What is the (Real Option) Value of a College Degree?*

JEFFREY R. STOKES†
313 Curris Business Building
Department of Finance
College of Business Administration
University of Northern Iowa, Cedar Falls, IA 50614-0124

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Abstract

The value of a college degree is often quantified as the difference in earnings between those with and without a degree. The research presented here operationalizes this idea in two important ways. First, since future income and tuition are uncertain, a contingent claims model is developed and the appropriate discount rate for valuing future earnings is, therefore, endogenized given an economy that does not permit arbitrage. Second, the model is sensitive to the valuation of the real option to obtain an advanced degree in addition to the valuation of the earnings for an individual with an undergraduate degree. In this framework, the value of a high school diploma is shown to be the sum of: (1) capitalized earnings, (2) the real option to obtain an undergraduate degree, and (3) the embedded or compound real option to obtain an advanced degree. Numerical examples are presented that demonstrate the performance and key drivers of the model. One important finding is that by ignoring the real options to further one's education, the value of a college degree is likely significantly understated.

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* Any errors or omissions are the sole responsibility of the author.

† Associate Professor of Finance, phone: 319-273-2929, email: jeffrey.stokes@uni.edu

1 Introduction

U.S. Census Bureau data show that the level of education has risen steadily in the U.S. over the last 70 years. For example, in 1940 less than 25% of people aged 25 and over had a high school diploma or higher and less than 5% had a bachelor's degree or higher. By 2008, those percentages had increased to 85% and about 28%. In addition, these data are used by Cheeseman Day and Newburger (2002) to show that there is a strong positive relationship between income and education in that each successive degree increases median earnings for the recipients of the degree. Sanchez and Laanan (1997) examine state unemployment records in California and draw similar conclusions.

What has also been increasing steadily over time is the cost of obtaining a college degree which includes tuition, fees, books, etc.[1] For example, the data reported in *2011 Trends in College Pricing* show that average published tuition and fees have increased at private nonprofit four-year institutions at an average annual rate of 6.54% since the 1981-82 academic year. Similarly, average published tuition and fees have increased at pubic four-year institutions at an average annual rate of 7.84% over the last 10 years alone. Such values are clearly in excess of the average annual rate of inflation in the U.S. and demonstrate that even though the U.S. population is becoming more educated, they are paying more it.

Faced with what seems like ever increasing costs, Kelly (2010) recently asked the question: is getting a college degree still worth it? Clearly the research cited above suggests that getting college degrees increases (median) earnings and so it would seem that the answer is yes. Even so, the decision perhaps necessitates a careful consideration of the tradeoff between incrementally higher earnings on the one hand and the increasing cost of education on the other. This also suggests that consideration of such a tradeoff is important for putting a monetary value on higher education. Valuation in this context is difficult because the future costs and returns associated with obtaining a degree are uncertain.

There has been surprisingly little research devoted to methodological considerations for determining the value of a college degree. Synthetic work-life estimates are reported by Cheeseman Day and Newburger (2002) and Julian and Kominski (2011) in which median earnings reported for eight different age groups are summed across those groups. The resulting estimates seek to place a value on the work-life earnings of a typical

[1] The cost of room and board has also steadily increased but is not considered here since living expenses such as these are incurred whether an individual goes to college or not.

individual with a particular level of education. Porter (2002) suggests that the earnings discrepancy between high school graduates and college graduates gives some indication of the value of a college education. Indeed, basic financial intuition would suggest that capitalizing lifetime earnings both with and without a degree may provide evidence of the value of having (or not having) a degree.

Even so, critical aspects of valuation are missing by just capitalizing earnings. For example, an appropriate discount rate must be determined. Also, in the U.S. going to college is basically a right but definitely not an obligation and the investment in education is an irreversible one. Lastly, a high school diploma is required to earn an undergraduate degree and an undergraduate degree is required for an advanced degree. These features suggest that a real option approach to valuation is more appropriate. More specifically, the value of a college education is equal to capitalized salary plus the value of the option to obtain an advanced degree. Similarly, the value of a high school diploma is equal to capitalized wages plus the value of the option to obtain an undergraduate degree and the embedded or compound option to obtain an advanced degree.

In this paper, we develop such a real option model and empirically demonstrate its use for valuing earnings and educational options. The paper is organized as follows. In section 2, we make the basic assumptions required of the valuation methodology, value the earnings streams for individuals with different levels of education, and develop and analytically solve the option valuation models in a contingent claims framework. In section 3, we discuss data to be used in the empirical implementation of the model. Presented in section 4 are some empirical results and discussion associated with the model and concluding comments are presented in section 5.

