Valuing Managerial Flexibility in Technology R&D

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Abstract

Developing better technologies to adapt to a changing world is crucial for the growth of a knowledge-based economy. Indeed, new products in the areas of energy, pharmaceuticals, and telecommunications generate significant economic benefits. However, in order to maximise those benefits, firms need to devise research and development (R&D) strategies that are responsive to market conditions. We take the real options approach to provide insights about the value of flexibility in managing technology R&D programmes from the perspective of a knowledge-based firm. Specifically, we account for managerial discretion over the pace of R&D effort and timing of new product launches by solving the firm's optimal stopping time problem.

1 Introduction

Firms in most open economies in the world rely heavily upon innovation for growth. This applies to not only established sectors such as financial services and pharmaceuticals but also nascent ones such as energy technologies and telecommunications. However, firms that manage technology research and development (R&D) programmes face challenges because of the numerous uncertainties under which manifold decisions are made. In order to maximise the benefits from innovation, firms have to manage R&D programmes such that new product versions are ready for launch when market conditions are favourable. Thus, an economic analysis of technology R&D management should consider uncertainty in prices and innovation along with discretion over R&D effort and new product launches.

Our main objective is to provide insights about technology R&D management by answering the following questions:

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- How does managerial flexibility to control R&D effort by switching between active and suspended states affect project value?
- How does the discretion to defer launching new product versions affect project value and interact with the aforementioned switching options?
- How do project values and switching/launch decisions change when the firm develops a (potentially infinite) sequence of new product versions?

The structure of this paper is as follows:

- Section 2 reviews the literature in order to provide context for our paper's contribution
- Section 3 outlines the assumptions for the model and sets up the decision-making problem of the firm
- Section 4 solves the suite of problems analytically in order to provide insights about decision making and managerial flexibility
- Section 5 illustrates the main results numerically in order to synthesise the intuition
- Section 6 summarises the research, discusses its limitations, and offers directions for future work in this area

2 Related Work

The R&D decisions of a firm are amenable to real options analysis because there is discretion over the timing and scale of a project's phases. We build on the related literature in this area in order to examine our problem of optimal R&D management under market and technological uncertainty when there are repeated (and potentially infinite) innovations. Some of the related work may be summarised as follows:

- [3] describe the real options framework and introduces a problem in which repeated decisions are made due to both price and physical uncertainties
- [7] assess alternative migration strategies for adopting new technologies via the real options approach
- [9] examine how a firm may use flexibility over suspension and launch timing of its R&D programme for a single innovation
- [5], [8], and [4] explore repeated options in order to illustrate how the optimal decision rule is different from the single-investment case

- [2] and [6] examine the impact of investment lags
- [1] introduce technology adoption in face of rivalry

3 Problem Description

We assume that a firm has the perpetual right, but not the obligation, to invest in a product along with an ongoing R&D programme that will yield (potentially an infinite stream of) successively upgraded versions of the same product. The deterministic investment cost of launching the initial version of the product together with the R&D programme is I (in \$). This may be thought of as a lump-sum cost of establishing a new product line and R&D laboratory, viz., facility construction, installation of new equipment, hiring costs for new staff members, etc. Once the product is launched, the price that it earns at time $t \geq 0$, P_t (in \$/annum), is exogenous and evolves according to the following geometric Brownian motion (GBM):

$$dP_t = \alpha P_t dt + \sigma P_t dz_t \tag{1}$$

We assume that $P_0 \equiv P > 0$ and all revenues are discounted at the exogenous annual discount rate, $\rho > \alpha$.

As long as R&D is ongoing, the firm incurs variable cost at a rate k (in \$/annum), which is related to staff salaries and laboratory expenses for the facility. As long as the R&D programme is active, we assume that it delivers an improved version of the existing technology according to a Poisson process at rate $\lambda \geq 0$ (in annum^{-1}). For example, $\lambda = 0.2$ means that an innovation will be provided on average every five years. The new version of the technology is more efficient in the sense that the instantaneous revenues associated with it at time t are $P_t(1+u)$, where $0 \leq u \leq 1$ is a deterministic efficiency gain. Initially, we assume that the new version of the technology is immediately adopted upon delivery at fixed cost $K < I$ (in \$), which may be thought of as the cost of upgrading the network in case of distributed energy and mobile telecommunications or simply providing support to customers to handle the switchover. Once the new version of the technology is installed, the R&D programme continues *ad infinitum* with a stream of innovations.

In this set up, the only flexibility that the firm has is in determining the timing of the initial investment that includes the embedded R&D programme. However, additional flexibilities may also be rational to explore. For example, a firm may be reluctant to continue an R&D programme in a poor market environment. Thus, for low levels of revenues, the firm may prefer to mothball its R&D laboratory only to resume its operations once the revenues are sufficiently high. This type of flexibility (with infinite suspension and resumption options costlessly available) is the *switching* option.

Another type of flexibility concerns the launch of the improved product once delivered by the R&D laboratory. Indeed, if the switching option is not available (for various reasons ranging from inflexible labour markets to the strategic dimension of not allowing human capital to dissipate), then the firm may delay the launch of the new version of the product in order to take advantage of market conditions. This *launching* option implies that the firm holds on to the improved technology and chooses the right time at which to introduce it to the market. In the meantime, ongoing R&D is suspended and resumes only once the improved product is launched, which is a more natural cycle for managing staff than suspending their projects prior to completion. Finally, both switching and launching flexibilities may be used together.

In order to gain analytical insights, we have several simplifying assumptions. First, there is only one source of market uncertainty, i.e., the price of the output, which follows a GBM. More realistically, the demand for the technology as well as R&D expenditures may be uncertain. Second, the price process may not be exogenous but influenced by the firm's decisions. Indeed, the market for smartphone platforms is dominated by a few major players, viz., Apple, Google, and Research in Motion, and it is likely that launch of a new product by one will affect the revenues earned by the entire industry. However, such strategic effects are not considered in this paper with a single firm that has no influence on the market price through its behaviour. Third, technological uncertainty is represented via a Poisson process, which may not be realistic due to its memoryless property. In fact, the rate of innovation may be affected by R&D expenditures, learning effects, and the behaviour of rivals. We capture this dependency only approximately by giving the firm control (in some instances) over the rate of the R&D process. Fourth, there is no lead time for a new version of the technology to be adopted, which may not be realistic in an industry with significant network effects such as energy or telecommunications. However, we have a fixed upgrading cost to capture (admittedly imperfectly) this effect. Finally, we assume that if a firm has the launching flexibility, then its R&D programme suspends once an innovation arrives and waits to be launched.

3.1 Case 1: No Embedded Flexibility

We first consider a single innovation's arrival when the firm has no embedded flexibility over its R&D programme. In terms of Fig. 1, the state-transition diagram terminates with a state 2 in which no further technology innovation is possible. Although unrealistic, this situation serves as a benchmark against which we may gauge decision making in the presence of infinite technology upgrades. Now, given that the firm is in state 1, it receives revenues based on a stochastic price, P_t, from the initial version of the technology and has expenditures at rate k for its ongoing R&D. Since the next generation of the technology will arrive according to a Poisson process with rate λ, the firm's expected present value (PV) given immediate adoption of the next generation of the technology upon arrival is based on a combined GBM with a Poisson process. Once the innovation arrives, the firm pays the fixed upgrade cost, K, to install the new technology and R&D terminates. Going back to state 0, the firm's decision-making problem is to determine the optimal price threshold, P_1^*, at which to invest in the new technology along with the associated R&D programme that maximises the expected PV of the whole project net of the investment cost, I.

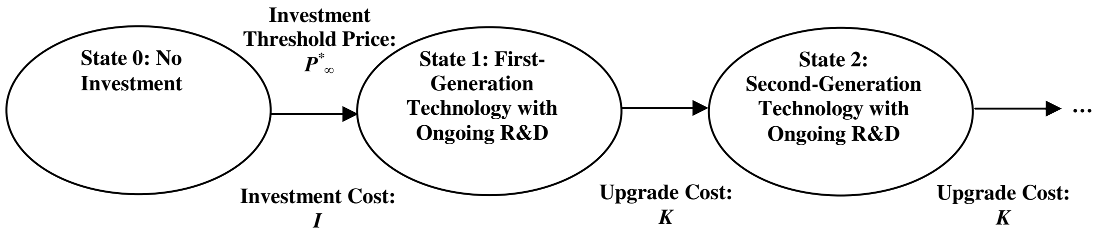

Figure 1: State-Transition Diagram without Embedded Flexibility

With infinite technological innovations, the transitions between states correspond exactly to the diagram indicated in Fig. 1, i.e., they indicate the version of the technology that is installed. Now, instead of there being a single arrival time, there is a series of independent and indentically distributed inter-arrival times. Since each innovation is automatically adopted upon arrival with upgrade cost K, the firm's expected PV with an ongoing (rather than a one-time) R&D programme and a sequence of infinite technological innovations is again static. Analogous to the single-innovation problem, the only decision that the firm has is over the timing of the initial investment. Again, its objective is to maximise the expected NPV of the project by determining the optimal threshold price, P_∞^*.

