathcal{B}, \quad x \in \mathcal{X} \tag{16}$$

$$0 \le -\frac{h_{x,n}}{G_x} + \sum_{\ell=1}^{L} \alpha_{i,j,n}(\ell) \theta_{i,j,n}(\ell) \le (1 - \Omega_{x,n}) M^{**} \quad , \forall n \in \mathcal{N}, \quad i, j \in \mathcal{B}, \quad x \in \mathcal{X} \tag{17}$$

$$\sum_{\ell=1}^{L} \theta_{i,j,n}(\ell) = \theta^+_{i,j,n} + \theta^-_{i,j,n} \quad , \forall n \in \mathcal{N}, \quad i, j \in \mathcal{B} \tag{18}$$

$$\theta_{i,n} - \theta_{j,n} = \theta^+_{i,j,n} - \theta^-_{i,j,n} \quad , \forall n \in \mathcal{N}, \quad i, j \in \mathcal{B}, \quad x \in \mathcal{X} \tag{19}$$

$$\theta^+_{i,j,n} \ge 0 \quad , \forall n \in \mathcal{N}, \quad i, j \in \mathcal{B}, \quad x \in \mathcal{X} \tag{20}$$

$$\theta^-_{i,j,n} \ge 0 \quad , \forall n \in \mathcal{N}, \quad i, j \in \mathcal{B}, \quad x \in \mathcal{X} \tag{21}$$

$$\theta^{ref} = 0. \tag{22}$$

This set of equations is a linearised approximation of the equations for power flows and losses

$$f_{i,j} = -B sin(\theta_i - \theta_j)$$
$$h_{i,j} = 2G(1 - cos(\theta_i - \theta_j)),$$

where B and G are the susceptance and conductance of the line and θ denotes the voltage angle of the bus.

The detailed explanation of the linearisation process can be found in Alguacil et al. (2003).

3 Numerical Study

3.1 Data

To study the impact of the timing of investments on network configuration, we will consider three different transmission network "building blocks": independent, radial and meshed designs. The choice of these building blocks is based on the studies by Delgado and Claro (2011) and Van Mieghem (2007).

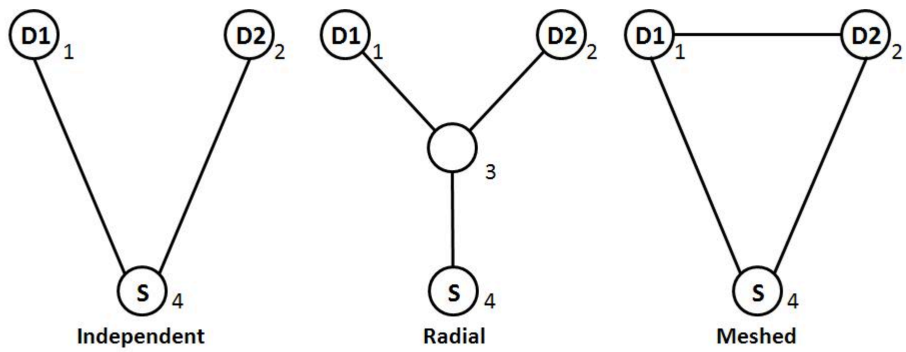

Figure 2: "Building blocks" of network design

Figure 2 shows a representation of the three network designs. Circles represent buses, and connections represent allowed corridors. The number on the right side of each bus is the corresponding index value. A corridors is denoted by the pair of index values of the buses it connects.

The independent design features two separate corridors that serve two demands. Even if operationally one might not find any synergies among the two investments, as they can be seen as two separate networks, depending on demand correlations it is possible to observe a portfolio effect, which makes the whole investment, in a financial perspective, less risky.

Like the previous design, the radial design also has different corridors for the two demands, but additionally it includes a common corridor between the generation bus and an intermediate bus. This way it may be possible to benefit not only from financial advantages, but also from operational advantages, e.g., of the pooling of demands in the shared corridor, which might compensate for an increase in the total distance in which power is transported. As we will be considering constant distances between generation and demand buses, and this design implies not using the shortest distances between them, higher power losses are expected.