2 Model Prerequisites and Valuation

In this section, the underlying state variables and sources of uncertainty that affect the college and advanced degree options are specified and discussed. Unless otherwise noted, by college degree we mean an undergraduate (four-year) B.S. or B.A. degree. By advanced degree, we mean any degree beyond an undergraduate degree (e.g. an M.B.A. or Ph.D.) including professional degrees such as J.D. or M.D.[2] The key point here is that the undergraduate degree is a requirement for pursuing an advanced degree. The option valuation model is then specified and solved analytically and some important results are then presented. It should also be noted that in what follows, only the financial incentive

[2] Advanced degree and graduate degree are used interchangeably throughout.

associated with exercising educational options are modeled. As Porter (2011) and Kelly (2010) both note, individuals consider pursuing higher education for non-financial reasons as well. Even so, financial incentives are more readily quantifiable and represent a significant consideration for most if not all individuals faced with a college attendance decision. In addition, Carnevale *et al*. (2011) finds that, not surprisingly, different majors have different economic value. However, our model makes no distinction between majors since the major contribution is the idea that education is best valued as a real (sequential) compound option.

2.1 Stochastic state variables

For the valuation model to follow, two stochastic state variables are modeled. Let $g(t) = g_t$ represent the time t salary facing an individual with an advanced degree. We assume that g_t can be characterized by a first-order stochastic differential equation and evolves over time as a geometric Brownian motion so that $dg_t/g_t = \mu_g dt + \sigma_g dz_t^g$ where the instantaneous rate of salary growth is μ_g, σ_g is the instantaneous volatility of the rate of growth, and z_t^g is a $\mathbb{P}$-Brownian motion where $dz_t^g \sim N(0, dt)$.[3]

Let $\omega_t^g = z_t^g + \lambda_g t$ be a $\mathbb{Q}$-Brownian motion where $\mathbb{Q}$ is the risk-neutral measure and λ_g is a constant so that $d\omega_t^g = dz_t^g + \lambda_g dt$. Using this result, the risk-neutral diffusion for g_t can be written as

(1) $dg_t = (\mu_g - \lambda_g \sigma_g) g_t dt + \sigma_g g_t d\omega_t^g$.

In (1) the interpretation of λ_g is that it is the market price of risk associated with advanced degree salary. To simplify the notation somewhat, in what follows let $\delta_g = \mu_g - \lambda_g \sigma_g$ be the risk neutral drift for the diffusion in (1).

To promote tractability, in the analysis that follows g_t forms the basis for all salaries considered in the model. For example, we assume that $b_t = \alpha_b g_t$ is the salary facing an individual with a bachelor's degree while $h_t = \alpha_h g_t$ is the wage for an individual with a high school diploma (i.e. with no college education). In general, $0 \leq \alpha_h \leq \alpha_b \leq 1$ with the implication that under strict inequality, individuals with high school diplomas earn less than those with bachelor's degrees and individuals with bachelor's degrees earn less than those with graduate degrees.

[3] Here, $\mathbb{P}$ is the natural probability measure.

Next, let c_t represent the (stochastic) cost of a graduate degree (e.g. tuition, fees, books, etc.). The evolution of c_t is also modeled as a first-order stochastic differential equation, namely,

(2) $dc_t = \delta_c c_t dt + \sigma_c c_t d\omega_t^c$.

In (2), we also let $\delta_c = \mu_c - \lambda_c \sigma_c$ be the risk-neutral drift where the parameters are defined analogously to those for equation (1). In addition, it is assumed for simplicity and tractability that the cost of an undergraduate degree, u_t, is a linear function of the cost of an advanced degree so that $u_t(c_t) = \gamma c_t$ where $\gamma < 1 (> 1)$ indicates that undergraduate tuition and fees are less (more) than those associated with graduate school.

Lastly, define $k_t^g(g_t, c_t) = g_t/c_t$ as the ratio of salary-to-cost for a graduate degree. Consistent with real option theory as discussed in Dixit and Pindyck (1994), it is assumed that there is a value, k_g^*, such that when $k_t^g \geq k_g^*$, there is a financial incentive for an individual with an undergraduate degree to exercise their option to pursue graduate study. Similarly, define $k_t^b(g_t, c_t) = \alpha_b g_t/\gamma c_t$ as the ratio of salary-to-cost for an undergraduate degree. Here we also assume that there is a value, k_b^*, such that when $k_t^b \geq k_b^*$, there is a financial incentive for an individual with a high school diploma to exercise their option to pursue undergraduate study.