3.2 Case 2: Switching Flexibility

If the firm has the flexibility to switch the R&D on and off costlessly once the programme has been initiated from state 0, then an embedded decision is to determine the endogenous price thresholds for switching between states i and i' in the context of Fig. 2. As in the case without flexibility, however, the firm automatically adopts the new product version when it arrives. Thus, the flexibility to switch between R&D on and off states gives the firm indirect control over the adoption of an upgrade.

With a single innovation to arrive, the firm's decision in state 1 is to determine the switching threshold price, S_1^*, below which it will suspend R&D and, thus, enter state 1'. Once in state 1', the firm will wait until the price rises above S_1^* before resuming R&D. However, if the firm is in state 1 and the innovation occurs, then the new technology is immediately adopted, thereby moving the firm to state 2. Hence, the firm's decision in state 0 is to select the optimal price threshold, P_1^*, at which to initiate such a project.

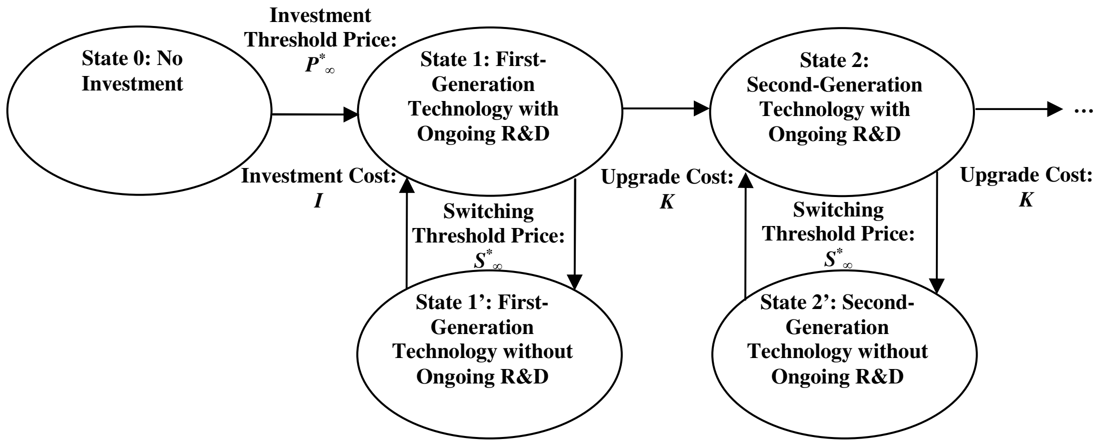

Figure 2: State-Transition Diagram with Switching Flexibility

With infinite innovations, the firm faces the endogenous R&D switching decision in each main state i. However, intuition suggests that due to costless switching and the similarity of cash flows, there is a common switching threshold, S_∞^*, for each generation of the technology installed. Thus, it suffices to solve the switching problem at any one of the main states. By substituting this into the firm's value function in state 1, the optimal entry decision from state 0 consists of determining the optimal price threshold, P_∞^*.

3.3 Case 3: Launching Flexibility

An option to delay the launch of a new version of the technology provides the firm with the direct control over transition between states that is lacking in Sect. 3.2. Now, once an innovation arrives, ongoing R&D stops, and the firm decides on the optimal time to launch the new version (see Fig. 3). With a single innovation, there is only one such decision to make, i.e., the firm must determine the optimal price threshold, L_1^*, in order to maximise the expected NPV from the new version of the technology. Backing up to state 0, the firm decides the optimal investment threshold, P_1^*, at which to launch the initial version of the technology along with an R&D programme that produces a single innovation.

With infinite innovations, there are infinite launching decisions. However, due to the similarity of cash flows between states i' and $(i+1)'$, there is a common launching threshold, L_∞^*. Again, from state 0, the firm determines the optimal investment threshold, P_∞^*.

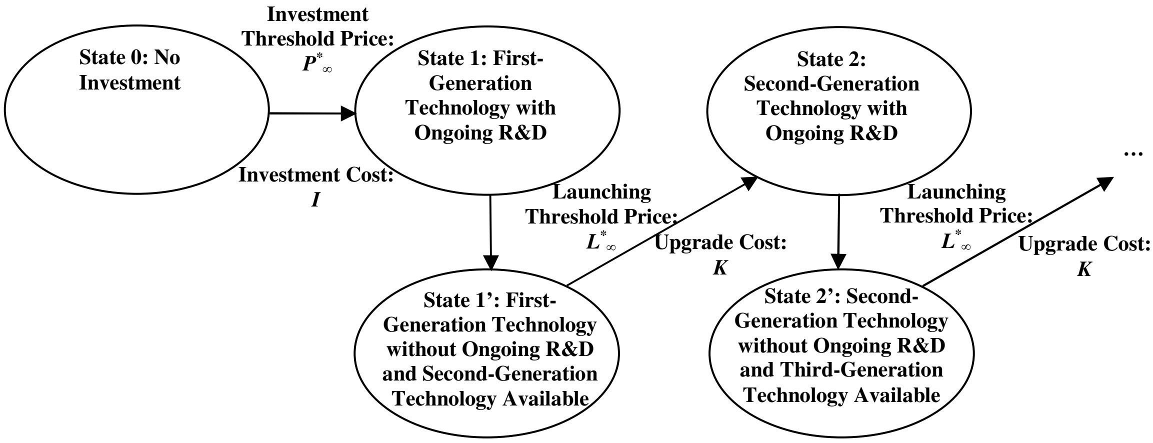

Figure 3: State-Transition Diagram with Launching Flexibility

3.4 Case 4: Both Switching and Launching Flexibilities

With both forms of flexibility, the firm decides on optimal switching and launching decisions. Whether with a single innovation or infinite ones, a firm with an active R&D programme in state i may decide to suspend it (and enter state i') if the price drops below a threshold. However, if the innovation occurs prior to suspension (taking the firm to state i''), then R&D is stopped until the price is high enough to justify the launch of the new version of the technology. After the launch, the firm's R&D programme resumes in state $i+1$ (see Fig. 4). How these flexibilities interact is the subject of our investigation in Section 4.4.

4 Analytical Models

4.1 Model 1: No Embedded Flexibility

For reference, we begin with the case without embedded flexibility. Corresponding to the state-transition diagram in Fig. 1, the firm is passive after entering state 1 and simply lets the R&D programme run with immediate adoption of the new version of the technology when available. With just a single innovation, i.e., there are no transitions beyond state 2, the firm's expected PV in state 1 is determined via the law

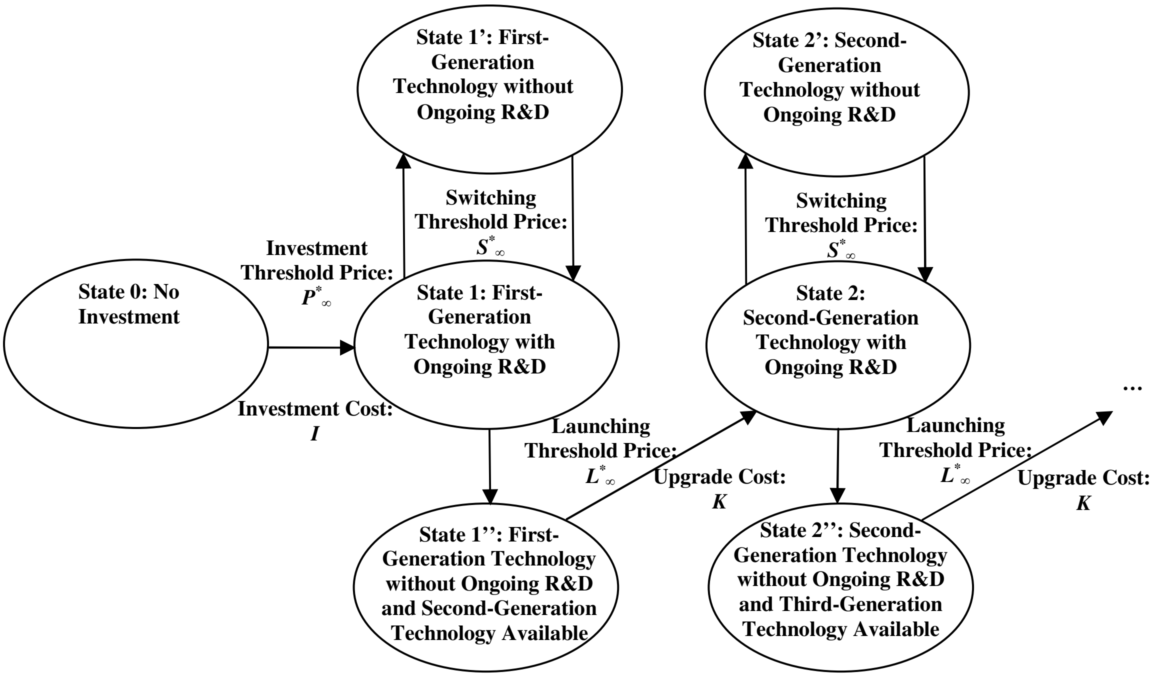

Figure 4: State-Transition Diagram with Switching and Launching Flexibilities

of iterated expectations:

$$
\begin{aligned}
V_1(P) &= \mathbb{E}_P\left[\int_0^{X_1}(P_t-k)e^{-\rho t}dt - Ke^{-\rho X_1} + \int_{X_1}^{\infty} P_t(1+u)e^{-\rho s}dt\right] \\
\Rightarrow V_1(P) &= \mathbb{E}_P\left[\mathbb{E}_P\left[\int_0^{X_1}(P_t-k)e^{-\rho t}dt - Ke^{-\rho X_1} + \int_{X_1}^{\infty} P_t(1+u)e^{-\rho s}dt \mid X_1\right]\right] \\
\Rightarrow V_1(P) &= \mathbb{E}_P\left[\frac{P}{\rho-\alpha}\left(1-e^{-(\rho-\alpha)X_1}\right) - \frac{k}{\rho}\left(1-e^{-\rho X_1}\right) - Ke^{-\rho X_1} + \frac{P(1+u)}{\rho-\alpha}e^{-(\rho-\alpha)X_1}\right] \\
\Rightarrow V_1(P) &= \frac{P(\rho-\alpha+\lambda(1+u))}{(\rho-\alpha)(\rho+\lambda-\alpha)} - \frac{k}{\rho+\lambda} - \frac{\lambda K}{\rho+\lambda} \qquad (2)
\end{aligned}
$$

Here, X_1 is an exponentially distributed random variable with rate λ that indicates the time until the arrival of the innovation, and the expectation is conditional on the initial price level, P. In order to understand the expression in Eq. (2) intuitively, it is instructive to consider limiting cases where either $\lambda = 0$ (the R&D never delivers an innovation) or $u = 0$ (the innovation does not improve the technology's performance). In the former, the firm's expected PV reduces to $\frac{P}{(\rho-\alpha)} - \frac{k}{\rho}$, i.e., the firm simply obtains the PV of revenues from the existing technology and pays for an unsuccessful R&D programme forever. When $u = 0$, the firm's expected PV becomes $\frac{P}{(\rho-\alpha)} - \frac{k}{\rho+\lambda} - \frac{\lambda K}{\rho+\lambda}$. In this situation, the firm's PV of revenues are the same, but its expected R&D costs are lower because once the new technology arrives, it is adopted, thereby ending the stream of expenditure. Furthermore, it also pays the discounted upgrade cost, K.

Going back to state 0, the firm's decision-making problem is to find the optimal investment threshold price, P_1^*, at which to invest in order to receive a project with expected PV given in Eq. 2. Using the Bellman equation and Itô's lemma in a dynamic programming framework, the firm's value in state 0 is (see [3]):

$$F_1(P) = A_{11} P^{\beta_1} \tag{3}$$

where β_1 is the positive root of the characteristic quadratic $\frac{1}{2}\xi(\xi-1)+\alpha\xi-\rho=0$. Via the following value-matching (VM) and smooth-pasting (SP) conditions, we obtain the optimal investment threshold price, P_1^*, as well as the endogenous constant, A_{11}:

$$F_1(P_1^*) = V_1(P_1^*) - I \tag{4}$$

$$F_1'(P_1^*) = V_1'(P_1^*) \tag{5}$$

Thus, we obtain the following:

$$P_1^* = \left(\frac{\beta_1}{\beta_1-1}\right)\frac{(\rho-\alpha)(\rho+\lambda-\alpha)}{\rho-\alpha+\lambda(1+u)}\left\{\frac{k}{\rho+\lambda}+\frac{\lambda K}{\rho+\lambda}+I\right\} \tag{6}$$

$$A_{11} = \frac{\left(P_1^*\right)^{1-\beta_1}(\rho-\alpha+\lambda(1+u))}{\beta_1(\rho+\lambda-\alpha)(\rho-\alpha)} \tag{7}$$

With infinite innovations, the firm's value function in state 1 reflects passive adoption of subsequent technology arrivals each with the independent and identically distributed inter-arrival time, $X_i \sim exp(\lambda)$. Furthermore, $X_0 \equiv 0$, and $Y_n \equiv \sum_{i=0}^{n} X_i$ represents the calendar time at which the nth innovation is available. The firm's expected PV in state 1 becomes the following:

$$\begin{aligned}
V_\infty(P) &= \mathbb{E}_P\left[\sum_{i=0}^{\infty}\int_{Y_i}^{Y_i+X_{i+1}}(P_t-k)(1+u)^i e^{-\rho t}dt - \sum_{i=1}^{\infty} K e^{-\rho Y_i}\right] \\
\Rightarrow V_\infty(P) &= \mathbb{E}_P\Bigg[\mathbb{E}_P\left[\int_0^{X_1}(P_t-k)e^{-\rho t}dt - Ke^{-\rho X_1} \mid X_1\right] \\
&\quad + \mathbb{E}_P\left[\int_{X_1}^{X_1+X_2}(P_t(1+u)-k)e^{-\rho t}dt - Ke^{-\rho(X_1+X_2)} \mid X_1, X_2\right] \\
&\quad + \mathbb{E}_P\left[\int_{X_1+X_2}^{X_1+X_2+X_3}(P_t(1+u)^2-k)e^{-\rho t}dt - Ke^{-\rho(X_1+X_2+X_3)} \mid X_1, X_2, X_3\right] + \cdots\Bigg] \\
\Rightarrow V_\infty(P) &= \frac{P}{\rho+\lambda-\alpha}\sum_{i=0}^{\infty}\left[\frac{(1+u)\lambda}{\rho+\lambda-\alpha}\right]^i - \frac{k}{\rho+\lambda}\sum_{i=0}^{\infty}\left(\frac{\lambda}{\rho+\lambda}\right)^i - K\sum_{i=1}^{\infty}\left(\frac{\lambda}{\rho+\lambda}\right)^i \\
\Rightarrow V_\infty(P) &= \frac{P}{\rho-u\lambda-\alpha} - \frac{k}{\rho} - \frac{\lambda K}{\rho} \qquad (8)
\end{aligned}$$

Now, if $\lambda = 0$, then the expected PV in Eq. 8 becomes $\frac{P}{(\rho-\alpha)} - \frac{k}{\rho}$, which corresponds to the cash flows with the current technology installed forever along with ongoing R&D expenditures but no subsequent arrival of an innovation. Similarly, if $u = 0$, then the expected PV is $\frac{P}{(\rho-\alpha)} - \frac{k}{\rho} - \frac{\lambda K}{\rho}$, i.e., the firm's revenue stream does not improve but R&D expenditures last forever with occasional payment of the upgrade cost.

Analogous to the single-innovation situation, the firm's objective in state 0 is to select the optimal price threshold, P_∞^*, at which to invest. Thus, its value function is similar to that in Eq. 3:

$$F_\infty(P) = A_{1\infty}P^{\beta_1} \tag{9}$$

The VM and SP conditions are now:

$$F_\infty(P_\infty^*) = V_\infty(P_\infty^*) - I \tag{10}$$

$$F'_\infty(P_\infty^*) = V'_\infty(P_\infty^*) \tag{11}$$

Thus, we obtain the following:

$$P_\infty^* = \left(\frac{\beta_1}{\beta_1 - 1}\right)(\rho - u\lambda - \alpha)\left\{\frac{k}{\rho} + \frac{\lambda K}{\rho} + I\right\} \tag{12}$$

$$A_{1\infty} = \frac{(P_\infty^*)^{1-\beta_1}}{\beta_1(\rho - \alpha - \lambda u)} \tag{13}$$

4.2 Model 2: Switching Flexibility

In terms of Fig. 2, an active firm that has a single innovation is in state 1 and must decide the optimal price threshold, S_1^*, below which to suspend its R&D programme. At this point, both the output price, currently $P > S_1^*$, and the remaining time until the innovation's arrival, X_1, are evolving stochastically. Indeed, if the R&D programme succeeds in providing the innovation before the price drops to the suspension trigger, then the new version of the technology is immediately adopted, and the firm transits to state 2, thereby effectively terminating the decision-making problem. However, if the output price drops to the suspension trigger before the arrival of the innovation, then the firm transits to state 1' and remains there until the output prices rises to the same trigger, S_1^*, which re-activates the R&D programme. This common trigger for this optimal control problem is justified because the switches are costless and infinite in number (see [11]). If either of these assumptions does not hold, then the optimal price trigger for R&D suspension will be different from that for its resumption.