The meshed design is also similar to the independent design, but it considers an additional corridor between the two demand buses. This additional corridor becomes valuable if situations of overcapacity in one demand bus and undercapacity in the other demand bus are likely to occur. Investing in this new added corridor will allow energy to flow between demand buses taking advantage of already existing capacity, but it will only be interesting if that benefit outweighs the increase in losses, as in the case of the radial design.

Corridor 1-4	500
Corridor 2-4	500
Corridor 3-4	300
Corridor 1-3	215
Corridor 2-3	215
Corridor 1-2	200

Table 1: Corridor lengths (in kilometers)

Table 1 summarizes the corridor lengths that we considered for this numerical case. The lengths of corridors were chosen such that 1-2 is lower than 1-4 and 2-4, and additional value exists for the meshed design. Corridors 1-4 and 2-4 are equal, which allows us to find symmetric results. As for the corridor 3-4, its length was chosen to allow the Euclidean distances in 1-3 and 2-3 to be close to a round number.

We use similar technical data for all lines, the differences depending only on the corridor length. We define a unit of demand as 50 MVA, and base voltage as 220 kV, which results in a base impedance of 968 Ω. Line resistance and conductance are respectively 0.07 Ω/km and 0.05 Ω/km, and the line cost is 70.00 €/(MW·km). Energy cost is 0.0003€/MWh which corresponds to 131.40€/unit. The price is 248.33€/unit, to yield a 0.5 critical fractile (Van Mieghem 2007), assuming a 25 year annuity on the investment cost of corridors 1-4 and 2-4. The load curtailment cost is assumed to be 12 times the charged price, 2980.00€/unit.

Susceptance and conductance are calculated using line resistance (R) and reactance (X), as follows:

$$B_x = -\frac{X_x}{R_x^2 + X_x^2}$$
$$G_x = \frac{R_x}{R_x^2 + X_x^2}$$

The binomial tree has two time steps, and reflects a long-term planning schedule, with a Δt of 5 years. We define both μ_1 and μ_2 as 5%, the discount rate as 5%, and for the perpetuity we assume a 0% growth rate.

3.2 Network Value

We perform this set of computational experiments with the IBM ILOG CPLEX Optimizer, seeking to study the impact of demand uncertainty and correlation in network configuration and value. We optimize each design, considering increases in the standard deviations of the demands, from 0% to 30% of the mean, in steps of 10%, and in correlation, from -1.0 to 1.0, in steps of 0.5. We initially assume that σ_1 and σ_2 are equal.

For the independent design, as can be observed in Figure 3, demand correlation does not affect the network value, since the lines are completely independent. Increasing demand uncertainty increases the expanded net present value, however this does not happen in a monotonous way. For standard deviations of 0% and 10%, the discrete nature of the investments, in situations of load curtailment, leads to the possibility that slight increases in demand will not translate into profitability when adding a new line to a corridor, with the extra demand resulting only in an increase in costs. However, when uncertainty increases, that additional investment can become profitable, by reducing load curtailment costs and increasing the amount of demand fulfilled.