Because an undergraduate degree is a requirement for pursuing an advanced degree, the option to obtain an undergraduate degree must be exercised first and the option to obtain a graduate degree is an embedded or compound option as a result. The sequential nature of events means it will be necessary to first value the option to obtain a graduate degree and then subsequently value the option to obtain an undergraduate degree.

2.2 Earnings valuation

It will prove useful to value the income streams associated with each degree before tackling the more difficult problem of valuing the options. Here, we demonstrate the valuation of earnings for an individual with an undergraduate degree and then simply state the result for the valuation of other earnings where they are analogous. An individual who earns an undergraduate degree at, say, time t' begins earning income, b_t, worth $B_t(b_t, t)$ for all $t' \leq t \leq T$ where T represents the time when the individual retires. The fundamental valuation equation for this earnings stream can then be found by equating the sum of the income flow and the expected (risk-neutral) capital gain on B_t with the risk free return on B_t. More clearly,

(3) $b_t dt + \mathbb{E}^{\mathbb{Q}}(dB_t) = rB_t dt$

where r is the instantaneous risk free rate of return.

Applying Ito's lemma to B_t to compute the capital gain required by (3) results in

(4) $$dB_t = \frac{\partial B_t}{\partial b_t} dg_t + \frac{1}{2}\frac{\partial^2 B_t}{\partial b_t^2} db_t^2 + \frac{\partial B_t}{\partial t}.$$

Substituting using the risk-neutral diffusion for b_t,[4] taking the expectation of the result, and substituting into (3) gives

(5) $$\left(\frac{{\sigma_g}^2 b_t^2}{2}\right)\frac{\partial^2 B_t}{\partial b_t^2} + \delta_g b_t \frac{\partial B_t}{\partial b_t} + \frac{\partial B_t}{\partial t} + b_t = rB_t.$$

Equation (5) is a second-order, linear, parabolic PDE, the solution of which represents the value of lifetime undergraduate earnings. The boundary conditions for (5) must ensure that: (1) the value of the earnings equals zero whenever the income itself is zero, (2) the value of the earnings is bounded, and (3) the value of the income stream is zero at retirement. Mathematically, these conditions imply that

(6) $B_t(0,t) = 0, B_t(\infty, t) < \infty$ and $B_t(b_t, T) = 0$.

A solution to (5) that meets the boundary conditions in (6) is

(7) $$B_t(b,t) = \frac{b_t}{r - \delta_g}\left[1 - e^{-(r-\delta_g)(T-t)}\right], \forall\, t' \le t \le T.$$

Notice that as $T \to \infty$, the term involving e approaches zero so that

(8) $$B_t(b_t, t) = B_t(b_t) = \frac{b_t}{r - \delta_g}.$$

Equation (8) is recognized as the present value of a perpetuity of b_t dollars in income where the discount rate is the risk free rate adjusted for the risk-neutral rate of growth in earnings. As a practical matter, there is little difference between (7) and (8) for reasonably large T and typical discount rates.[5] However, tractability is greatly improved if we assume (8) is a reasonable approximation to (7). Therefore, from here forward we assume that T >> 0 and work within an infinite planning horizon framework.

Equation (8) holds for individuals who successful complete their undergraduate studies. In fact, this is the value of the earnings that would be given up (along with the cost of tuition, fees, books, etc.) if the individual decides to pursue an advanced degree. However, pursue is the operative word here in that not all individuals who begin college

[4] Since $b_t = \alpha_b g_t$ it follows that $db_t = (\mu_g - \lambda_g \sigma_g) b_t dt + \sigma_g b_t d\omega_t^g$ using (1) above.

[5] For example, the present value of a $40K income for 45 years at a discount rate of 10% is $445K using (7) and $450K using (8), a difference of only $5K.

finish college whether at the undergraduate or graduate levels. To accommodate this reality, let $G_t(g_t, q_t)$represent the present value of graduate earnings.[6] The new argument in $G_t(g_t, q_t)$ is a Poisson random variable, q_t, where $dq_t = 1$ with probability $\pi_g dt$ and $dq_t = 0$ with probability $1 - \pi_g dt$. The Poisson variable is meant to proxy any of those situations that result in an individual not being able to complete advanced degree studies (e.g. failing classes or comprehensive examinations). In addition, for an individual who earned their undergraduate degree but had poor grades, π_g might equal one or be very close to one indicating a very high probability of failure if such an individual attempts to obtain an advanced degree.