For the single-innovation situation, we, thus, obtain the firm's expected PV in states 1 and 1' by conditioning on what happens in the next dt time units, applying Itô's lemma, using the fact that higher-order terms involving dt go to zero in the limit, and solving the resulting ordinary differential equation

(ODE). Beginning with state 1', i.e., $P < S_1^*$, we have the following:

$$\begin{aligned}
V_1(P) &= Pdt + (1-\rho dt)\mathbb{E}_P\left[V_1(P+dP)\right] \\
\Rightarrow \quad & V_1(P) = Pdt + (1-\rho dt)\mathbb{E}_P\left[V_1(P) + V_1'(P)dP + \frac{1}{2}V_1''(P)\left(dP\right)^2\right] \\
\Rightarrow \quad & V_1(P) = Pdt + (1-\rho dt)\left[V_1(P) + V_1'(P)\alpha P dt + \frac{1}{2}V_1''(P)\sigma^2 P^2 dt\right] \\
\Rightarrow \quad & \frac{1}{2}\sigma^2 P^2 V_1''(P) + \alpha P V_1'(P) - \rho V_1(P) + P = 0 \\
\Rightarrow \quad & V_1(P) = \frac{P}{\rho-\alpha} + B_{11}P^{\beta_1} \qquad (14)
\end{aligned}$$

Intuitively, the first line of Eq. 14 states that the expected PV of a firm in state 1' consists of the revenues earned in the next dt time units plus the discounted conditional expected value of the firm dt time units hence when the price will have changed by dP. The solution in the final line of Eq. 14 indicates that the firm's expected PV in state 1' is the expected PV of revenues from perpetual deployment of the incumbent technology plus an option-like term to reflect the possibility of resuming R&D that will eventually lead to an upgrade. For this reason, the other branch of the solution to the ODE, $B_{21}P^{\beta_2}$, must vanish as P goes to zero, which implies $B_{21} = 0$ (where β_2 is the negative root of the characteristic quadratic $\frac{1}{2}\xi(\xi-1) + \alpha\xi - \rho = 0$). This leaves B_{11} as a positive endogenous constant.

We proceed similarly to obtain $V_1(P)$ in state 1, i.e., when $P \geq S_1^*$:

$$\begin{aligned}
V_1(P) &= (P-k)dt + (1-\rho dt)(1-\lambda dt)\mathbb{E}_P\left[V_1(P+dP)\right] + (1-\rho dt)\lambda dt \mathbb{E}_P\left[\frac{P(1+u)}{\rho-\alpha} - K\right] \\
\Rightarrow \quad & V_1(P) = (P-k)dt + (1-(\rho+\lambda)dt)\left[V_1(P) + V_1'(P)\alpha P dt + \frac{1}{2}V_1''(P)\sigma^2 P^2 dt\right] \\
& +\lambda dt\left[\frac{P(1+u)}{\rho-\alpha} - K\right] \\
\Rightarrow \quad & \frac{1}{2}\sigma^2 P^2 V_1''(P) + \alpha P V_1'(P) - (\rho+\lambda)V_1(P) + P - k + \frac{\lambda P(1+u)}{\rho-\alpha} - \lambda K = 0 \\
\Rightarrow \quad & V_1(P) = \frac{(\rho-\alpha+\lambda(1+u))P}{(\rho+\lambda-\alpha)(\rho-\alpha)} - \frac{k}{\rho+\lambda} - \frac{\lambda K}{\rho+\lambda} + D_{21}P^{\delta_2} \qquad (15)
\end{aligned}$$

where δ_2 (δ_1) is the negative (positive) root of $\frac{1}{2}\xi(\xi-1)\sigma^2 + \alpha\xi - (\rho+\lambda) = 0$. The first line of Eq. 15 indicates that the expected PV of the firm in state 1 is the revenues minus the R&D expenditures in the next dt time units plus the discounted expected value of the firm dt time units hence conditioning on whether an innovation occurs in that interval. With probability $1 - \lambda dt$, an innovation will not occur, in which case the firm's expected value in dt time units is the same as that as in the first line of Eq. 14. However, if an innovation occurs (with probability λdt), then the firm enters state 2, where its expected PV is simply the discounted perpetual earnings from the upgraded technology minus the upgrade cost. After expanding this expression, we obtain the ODE in the next-to-last line of Eq. 15, the homogenous part of which has solution $D_{11}P^{\delta_1} + D_{21}P^{\delta_2}$. Meanwhile, a particular solution is of the form $aP + bk + cK + d$, where a, b, c, and d are constants to be determined. This particular solution corresponds to the expected PV of a firm in state 1 that does not have the switching flexibility and immediately adopts the innovation upon arrival, which is precisely the result of Section 4.1 (see Eq. 2). Thus, the remaining option-like term in the last line of Eq. 15 represents the possibility of suspending R&D if the output price drops. Since R&D suspension is unlikely when the price increases, the option-like term must go to zero as the price approaches infinity, which implies that $D_{11} = 0$ and D_{21} is a positive endogenous constant.

In order to specify $V_1(P)$ completely, the unknowns B_{11}, D_{21}, and S_1^* need to be found. The two equations corresponding to VM and SP conditions may be written at $P = S_1^*$. A third equation relating the second derivatives of $V_1(P)$ at the same threshold is also used to describe the optimal switching. This equation is known as the super-contact (SC) condition and intuitively means that the marginal values of being in states 1 and 1' are equal at the optimal threshold (see [11]). Hence, we have the following three equations that may be used to find the three unknowns, albeit numerically rather than analytically:

$$V_1(S_1^{*-}) = V_1(S_1^{*+}) \tag{16}$$

$$V_1'(S_1^{*-}) = V_1'(S_1^{*+}) \tag{17}$$

$$V_1''(S_1^{*-}) = V_1''(S_1^{*+}) \tag{18}$$

Finally, the value of the option to invest from state 0 in a project with a single R&D innovation is again given as:

$$F_1(P) = A_{11} P^{\beta_1} \tag{19}$$

However, finding A_{11} and P_1^* via VM and SP conditions analogous to those in Eqs. 4 and 5 is not as straightforward. This is because it may optimal to enter state 1' from state 0, i.e., it is possible for $P_1^* < S_1^*$. Effectively, the firm may want to benefit from the revenues provided by an existing version of the technology without the need for R&D expenditures just yet. In any case, the following VM and SP conditions apply:

$$F_1(P_1^*) = V_1(P_1^*) - I \tag{20}$$

$$F_1'(P_1^*) = V_1'(P_1^*) \tag{21}$$

If it is the case that $P_1^* < S_1^*$, then we obtain the following analytically since the form of $V_1(P)$ from Eq. 14:

$$P_1^* = \left(\frac{\beta_1}{\beta_1 - 1}\right)(\rho - \alpha)I \tag{22}$$

$$A_{11} = B_{11} + \frac{(P_1^*)^{1-\beta_1}}{\beta_1(\rho-\alpha)} \tag{23}$$

On the other hand, if $P_1^* \geq S_1^*$, then $V_1(P)$ is described by Eq. 15, and S_1^* must be found numerically.

With infinite innovations, the firm's problem continues from state 2 onward. However, the optimal switching decision between any two states i and i' may be analysed similarly since engaging in R&D creates the possibility of increasing instantaneous revenues from $P(1+u)^i$ to $P(1+u)^{i+1}$, i.e., by a factor of $1+u$ just as in the transition from state 1 to 2. As a result, a common switching threshold, S_∞^*, exists for all states, which means that we could analyse the situation in states 1 and 1' without loss of generality. Thus, for $P < S_\infty^*$, the firm's value function in state 1' is as follows:

$$\begin{aligned}
V_\infty(P) &= Pdt + (1-\rho dt)\mathbb{E}_P\left[V_\infty(P+dP)\right] \\
\Rightarrow \quad & V_\infty(P) = Pdt + (1-\rho dt)\mathbb{E}_P\left[V_\infty(P) + V'_\infty(P)dP + \frac{1}{2}V''_\infty(P)\left(dP\right)^2\right] \\
\Rightarrow \quad & V_\infty(P) = Pdt + (1-\rho dt)\left[V_\infty(P) + V'_\infty(P)\alpha P dt + \frac{1}{2}V''_\infty(P)\sigma^2 P^2 dt\right] \\
\Rightarrow \quad & \frac{1}{2}\sigma^2 P^2 V''_\infty(P) + \alpha P V'_\infty(P) - \rho V_\infty(P) + P = 0 \\
\Rightarrow \quad & V_\infty(P) = \frac{P}{\rho-\alpha} + B_{1\infty}P^{\beta_1}
\end{aligned} \tag{24}$$

The intuition for Eq. 24 is similar to that for Eq. 14.