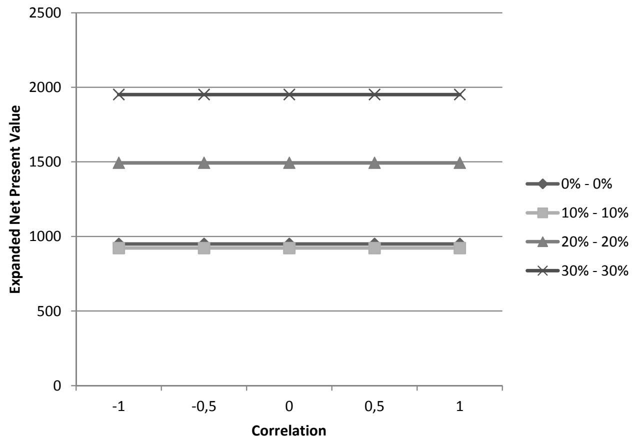

Figure 3: Independent design with equal standard deviations

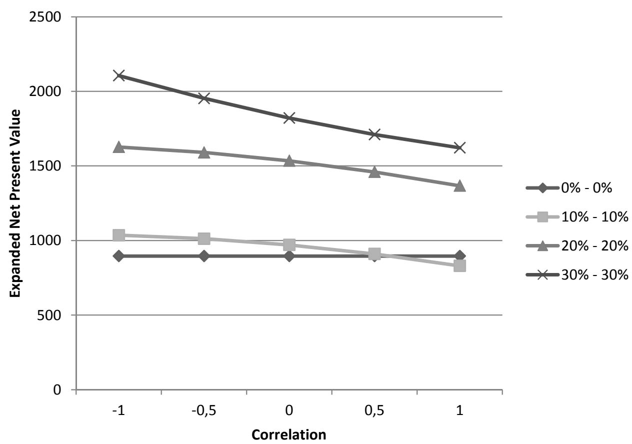

Figure 4: Radial design with equal standard deviations

The results for the radial design, presented in Figure 4, show that correlation now does play a role in the project value. Comparing this design with the previous, it is evident that as correlation gets more negative the expanded net present value tends to increase even if the total distance that energy travels and losses are higher. The usage of the common corridor tends to stabilize at lower correlations, reducing the likelihood of overcapacity or undercapacity. Additionally, for high correlations this design can perform worse than the independent design, as the benefits of pooling power demands disappear when they tend to have the same behaviour, and losses increase due to the increase of total length between the generation bus and demand buses. The effect reported for the independent design, related to discrete capacity investments and load curtailment costs, can also be seen with this design, for higher correlations.

In the meshed design, whose results are displayed in Figure 5, although the general trend is a reduction of value with increasing correlations, in the case of a 30% standard deviation, the maximum value is obtained with a -0.5 demand correlation, and in the case of 20%, the evolution of value is convex.

We also performed a set of computational experiments with different standard deviations, starting from equal values of 20%, and then introducing variations of +5% in one standard deviation, and -5% in the other. For lower variations, the value of the network tends to decrease, but as the variations increase, up to the

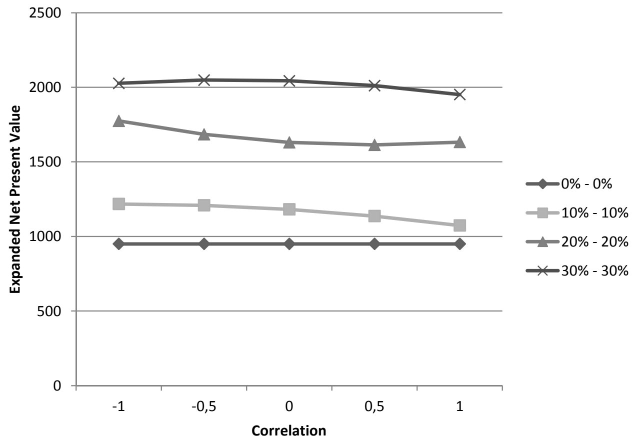

Figure 5: Meshed design with equal standard deviations

point where one demand is deterministic, the value of the network starts to increase again. In the meshed design it is also possible to see more clearly how an increase in correlation does not always lead to a decrease in value. For standard deviations of 5%-35%, we find a convex relation, and for 15%-25%, the value is always increasing with correlation.

3.3 Network Configuration

Under the settings used for this numerical study, for each design and level of uncertainty, correlation does not have an impact on the evolution of the network configuration in the scenario tree, although it has an impact on the network value. The changes in value are in fact determined only by the changes in node probabilities, as the correlation changes. This is related to the fact that the network investments take place as late as possible, under these settings.

To illustrate this, we present in Figure 6 the possible evolutions of network configurations when the demands have the same standard deviation of 10%. The corresponding probabilities are included in Tables 2, 3, and 4.

Comparing the independent and radial networks, the evolutions of the configurations of the corridors that connect to demand buses are exactly the same. For the shared corridor in the radial design, where the demands are pooled, the diversity of alternative evolutions in the scenario tree is higher. This is evidence of a finer adjustment to demand, which translates into benefits that even outweigh the additional costs of losses related to the higher length of the corridors from generation to demand.