Perhaps more importantly, when the Poisson event hits (i.e. when $dq_t = 1$), the value of graduate earnings drops to that of undergraduate earnings. The implication of this feature of the model in terms of valuation can be found by applying Ito's lemma to $G_t(g_t, q_t)$ yielding

$$(9) \quad dG_t = \frac{\partial G_t}{\partial g_t} dg_t + \frac{1}{2}\frac{\partial^2 G_t}{\partial g_t^2} dg_t^2 + [G_t(b_t) - G_t(g_t)]dq_t.$$

Notice the last term in (9) suggests that if the Poisson event happens (i.e. $dq_t = 1$), there is a financial loss equal to $G_t(b_t) - G_t(g_t)$ whereas if the event does not happen, $dq_t = 0$, and the term vanishes. Since $b_t = \alpha_b g_t$, we have $G_t(b_t) = G_t(\alpha_b g_t) = \alpha_b G_t(g_t)$ as long as $G_t(g_t)$ is homogeneous of degree zero. Therefore, the valuation equation can be written as

$$(10) \quad \left(\frac{{\sigma_g}^2 g_t^2}{2}\right)\frac{\partial^2 G_t}{\partial g_t^2} + \delta_g g_t \frac{\partial G_t}{\partial g_t} + g_t = v_g G_t,$$

where $v_g = r + (1 - \alpha_b)\pi_g$. Notice that if $\pi_g = 0$, $v_g = r$ and equation (10) is entirely analogous to equation (5), the value of earnings when there is no probability of failure. Stated differently, successfully obtaining an undergraduate degree is irreversible in that the earnings that result are valued without the probability of failure.

A solution to equation (10) that meets boundary conditions similar to those presented in (6) but for $G_t(g_t)$ is

$$(11) \quad G_t(g_t) = \frac{g_t}{v_g - \delta_g}.$$

In (11), the discount rate consists of the risk free rate adjusted for the probability of failure and the risk-neutral rate of growth in earnings.

[6] Given the assumption of a sufficiently long planning horizon, G_t does not depend explicitly on *t*.

Analogous to equation (11) is the value of the baccalaureate salary when viewed from the prospective of an individual with a high school diploma having yet earned their undergraduate degree. Given the preceding results, we simply state the result without the derivation as

$$(12) \quad \hat{B}_t(b_t) = \frac{b_t}{v_b - \delta_g}, \quad v_b = r + (1 - \alpha_h)\pi_b,$$

where π_b is the probability of some Poisson event happening that precludes matriculation for the undergraduate. Notice that the difference between $B_t(b_t)$ and $\hat{B}_t(b_t)$ is simply timing with the former applicable for the individual who has earned the undergraduate degree and the latter applicable for the individual who has not. In addition, for many individuals, $\pi_g > \pi_b$.

Lastly, the value of the perpetual wage associated with a high school diploma is given by

$$(13) \quad H_t(h_t) = \frac{h_t}{r - \delta_g},$$

where $h_t = \alpha_h b_t = \alpha_h \alpha_b g_t$. Here the appropriate discount rate is simply the risk free rate adjusted for the rate of growth in the wage facing an individual with a high school diploma.

Equations (8), (11), (12), and (13) are critical building blocks for the option values that follow in that the exercise boundaries k_g^* and k_b^* depend on them as do the option values, and therefore the value of the income streams themselves. For example, an individual with an undergraduate degree has earnings valued using (8) and must decide whether to pay c_t, and give up (8) in exchange for (11) owing to the probability of failure given by π_g. Similarly, an individual with a high school diploma must decide whether to pay γc_t and give up (13) in exchange for (12) given π_b. Of course both of these individuals also possess options as well. The individual with the high school diploma, for example, possesses the option to pursue an undergraduate degree and the embedded option to pursue graduate study. In the next section, we undertake the valuation of these options.

2.3 Contingent claims framework

Since the graduate option must be valued first, let $F_b(g_t, c_t)$ represent the value of undergraduate earnings and the value of the option to pursue an advanced degree possessed by an individual with an undergraduate degree. Mathematically, we have

$$(14) \quad F_t^b(g_t, c_t) = B_t(\alpha_b g_t) + V_t^g(g_t, c_t, q_t).$$

where $B_t(\alpha_b g_t)$ is given by (8). In (14) V_t^g represents the value of the option to obtain an advanced degree and necessarily depends on the graduate salary, g_t, the cost of obtaining the degree, c_t, and the Poisson variable that determines the likelihood of failure ($dq_t = 1$) or success ($dq_t = 0$).