For $P \geq S_\infty^*$, we obtain $V_\infty(P)$ again by conditioning on what happens in the next dt time units:

$$\begin{aligned}
V_\infty(P) &= (P-k)dt + (1-\rho dt)(1-\lambda dt)\mathbb{E}_P\left[V_\infty(P+dP)\right] \\
& +(1-\rho dt)\lambda dt \mathbb{E}_P\left[V_\infty(P(1+u)) - K\right] \\
\Rightarrow \quad & V_\infty(P) = (P-k)dt + (1-(\rho+\lambda)dt)\left[V_\infty(P) + V'_\infty(P)\alpha P dt + \frac{1}{2}V''_\infty(P)\sigma^2 P^2 dt\right] \\
& +\lambda dt\left[V_\infty(P(1+u)) - K\right] \\
\Rightarrow \quad & \frac{1}{2}\sigma^2 P^2 V''_\infty(P) + \alpha P V'_\infty(P) - (\rho+\lambda)V_\infty(P) + P - k + \lambda V_\infty(P(1+u)) - \lambda K = 0 \\
\Rightarrow \quad & V_\infty(P) = \frac{P}{\rho-\alpha-\lambda u} - \frac{k}{\rho} - \frac{\lambda K}{\rho} + D_{2\infty}P^{\gamma_2}
\end{aligned} \tag{25}$$

where γ_2 (γ_1) is the negative (positive) root of $\frac{1}{2}\xi(\xi-1)\sigma^2 + \alpha\xi - (\rho+\lambda) + \lambda(1+u)^\gamma = 0$. The first line of Eq. 25 differs from that Eq. 15 in the third term. With infinite innovations, the firm's value conditional on a Poisson arrival in the next dt time units looks the same but with a higher initial output price and the deduction of the fixed upgrade cost, K. Consequently, the final line of Eq. 25 is similar to that of Eq. 2 but with an extra term, $D_{2\infty}P^{\gamma_2}$, that reflects the option value of suspending the R&D programme due to a decrease in the price. It should be noted that $\gamma_2 \neq \delta_2$ because the combined GBM-PP describing the output price persists even after an innovation arrives.

In order to determine $B_{1\infty}$, $D_{2\infty}$, and S_∞^*, VM, SP, and SC conditions analogous to those in Eqs. 16 through 18 are set up and solved numerically. Finally, the value of the investment opportunity with infinite innovations is:

$$F_\infty(P) = A_{1\infty} P^{\beta_1} \tag{26}$$

As described in Eqs. 20 and 21, $A_{1\infty}$ and P^*_∞ may be obtained. However, the solution is obtained analytically only if $P^*_\infty < S^*_\infty$.

4.3 Model 3: Launching Flexibility

With the launching flexibility, the firm is able to hold back a new version of the technology from the market when it arrives (see Fig. 3). In a situation with a single innovation, the firm compares the current output price to the launching threshold, L_1^*. If an innovation occurs when $P \geq L_1^*$, then the firm immediately launches the new version of the technology, i.e., proceeds to state 2. Consequently, its value function in state 1 is different depending on the current price. However, if $P < L_1^*$, then the occurrence of an innovation takes the firm to state 1', where it waits until the output price is high enough. For $P < L_1^*$, the firm's value function in state 1 is obtained by conditioning on what happens in the next dt time units as follows:

$$\begin{aligned}
V_1(P) \quad & = (P-k)dt + (1-\rho dt)(1-\lambda dt)\mathbb{E}_P\left[V_1(P+dP)\right] \\
& +(1-\rho dt)\lambda dt\left[E_{11}P^{\beta_1} + \frac{P}{\rho-\alpha}\right] \\
\Rightarrow \quad & V_1(P) = (P-k)dt + (1-(\rho+\lambda)dt)\left[V_1(P) + V_1'(P)\alpha P dt + \frac{1}{2}V_1''(P)\sigma^2 P^2 dt\right] \\
& +\lambda dt E_{11}P^{\beta_1} + \frac{\lambda dt P}{\rho-\alpha} \\
\Rightarrow \quad & 0 = P - k + V_1'(P)\alpha P + \frac{1}{2}V_1''(P)\sigma^2 P^2 - (\rho+\lambda)V_1(P) + \lambda E_{11}P^{\beta_1} + \frac{\lambda P}{\rho-\alpha} \\
\Rightarrow \quad & V_1(P) = \frac{P}{\rho-\alpha} - \frac{k}{\rho+\lambda} + E_{11}P^{\beta_1} + B_{11}P^{\delta_1} \qquad (27)
\end{aligned}$$

Intuitively, the first line of Eq. 27 indicates that the firm's instantaneous cash flows consist of revenues from the installed technology and R&D expenditures plus the usual value dt time units later given no innovation in that interval plus the expected PV given an innovation occurs in the next dt time units. In the latter case, the firm's R&D programme is suspended until the price increases to the launching threshold. Thus, its expected PV consists of perpetual income from the current version of the technology plus the option to launch the new version. The components of the value function in the last line of Eq. 27 reflect the expected PV from using the current version of the technology forever minus the PV of the R&D expenditure until an innovation occurs plus the option value to launch the new version of the technology plus an option-like term, $B_{11}P^{\delta_1}$, that adjusts for the possibility of a price increase that takes the firm into a region from where it would immediately launch the new version of the technology once it arrives. For this reason, B_{11} is a negative endogenous constant because it corrects for the term, $\frac{P}{\rho-\alpha}$, which assumes that the current version of the technology will remain in place forever. This supposition becomes increasingly unlikely as the price increases.

For $P \geq L_1^*$, the firm's value function in state 1 is as follows:

$$\begin{aligned}
V_1(P) \quad &= (P-k)dt + (1-\rho dt)(1-\lambda dt)\mathbb{E}_P\left[V_1(P+dP)\right] \\
&+(1-\rho dt)\lambda dt\left[\frac{P(1+u)}{\rho-\alpha}-K\right] \\
\Rightarrow \quad & V_1(P) = (P-k)dt + (1-(\rho+\lambda)dt)\left[V_1(P)+V_1'(P)\alpha P dt + \frac{1}{2}V_1''(P)\sigma^2P^2dt\right] \\
&+\frac{\lambda dt P(1+u)}{\rho-\alpha} - \lambda K dt \\
\Rightarrow \quad & 0 = P - k + V_1'(P)\alpha P + \frac{1}{2}V_1''(P)\sigma^2P^2 - (\rho+\lambda)V_1(P) + \frac{\lambda P(1+u)}{\rho-\alpha} - \lambda K \\
\Rightarrow \quad & V_1(P) = \frac{P(\rho-\alpha+\lambda(1+u))}{(\rho+\lambda-\alpha)(\rho-\alpha)} - \frac{k}{\rho+\lambda} - \frac{\lambda K}{\rho+\lambda} + D_{21}P^{\delta_2} \qquad (28)
\end{aligned}$$

The first line of Eq. 28 indicates that conditional on the innovation's arrival, the firm's value function simply becomes the expected PV of perpetually using the new version of the technology minus the upgrading cost. Thus, it reflects the case of immediate launching upon an innovation's arrival. Correspondingly, the final line of Eq. 28 is similar to that in Eq. 2 plus an option-like term that reflects the possibility of moving into the region where immediate launch of the innovation is not optimal.

We first solve for L_1^* and E_{11} via VM and SP conditions between the last terms in the first lines of Eqs. 27 and 28:

$$E_{11}(L_1^*)^{\beta_1} + \frac{L_1^*}{\rho-\alpha} = \frac{L_1^*(1+u)}{\rho-\alpha} - K \qquad (29)$$

$$\beta_1 E_{11}(L_1^*)^{\beta_1-1} + \frac{1}{\rho-\alpha} = \frac{1+u}{\rho-\alpha} \qquad (30)$$

Thus, we obtain the following:

$$L_1^* = \left(\frac{\beta_1}{\beta_1-1}\right)(\rho-\alpha)K \qquad (31)$$

$$E_{11} = \frac{(L_1^*)^{1-\beta_1}u}{\beta_1(\rho-\alpha)} \qquad (32)$$

The remaining constants, B_{11} and D_{21} are found analytically via VM and SP conditions between the two branches of $V_1(P)$ at L_1^*. Finally, the value of the option to invest from state 0 has a similar form as in Eq. 19.

With infinite innovations, the firm's value function in sate 1 is again found by conditioning on what happens in the next dt time units. First, for $P < L^*_\infty$:

$$\begin{aligned}
V_\infty(P) &= (P-k)dt + (1-\rho dt)(1-\lambda dt)\mathbb{E}_P\left[V_\infty(P+dP)\right] \\
&+(1-\rho dt)\lambda dt\left[E_{1\infty}P^{\beta_1} + \frac{P}{\rho-\alpha}\right] \\
\Rightarrow \quad & V_\infty(P) = (P-k)dt + (1-(\rho+\lambda)dt)\left[V_1(P) + V'_\infty(P)\alpha P dt + \frac{1}{2}V''_\infty(P)\sigma^2P^2dt\right] \\
&+\lambda dt E_{1\infty}P^{\beta_1} + \frac{\lambda dt P}{\rho-\alpha} \\
\Rightarrow \quad & 0 = P - k + V'_\infty(P)\alpha P + \frac{1}{2}V''_\infty(P)\sigma^2P^2 - (\rho+\lambda)V_\infty(P) + \lambda E_{1\infty}P^{\beta_1} + \frac{\lambda P}{\rho-\alpha} \\
\Rightarrow \quad & V_\infty(P) = \frac{P}{\rho-\alpha} - \frac{k}{\rho+\lambda} + E_{1\infty}P^{\beta_1} + B_{1\infty}P^{\delta_1} \qquad (33)
\end{aligned}$$

Similarly, for $P \geq L^*_\infty$:

$$\begin{aligned}
V_\infty(P) &= (P-k)dt + (1-\rho dt)(1-\lambda dt)\mathbb{E}_P\left[V_\infty(P+dP)\right] \\
&+(1-\rho dt)\lambda dt\left[V_\infty(P(1+u)) - K\right] \\
\Rightarrow \quad & 0 = P - k + V'_\infty(P)\alpha P + \frac{1}{2}V''_\infty(P)\sigma^2P^2 - (\rho+\lambda)V_\infty(P) \\
&+V_\infty(P(1+u)) - \lambda K \\
\Rightarrow \quad & V_\infty(P) = \frac{P}{\rho-\alpha-\lambda u} - \frac{k}{\rho} - \frac{\lambda K}{\rho} + D_{2\infty}P^{\gamma_2} \qquad (34)
\end{aligned}$$

In order to determine L^*_∞, $E_{1\infty}$, $B_{1\infty}$, and $D_{2\infty}$, VM and SP conditions as in Eqs. 29 and 30 are used along with VM and SP conditions between the two branches of $V_\infty(P)$ at $P = L^*_\infty$. Again, the value of the option to invest from state 0 has a similar form as in Eq. 19.