As for the independent and meshed networks, the evolutions of the configurations are the same, except for the fact that on the second time step a truly meshed configuration may emerge, with a corridor connecting both demand buses and a lower investment in one of the other corridors. The investment costs for this configuration are lower, and allow the investor to use the structure already in place, that might be underused by a possible decrease in one of the demands, to meet the other demand. The differences found on the probabilities of corridors 1-4 and 2-4 on the second time step are due to the new corridor. As the configuration is symmetric, swapping the probabilities of 1-4 and 2-4 would yield the same results.

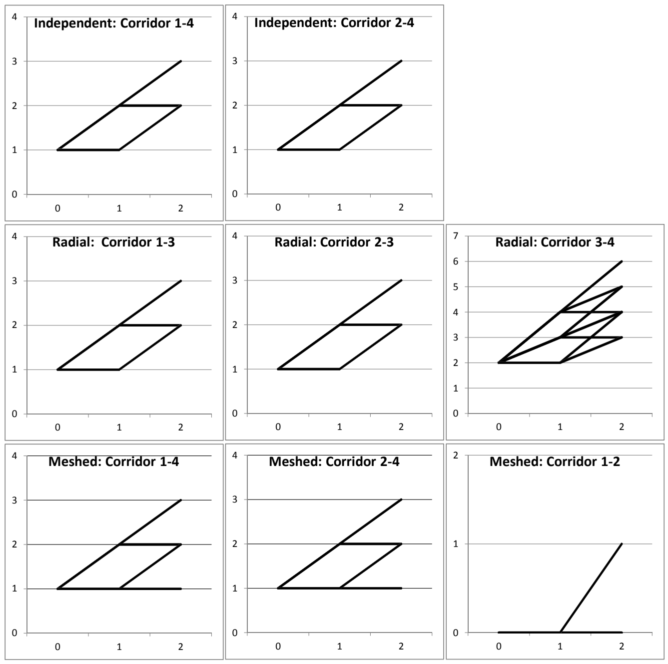

Figure 6: Possible number of lines by configuration, corridor and time step ($\sigma_1 = \sigma_2 = 10\%$)

Corridor		$1-4$				$2-4$			
Time Step		1		2		1		2	
Number of Lines		1	2	2	3	1	2	2	3
Correlation	-1.0	50%	50%	75%	25%	50%	50%	75%	25%
	-0.5	50%	50%	75%	25%	50%	50%	75%	25%
	0.0	50%	50%	75%	25%	50%	50%	75%	25%
	0.5	50%	50%	75%	25%	50%	50%	75%	25%
	1-0	50%	50%	75%	25%	50%	50%	75%	25%

Table 2: Probabilities of total number of lines by corridor and time step for the independent design($\sigma_1 = \sigma_2 = 10\%$)

Corridor		$1-3$				$2-3$				$3-4$						
Time Step		1		2		1		2		1			2			
Number of Lines		1	2	2	3	1	2	2	3	2	3	4	3	4	5	6
Correlation	-1.0	50%	50%	75%	25%	50%	50%	75%	25%	0%	100%	0%	0%	100%	0%	0%
	-0.5	50%	50%	75%	25%	50%	50%	75%	25%	13%	75%	13%	20%	59%	19%	2%
	0.0	50%	50%	75%	25%	50%	50%	75%	25%	25%	50%	25%	31%	38%	25%	6%
	0.5	50%	50%	75%	25%	50%	50%	75%	25%	38%	25%	38%	33%	34%	19%	14%
	1.0	50%	50%	75%	25%	50%	50%	75%	25%	50%	0%	50%	25%	50%	0%	25%

Table 3: Probabilities of total number of lines by corridor and time step for the radial design($\sigma_1 = \sigma_2 = 10\%$)

Corridor		$1-4$					$2-4$					$1-2$	
Time Step		1		2			1		2			2	
Number of Lines		1	2	1	2	3	1	2	1	2	3	0	1
Correlation	-1.0	50%	50%	25%	50%	50%	50%	50%	25%	50%	25%	50%	50%
	-0.5	50%	50%	25%	50%	50%	50%	50%	23%	52%	25%	52%	48%
	0.0	50%	50%	25%	50%	50%	50%	50%	19%	56%	25%	56%	44%
	0.5	50%	50%	25%	50%	50%	50%	50%	11%	64%	25%	64%	36%
	1.0	50%	50%	25%	50%	50%	50%	50%	0%	75%	25%	75%	25%