The fundamental valuation equation for V_t^g can be found in a manner similar to the approach outlined above. Since the option pays no intermediate cash flows, we equate the expected (risk-neutral) capital gain on V_t^g with the risk free return on the option. More clearly,

(15) $\mathbb{E}^{\mathbb{Q}}\left(dV_t^g\right) = rV_t^g dt$

Applying Ito's lemma to V_t^g to compute the capital gain results in

(16) $$dV_t^g = \frac{\partial V_t^g}{\partial g_t} dg_t + \frac{\partial V_t^g}{\partial c_t} dc + \frac{1}{2}\left(\frac{\partial^2 V_t^g}{\partial {g_t}^2} d{g_t}^2 + 2\frac{\partial^2 V_t^g}{\partial g_t \partial c_t} dg_t dc_t + \frac{\partial^2 V_t^g}{\partial {c_t}^2} d{c_t}^2\right) - V_t^g dq.$$

Notice the last term in equation (16) reflects the complete loss of option value if the Poisson event happens implying failure. By substituting (1) and (2) into (16) and taking the expectation of the result, (15) can be expressed as

(17) $$\left(\frac{{\sigma_g}^2 g_t^2}{2}\right)\frac{\partial^2 V_t^g}{\partial g_t^2} + g_t c_t \sigma_g \sigma_c \rho_{gc} \frac{\partial^2 V_t^g}{\partial g_t \partial c_t} + \left(\frac{{\sigma_c}^2 c_t^2}{2}\right)\frac{\partial^2 V_t^g}{\partial c_t^2} + \delta_g g_t \frac{\partial V_t^g}{\partial g_t} + \delta_c c_t \frac{\partial V_t^g}{\partial c_t} - \xi_g V_t^g = 0,$$

where ρ_{gc} is the instantaneous correlation between the two stochastic state variables and $\xi_g = r + \pi_g$.

Equation (17) is a second-order, linear, elliptical PDE, the solution of which is the value of the option to obtain an advanced degree. There are essentially three boundary conditions required to solve (17). First, if the salary associated with getting a graduate degree ever fell to zero, the option to obtain the degree is worthless. Next, recall that when $k_t^g \geq k_g^*$ there is a financial incentive to exercise the option to obtain the graduate degree by paying the cost of obtaining the degree and giving up the present value of baccalaureate income in exchange for the present value of graduate income. Lastly, there must be a smooth transition at the boundary where exercising the graduate option is optimal. These conditions are operationalized in the next section.

2.4 Model solution

In what follows, the time subscript is suppressed where no confusion can arise thereby easing the notation somewhat. To solve (17), we use $k_g = g/c$ and let $\hat{V}(k_g) = V_g/c$ so that

$$(18)\quad \left(\frac{\sigma^2 k_g^2}{2}\right)\left(\frac{d^2\hat{V}_g}{dk_g^2}\right) + (\delta_g - \delta_c)k_g\left(\frac{d\hat{V}_g}{dk_g}\right) - (\xi_g - \delta_c)\hat{V}_g = 0,$$

results. Equation (18) is a second-order, linear, ODE where $\sigma^2 = \sigma_g^2 - 2\sigma_g\sigma_c\rho_{gc} + \sigma_c^2$. The relevant boundary conditions discussed above for (17) expressed in terms of the transformed equation (18) are

$$\hat{V}_g(0) = 0 \qquad (19.1)$$

$$(19)\quad \hat{V}_g(k_g \geq k_g^*) = \theta_g k_g^* - 1 \qquad (19.2)$$

$$\hat{V}_g{}'(k_g^*) = \theta_g \qquad (19.3)$$

where $\theta_g = \frac{1}{v_g - \delta_g} - \frac{\alpha_b}{r - \delta_g}$.

By solving the ODE (18) using the boundary conditions in (19) the solution to (17) when $k_g < k_g^*$ can be written as

$$(20)\quad V_g = g\Phi\, \theta_g{}^{\phi}\left(\frac{g}{c}\right)^{\phi - 1}$$

where $\Phi = \left(\frac{1}{\phi}\right)\left(\frac{\phi}{\phi - 1}\right)^{1-\phi}$ and

$$(21)\quad \phi = \frac{-\left(\delta_g - \delta_c - \sigma^2/_2\right) + \sqrt{\left(\delta_g - \delta_c - \sigma^2/_2\right)^2 + 2\sigma^2(\xi_g - \delta_c)}}{\sigma^2} > 1.$$

