

Sequential Investment in Pollution Control Equipment under Uncertainty

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Overview

This paper investigates the investment in pollution control equipment under uncertainty. We assume that a firm's output generates a pollution by-product and the pollution reduces the productivity of capital. The dynamics of pollution is assumed to be governed by stochastic differential equation. Then, the firm has to invest in a pollution control equipment in order to reduce the pollutant and it incurs equipment cost. The firm also pays environmental tax to use input. In this paper, we assume that the firm can invest it as needed. Then, the firm's problem is to choose the investment timing under uncertainty and is formulated a singular stochastic control problem.

We solve the firm's problem by using variational inequalities and obtain the optimal investment strategy which is described by the threshold to invest in the pollution control equipment. Furthermore, we conduct a comparative static analysis on some parameters.

Methods

We assume that a firm produce a product in order to maximize its profit. But the firm's output generates a pollution by-product and the pollution reduces the productivity of capital. Then, the firm considers investing in a pollution control equipment to reduce the pollutant. The firm pays environmental tax to use input. Then, the firm's problem is to choose the investment timing under uncertainty. In this paper, we formulate the firm's problem as a singular stochastic control problem.

From Skorohod Lemma, we naturally guess that, under an optimal investment strategy, the firm invests in pollution control equipment whenever the pollutant stock reaches a threshold. In order to verify this conjecture, we solve the firm's problem by using variational inequalities. The verification theorem says a solution to the variational inequalities is optimal.

Results

As a result of analysis, we obtain the optimal investment strategy which is described by the threshold to invest in the pollution control equipment. Furthermore, we provide the results of the comparative static analysis of the threshold.

Conclusions

In this paper, we investigate the investment in pollution control equipment under uncertainty

and derive the optimal investment strategy.

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