4.4 Model 4: Both Switching and Launching Flexibilities

With both switching and launching flexibilities in a situation with a single innovation, we assume that $S^*_1 < L^*_1$. Using the same principles as in Sections 4.2 and 4.3, we obtain the following:

$$V_1(P) = \begin{cases} \frac{P}{\rho-\alpha} + B_{11}P^{\beta_1}, & \text{if } P < S^*_1 \\ E_{11}P^{\beta_1} + \frac{P}{\rho-\alpha} + C_{11}P^{\delta_1} + C_{21}P^{\delta_2} - \frac{k}{\rho+\lambda}, & \text{if } S^*_1 \leq P < L^*_1 \\ D_{21}P^{\delta_2} + \frac{P(\rho-\alpha+\lambda(1+u))}{(\rho-\alpha)(\rho+\lambda-\alpha)} - \frac{k}{\rho+\lambda} - \frac{\lambda K}{\rho+\lambda}, & \text{otherwise} \end{cases} \qquad (35)$$

The unknowns, B_{11}, C_{11}, C_{21}, D_{21}, and S^*_1 are found via the VM and SP conditions similar to those in Eqs. 29 and 30 as well as the VM, SP, and SC conditions similar to those in Eqs. 16, 17, and 18. The value of the option to invest is as given in Eq. 19.

As for the situation with infinite innovations, we also assume that $S_\infty^* < L_\infty^*$. Following the practice of conditioning on what happens in the next dt time units, we obtain the following:

$$V_\infty(P) = \begin{cases} \frac{P}{\rho-\alpha} + B_{1\infty}P^{\delta_1}, & \text{if } P < S_\infty^* \\ E_{1\infty}P^{\beta_1} + \frac{P}{\rho-\alpha} + C_{1\infty}P^{\delta_1} + C_{2\infty}P^{\delta_2} - \frac{k}{\rho+\lambda}, & \text{if } S_\infty^* \leq P < L_\infty^* \\ D_{2\infty}P^{\gamma_2} + \frac{P}{\rho-u\lambda-\alpha} - \frac{k}{\rho} - \frac{\lambda K}{\rho}, & \text{otherwise} \end{cases} \quad (36)$$

5 Numerical Examples

We illustrate the concepts discussed in Section 4 via numerical examples. For all examples, we use the following data: $\rho = 0.04$, $\alpha = 0$, $\lambda = 0.20$, $I = 100$, $K = 10$, $k = 1$, $u = 0.10$, and $\sigma \in (0.06, 0.30)$.

5.1 No Flexibility

The value functions of a firm with a single and infinite innovations are given in Fig. 5. Not surprisingly, the latter situation facilitates investment by increasing the firm's value of investment opportunity. Figs. 6 and 7 perform sensivity analysis on the optimal investment threshold and option value coefficient in both situations with respect to the volatility. As in standard real options models, the value of waiting increases with uncertainty, which also increases the optimal investment threshold. Again, infinite innovations facilitate investment and increase option value.

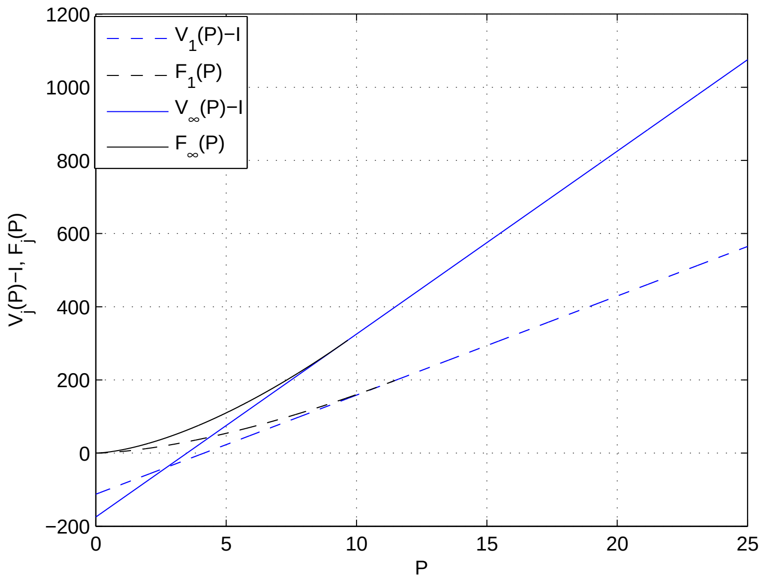

Figure 5: Value Curves without Flexibility

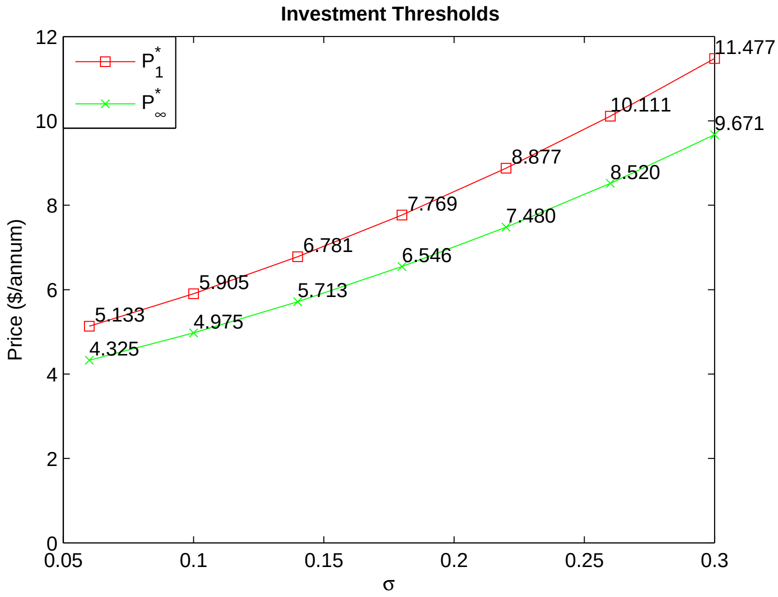

Figure 6: Optimal Investment Thresholds with Uncertainty

5.2 Switching Flexibility

With the flexibility to switch R&D on and off, the firm's value function in state 1 should be higher than the corresponding one in Section 5.1. From Fig. 8, the curve in $V_1(P)$ and $V_\infty(P)$ becomes apparent for low prices, thereby providing protection against the downside of doing R&D in unfavourable market conditions. Consequently, the value of the investment opportunity is higher than in a case without flexibility. Indeed, Figs. 9 and 10 indicate that investment is facilitated due to the increase in option value, which is quite intuitive. Likewise, both the optimal investment thresholds and the option value coefficients increase with uncertainty as does the optimal switching threshold price with a single innovation.