Therefore, the solution to F_b is

$$(22)\quad F_b = \begin{cases} \dfrac{\alpha_b g}{r - \delta_g} + g\Phi\,\theta_g{}^{\phi}\left(\dfrac{g}{c}\right)^{\phi-1} & k_g < k_g^* \qquad (22.1) \\[2ex] \dfrac{g}{v_g - \delta_g} - c & k_g \geq k_g^* \qquad (22.2) \end{cases}$$

Notice that equation (22) consists of two parts, one for each side of the boundary k_g^* where k_g^* is given by

$$(23)\quad k_g^* = \left(\frac{\phi}{\phi - 1}\right)\left(\frac{1}{\theta_g}\right).$$

As shown, the probability of failure affects the magnitude of k_g^*, as well as the option value. Notice also that in (23), if $\alpha_b = 1$, the individual with an undergraduate degree earns the same salary as an individual with an advanced degree. The implication is that $\theta_g = 0$, and $k_g^* = +\infty$ so that the threshold level that triggers the exercise of the graduate option will never be reached. Therefore $\alpha_b < 1$ is required for there to be a financial incentive to pursue an advanced degree.

Having analytically solved for the value of an undergraduate degree with the option to pursue a graduate degree, it remains to determine the value of a high school diploma with the option to get a bachelor's degree and the embedded option of a graduate degree. Here we take an approach similar to that presented above and let $F_h(b, u)$ represent the present value of the wage for an individual with a high school diploma along with the two option values so that

$$(24) \quad F_t^h(b_t, u_t) = \frac{\alpha_h b_t}{r - \delta_g} + V_t^b(b_t, u_t) + V_t^g\left(\frac{b_t}{\alpha_b}, \frac{u_t}{\gamma}\right).$$

Here, V_t^b represents the value of the option to obtain an undergraduate degree and V_t^g is given by (20) above and represents the value of the (compound) option to pursue a graduate degree.

Recall, that $b_t = \alpha_b g_t$ and $u_t = \gamma c_t$ so that V_t^b, being dependent on the same set of state variables as V_t^g, evolves analogously to (17). Suppressing the time subscript as before, an equation analogous to (18) results from letting $k_b = b/u$ and $\hat{V}_b(k_b) = V_b/u$. More clearly,

$$(25) \quad \left(\frac{\sigma^2 k_b^2}{2}\right)\left(\frac{d^2\hat{V}_b}{dk_b^2}\right) + \left(\delta_g - \delta_c\right)k_b\left(\frac{d\hat{V}_b}{dk_b}\right) - (\xi_b - \delta_c)\hat{V}_b = 0,$$

is a second-order, linear, ODE with $\xi_b = r + \pi_b$ and π_b represents the probability of failure to obtain an undergraduate degree.

Even though V_b and V_g depend on the same set of state variables, one of the boundary conditions for (25) is different than those for (18). In particular,

$$(26) \quad \begin{aligned} &\hat{V}_b(0) = 0 && (26.1)\\ &\hat{V}_b(k_b \geq k_b^*) = \theta_b k_b^* + \left(\frac{\Phi}{\gamma}\right)\left(\frac{\gamma}{\alpha_b}\right)^{\phi} \theta_g^{\phi} {k_b^*}^{\phi} - 1 && (26.2)\\ &\hat{V}_b{}'(k_b^*) = \theta_b + \phi\left(\frac{\Phi}{\gamma}\right)\left(\frac{\gamma}{\alpha_b}\right)^{\phi} \theta_g^{\phi} {k_b^*}^{\phi-1} && (26.3) \end{aligned}$$

where $\theta_b = \frac{1}{v_b - \delta_g} - \frac{\alpha_h}{r - \delta_g}$ and all other parameters are as defined previously.

The boundary condition (26.1) suggests that if undergraduate salary falls to zero, the option to obtain the degree is worthless and is analogous to (19.1) above for the graduate degree. However, (26.2) is different than (19.2) in that exercising the undergraduate option results in the present value of undergraduate income (after giving up the present value of the high school wage and paying the cost of education) but also yields the graduate option given by the rightmost term in (26.1). Equation (26.3) is the high contact condition that is analogous to (19.3).