Table 4: Probabilities of total number of lines by corridor and time step for the meshed design($\sigma_1 = \sigma_2 = 10\%$)

4 Conclusions

In this paper we propose a new model for multi-stage Transmission Network Expansion Planning under uncertainty that, to the best of our knowledge, is the first to address investment in capacity expansion as a portfolio of real options. As the expanded net present value objective function is developed assuming revenues for transmitted power, and costs for energy production and load curtailment, without considering specific regulatory obligations, the model can be regarded as a proof of concept for the design of power networks and other utility networks, like water or gas supply, which may be valued and configured following the same analysis logic and considering their specific technical aspects.

Our analysis confirms the value of network sources of operational flexibility, such as the shared corridor in the radial design, and the flexible corridor in the meshed design, in a multi-stage setting. We have also found that, even if increasing uncertainty tends to increase the network value, due to the discrete nature of line investments and the existence of load curtailments costs, value losses are possible for small increases in uncertainty. In addition, we have observed that an increase in correlation does not necessarily lead to a decrease in value - in some cases the maximum occurs at perfect correlation, uncorrelated demands, or other levels. This effect seems considerably more significant in the more flexible network designs.

Future developments may be directed to including in the model additional features that improve its relevance to practice, such as regulatory requirements, time lags for line construction, uncertainty in authorization of construction of new corridors, generation with variations in power supply, as is typical of wind farms, and seasonal demand fluctuations.

Acknowledgements

This work is financed by the ERDF - European Regional Development Fund through the COMPETE Programme (operational programme for competitiveness) and by National Funds through the FCT - Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) within project Flexible Design of Networked Engineering Systems /PTDC/SENENR/101802/2008.

References

Ahmed, S., King, A. J. and Parija, G.: 2003, A multi-stage stochastic integer programming approach for capacity expansion under uncertainty, *Journal of Global Optimization* **26**(1), 3–24.

Alguacil, N., Motto, A. L. and Conejo, A. J.: 2003, Transmission expansion planning: A mixed-integer lp approach, *IEEE Transactions on Power Systems* **18**(3), 1070–1077.

Brosch, R.: 2008, *Portfolios of Real Options*, Vol. 611 of *Lecture Notes in Economics and Mathematical Systems*, Springer.

Bustamante-Cedeño, E. and Arora, S.: 2008, Stochastic and minimum regret formulations for transmission network expansion planning under uncertainties, *Journal of the Operational Research Society* **59**, 1547–1556.

Chow, J. Y. J. and Regan, A. C.: 2011, Real option pricing of network design investments, *Transportation Science* **45**(1), 50–63.

Claro, J. and Sousa, J. P. d.: 2010, A multiobjective metaheuristic for a mean-risk multistage capacity investment problem, *Journal of Heuristics* **16**(1), 85–115.

Cox, J. C., Ross, S. A. and Rubinstein, M.: 1979, Option pricing: A simplified approach, *Journal of Financial Economics* **7**(3), 229–263.

de Neufville, R. and Scholtes, S.: 2011, *Flexibility in Engineering Design*, MIT Press.

Delgado, D. and Claro, J.: 2011, Transmission network expansion planning under demand uncertainty and risk aversion. Working Paper.

Dixit, A. K. and Pindyck, R. S.: 1994, *Investment Under Uncertainty*, Princeton University Press.

Marathe, R. R. and Ryan, S. M.: 2005, On the validity of the geometric brownian motion assumption, *The Engineering Economist* **50**(2), 159–192.

Trigeorgis, L.: 1993, The nature of option interactions and the valuation of investments with multiple real options, *Journal of Financial & Quantitative Analysis* **28**(1), 1–20.

Van Mieghem, J. A.: 2007, Risk mitigation in newsvendor networks: Resource diversification, flexibility, sharing, and hedging, *Management Science* **53**(8), 1269–1288.

Wang, T. and de Neufville, R.: 2005, Real options "in" projects, *9th Real Options Annual International Conference*.