However, with infinite innovations, the latter threshold is not monotonous with respect to the uncertainty. Intriguingly, it first increases with uncertainty as one would expect but then starts to decrease. Intuitively, we would expect greater uncertainty to make the firm more likely to switch R&D off from an active state and less likely to switch R&D on from a suspended state. Yet, here, we see a result that is rare in the real options literature and, to our knowledge, expressed fully only in [2]. There, a firm has the option to invest in a project that becomes active with a lag. While the firm is waiting for the project to launch, it also has the option to abandon the project. With a high enough lag, low costs of reversing the decision, and high uncertainty, the optimal investment threshold actually decreases with uncertainty. The explanation for this counterintuitive outcome is that unlike in the standard real options setting

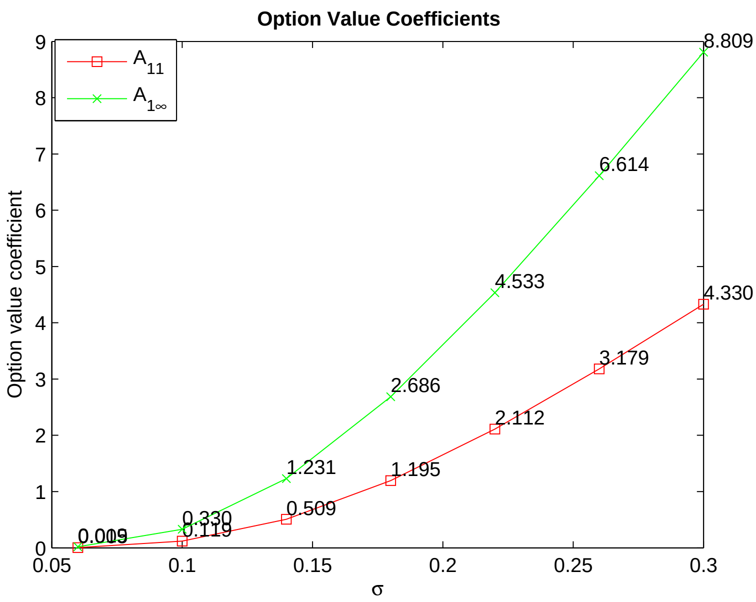

Figure 7: Option Value Coefficients with Uncertainty

(where the marginal benefit of delaying investment, which stems from the higher expected initial price and lower discounted investment cost, increases more with uncertainty than its marginal cost, which is related to the opportunity cost of forgone cash flows), here, the marginal cost of delaying is related to forgone revenues way out in the future. Plus, these revenues are bounded from below because of the option to abandon. Thus, for high enough uncertainty, the marginal cost may increase by more than the marginal benefit of delaying, thereby lowering the investment threshold. We have a similar dynamic at play here: from a state in which R&D is suspended, moving to an active state is like initiating investment in a project that will arrive with a lag (except that this lag is uncertain in our case). In fact, the optimal switching threshold increases monotonically with uncertainty in cases where the switching flexibility is less reversible, e.g., finite number of transitions.

5.3 Launching Flexibility

Instead of flexibility over R&D switching, the firm now has the flexibility to delay launching a new version of the technology. The value functions in state 1 have downside risk protection as in Section 5.2 (see Fig. 11). As expected, uncertainty causes the investment and launching thresholds to increase as well as the option value coefficients (see Figs. 12 and 13). Although the launching flexibility adds value relative to the case without any flexibility and enables the initial investment to proceed sooner, it is not as valuable as the switching flexibility. This may be due to a restrictive assumption in the model that

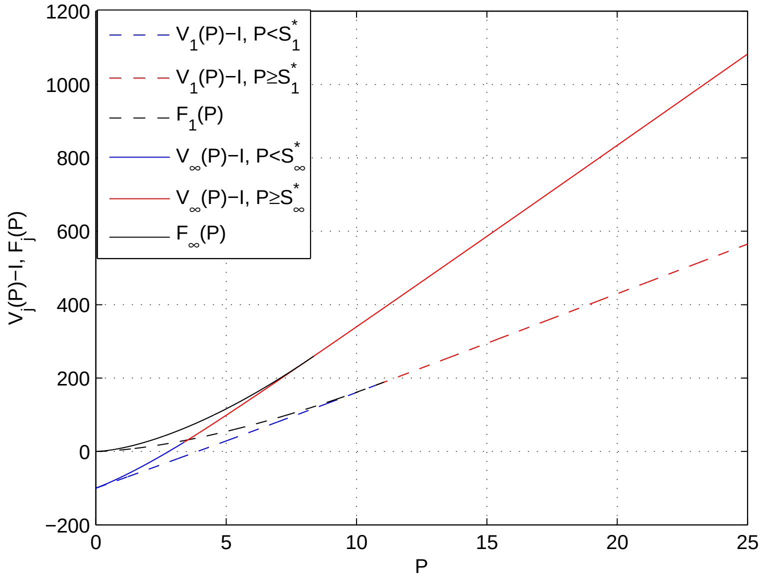

Figure 8: Value Curves with Switching Flexibility

stops R&D once an innovation arrives.

5.4 Both Switching and Launching Flexibilities

When both flexibilities are combined, they are more valuable than either flexibility on its own. The value curves in Fig. 14 for state 1 in either situation have three components. As uncertainty is increased, the investment and launching thresholds increase but are not higher than their corresponding values in Sections 5.2 and 5.3 (see Fig. 15). The optimal switching threshold is also lower than its corresponding value in Section 5.2, at least for the situation with infinite innovations. The non-monotonic effect due to the uncertainty is more pronounced now due to the added downside protection from the launching flexibility. Similarly, Fig. 16 indicates that the option value coefficients are higher for the situation with infinite innovations.

6 Conclusions

In this paper, we have examined the decision-making problem of an R&D-intensive firm using real options. The firm earns stochastic revenues from using the technology and has the incentive to continue R&D in order to increase revenues by improving the efficiency of the technology. Each subsequent version of the technology arrives randomly as long as R&D is ongoing. In order to see how flexibility over R&D

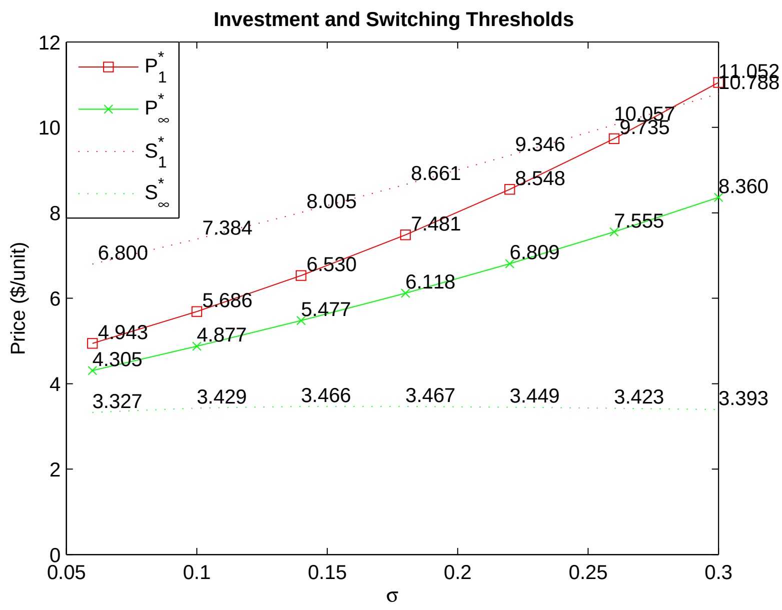

Figure 9: Optimal Investment Thresholds with Switching Flexibility and Uncertainty

switching and technology launching could be beneficial to the firm, we solve embedded optimal control problems. We find that most of the value of flexibility is captured by the switching option. However, the relationship between the switching threshold and uncertainty is not monotonic. This counterintuitive finding is explained by noting that the opportunity cost of not switching the R&D on is increased with uncertainty due to the reversibility of the switching.

For future work, this work's limitations may be addressed in several ways. First, the assumption of stopping R&D once a new version of the technology arrives in the case with launching flexibility could be relaxed. Second, there could a learning curve introduced to capture how innovation is influenced by funding and in stages. Third, a firm with an R&D division usually faces competition from rivals, which may hasten its decision to launch an innovation or to hasten the R&D effort.

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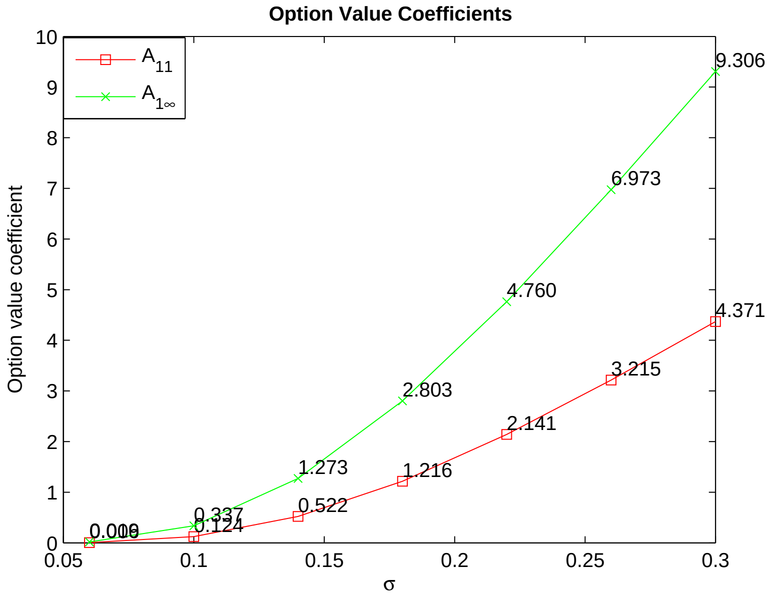