Given this, the solution for $F_h(b,u)$ is

$$(27) \quad F_h(b,u) = \begin{cases} \dfrac{\alpha_h b}{r-\delta_g} + \Omega\left(\dfrac{b}{u}\right)^{\psi} u + \Phi\left(\dfrac{b\theta_g}{\alpha_b}\right)^{\phi}\left(\dfrac{\gamma}{u}\right)^{\phi-1} & k_b < k_b^* \quad (27.1) \\ \dfrac{b}{v_b-\delta_g} + \Phi\left(\dfrac{b\theta_g}{\alpha_b}\right)^{\phi}\left(\dfrac{\gamma}{u}\right)^{\phi-1} - u & k_b \geq k_b^*. \quad (27.2) \end{cases}$$

In (27), $\Omega = \theta_b k_b^{*\,1-\psi} + \left(\frac{\Phi}{\gamma}\right)\left(\frac{\gamma}{\alpha_b}\right)^{\phi} \theta_g k_b^{*\,\phi-\psi} - k_b^{*\,-\psi}$ with

$\psi = \frac{-\left(\delta_g-\delta_c-\sigma^2/2\right)+\sqrt{\left(\delta_g-\delta_c-\sigma^2/2\right)^2+2\sigma^2(\xi_b-\delta_c)}}{\sigma^2}$>1 and the threshold level above which the option to obtain an undergraduate degree is exercised is given by

$$(28) \quad k_b^* = \frac{\psi}{(\psi-1)\theta_b - (\phi-\psi)\left(\frac{\Phi}{\gamma}\right)\left(\frac{\gamma}{\alpha_b}\right)^{\phi}\theta_g^{\phi}}.$$

Notice that if the undergraduate salary-to-cost ratio is low, the value of a high school diploma is equal to the capitalized wage associated with a high school diploma plus the value of the undergraduate option and the embedded graduate option that can only be exercised after completing the undergraduate degree. The third term in (27.1) in this case is the value of the graduate option found in equation (20) and rewritten in terms of the *b* and *u* state variables. When the undergraduate salary-to cost ratio is high, the option to pursue the undergraduate degree is exercised, and the value of the high school diploma is given by the capitalized earnings for a college graduate (adjusted for the probability of failure) plus the value of the option to obtain a graduate degree less the cost of the undergraduate education. If $\pi_b = \pi_g$, then $\alpha_h < \alpha_b$ ensures that $k_b^* < k_g^*$ and the options are exercised sequentially as required. In the more likely case that $\pi_b \neq \pi_g$ conditions ensuring that $k_b^* < k_g^*$ are not readily apparent. Therefore, we defer this aspect of the model to the empirical section of the paper.

3 Data and Empirical Application

To demonstrate the model in an empirical setting, we first make a few assumptions that will enable us to determine the magnitude of the market price of risk as it relates to the two sources of uncertainty modeled. To that end, we assume the existence of two freely traded securities, m_t and n_t, such that: (1) risk uncorrelated with changes in either security is not priced, (2) neither security has any cash payouts associated with them, and (3) each security follows geometric Brownian motion with drift α_j and volatility σ_j where $j = m, n$. Given these three basic assumptions and the constant risk free rate assumed above, it follows that $\lambda_i = \left(\frac{\alpha_j - r}{\sigma_j}\right)\rho_{ij}$ for $i = g, c$ and $j = m, n$. Alternatively, define β_{ij} as the beta between the state variable i and the security j. It follows that $\beta_{ij} = \frac{\rho_{ij}\sigma_i}{\sigma_j}$ so that $\lambda_i = \left(\frac{\alpha_j - r}{\sigma_i}\right)\beta_{ij}$. In either case, these assumptions allow us to express the unknown market price of risk for each state variable as a function of observable parameters associated with the two securities m_t and n_t. This also implies that the drift terms shown in equations (1) and (2) above are now given by $\delta_i = \mu_i - (\alpha_j - r)\beta_{ij}$ for $i = g, c$ and $j = m, n$.

The data used for the empirical application are a combination of assumed values and actual data collected from 2010 U.S. Census data and *2011 Trends in College Pricing*. According to the Census data, the median salary for individuals with a high school diploma working full-time was \$30,627 while that for a 4-year bachelor's degree was \$56,665 in 2009. Advanced degrees could be at the Master's, Doctoral, or Professional levels in this study. For no particular reason we chose the M.S. degree which had an estimated median salary for full-time workers of \$73,738 in 2009. Given these salaries, the coefficients that determine the fractional level of the high school wage and bachelor's degree are then: $\alpha_h = 0.4153$ and $\alpha_b = 0.7685$.

The probabilities of failure are arbitrarily set at 10% (probability of undergraduate failure) and 5% (probability of graduate failure). Sensitivity analysis around these values suggests that the magnitude of these probabilities, in general, has a significant impact on the threshold values that determine optimal exercise. For example, given the other data and parameter values in the model, a graduate probability of failure in excess of 6% suggests that the graduate option will never be exercised (i.e. infinite threshold value). This idea is explore more in more detail below.