Figure 10: Option Value Coefficients with Switching Flexibility and Uncertainty

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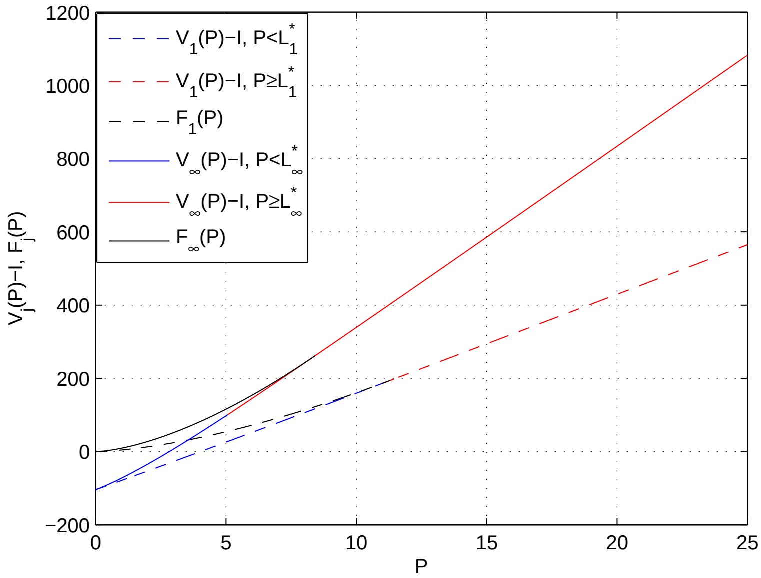

Figure 11: Value Curves with Launching Flexibility

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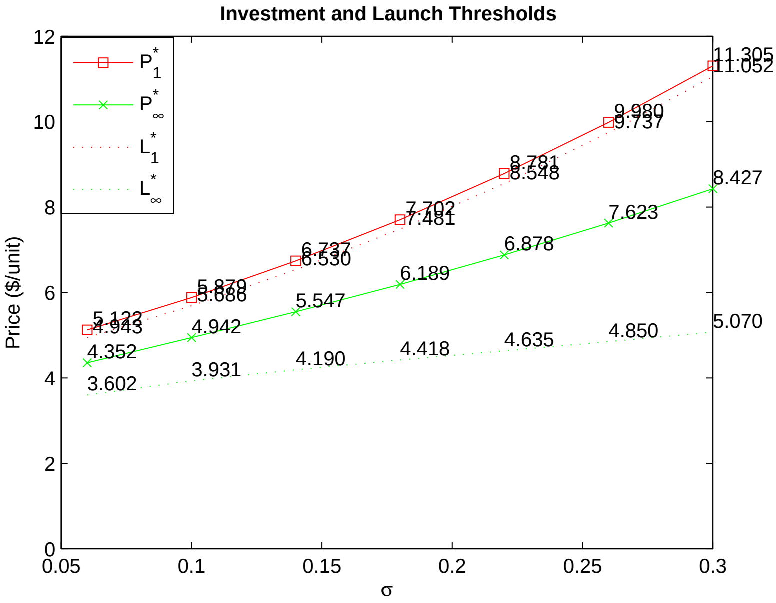

Figure 12: Optimal Investment Thresholds with Launching Flexibility and Uncertainty

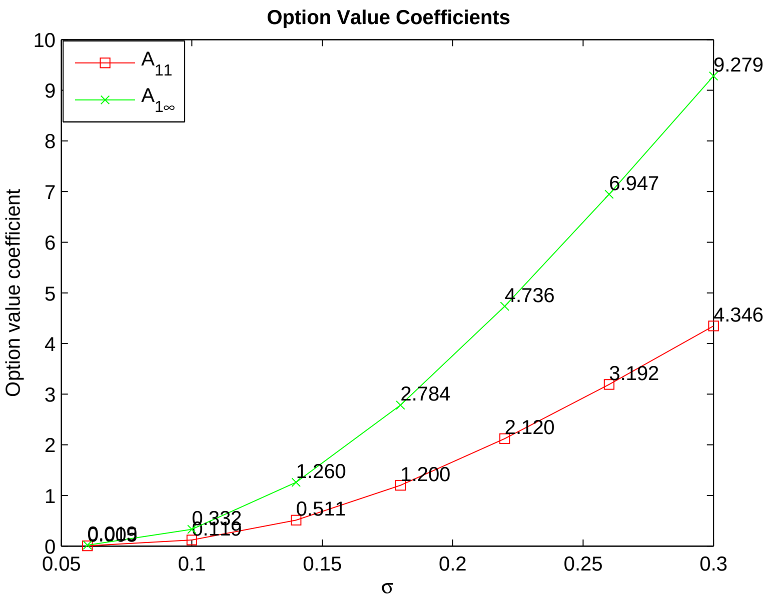

Figure 13: Option Value Coefficients with Launching Flexibility and Uncertainty

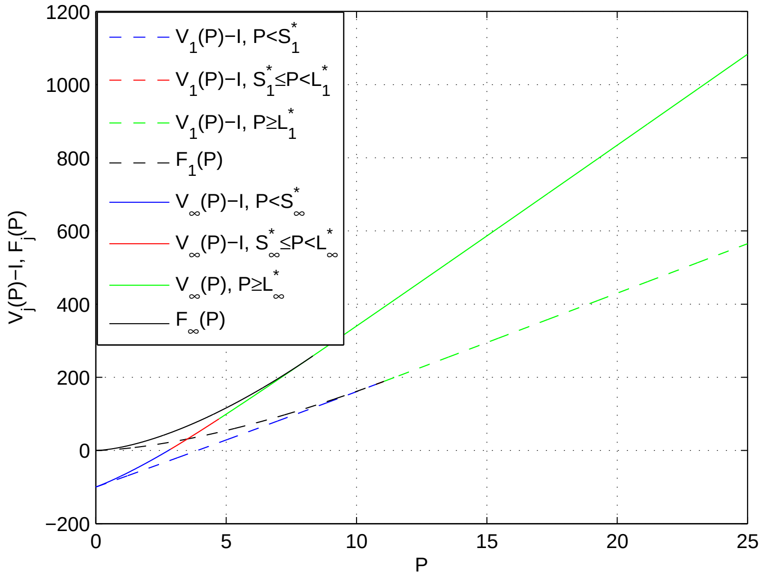

Figure 14: Value Curves with Switching and Launching Flexibilities

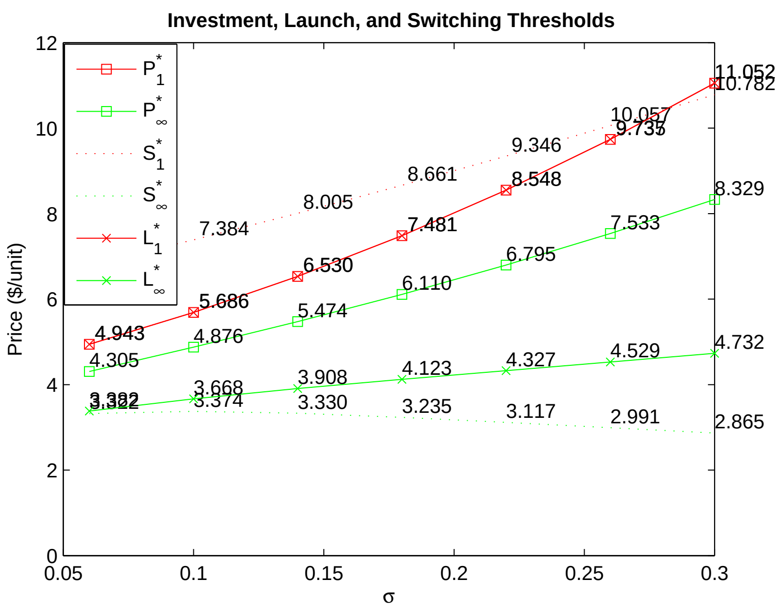

Figure 15: Optimal Investment Thresholds with Switching/Launching Flexibility and Uncertainty

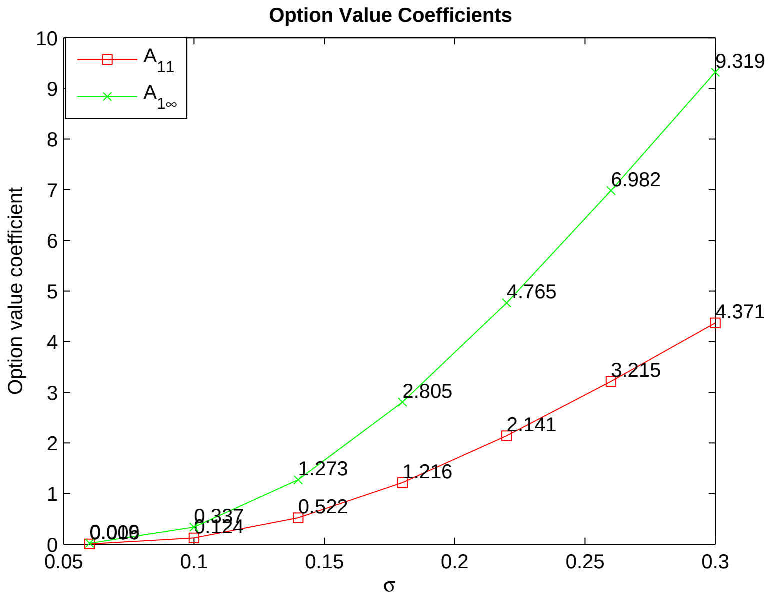

Figure 16: Option Value Coefficients with Switching/Launching Flexibility and Uncertainty