For two reasons the volatilities are assumed values and are set at 10% for salaries and 20% for tuition and fees. First, reported income and tuition data are gross averages and therefore lack the variation required to generate meaningful volatility estimates. For example, *2011 Trends in College Pricing* data show that tuition and fees have increased on average about 7.5% annually over the last 30 years at public four-year colleges. However, the average volatility computed from the average rates of growth is only about 2.5% per year over those years. This is because computing the volatility from the average rate of growth in this way dilutes the actual volatility. For example, California recently increased tuition and fees by 21% at public four-year institutions. The second reason for assuming these values is to measure the impact these parameters have on the value of undergraduate and graduate degrees. If tuition and fees are expected to continue to increase, perhaps with more volatility, it would be useful to see what impact these increases have on threshold levels and earnings and option values.

The average rate of growth in graduate salary was estimated from 2010 Census data to be about 3% annually. This value is likely institution and degree program specific and, therefore, also suffers from the aggregation issues discussed above. As noted above, the average annual rate of growth in tuition and fees at four-year public colleges was estimated at 7.5% from data contained in *2011 Trend in College Pricing*. The risk free rate is assumed to be a constant 3% throughout and undergraduate tuition and fees are assumed to be twice that of graduate tuition and fees. The hedging securities are assumed to have rates of return equal to 10% and 15% with risk that is 1.5 times these levels. Perfect positive correlation is assumed between the hedging securities and the two state variables (i.e. $\rho_{ij} = 1$) while the correlation between the two state variables themselves (i.e. $\rho_{gc} = 0$) is assumed to be zero.

4 Results and Discussion

Shown in Tables 1 through 4 are example results of the model using the basic set of data presented above. Given the quality of the data discussed above, the results are more illustrative than anything but do give some sense of how the model preforms. Table 1 shows the value of a high school wage earner's income which includes their undergraduate and graduate option values (see equation (27)) while Table 2 shows the value of an individual's baccalaureate salary including their graduate option value (see equation (22)). In either case and as expected, the value increases as wage or salary increases and decreases as tuition and fees increase. In Tables 3 and 4, the option values associated with undergraduate (V_b) and graduate (V_g) degrees are presented. In the tables, a zero value indicates that the option would be exercised since the ratio of earnings-to-cost is above the respective threshold. Also as expected the option values increase as wage/salary increases and decrease as tuition and fees increase.

Unfortunately the values reported in Tables 1 through 4 are not comparable to any other publishes estimates. For example, work-life estimates reported by Julian and

Kominski (2011) are not comparable since their methodology doesn't recognize an educational cost, discounting (in any conventional sense), or future uncertainty related to salaries or the cost of education. In addition, their results are stratified by gender and ethnicity.

While the estimates here necessarily understate earnings due to the assumption of a perpetual employment horizon, the impact of the undergraduate and graduate degree optionality is not necessarily insignificant depending on the income and tuition levels assumed. For example, using the option value of an undergraduate degree presented in Table 3, an individual faced with the prospect of an undergraduate salary close to the 2009 median would likely not exercise their option if the cost of tuition and fees is more than about $30K. For example, if tuition and fees were $60K, the option is worth about $27.5K and would not be exercised. Further, an undergraduate salary of $57.6K/year with $60K in tuition and fees translates to a graduate salary of $75K/year with graduate tuition of $30K. The graduate option in this case is worth an addition $22.7K. Therefore, there is about $50K in optionality that would not be accounted for using, for example, work-life estimates methodology.

Shown in Figure 1 is the impact that the probability of failure at both the undergraduate (π_g) and graduate (π_g) levels has on the undergraduate threshold value (k_b^*) that defines the optimal exercise boundary. As shown, the relationship is nonlinear with the threshold increasing at an increasing rate as the probability of failure increases. The implication is the intuitive result that there is less likelihood that the undergraduate option will be exercised the higher the probability of undergraduate failure. However, the relationship is most pronounced when there is a high probability of graduate failure. A probability of graduate failure in excess of 5% or so will nearly ensure that the undergraduate option is never exercised when the probability of undergraduate failure is around 10% or greater. For example, a graduate (undergraduate) failure probability of 5% (10%) would imply a salary-to-cost ratio in excess of 2.2 to justify exercising the option to go to college.

In general, there is a complex nonlinear relationship between the value of the undergraduate option and tuition and fee volatility. This relationship is shown in Figure 2 for the base values used to generate Tables 1 to 4, assuming $50K in tuition and fees and an undergraduate salary of $38.4K. Two cases are shown each with a probability of graduate failure equal to 5%, but with two levels for the probability of undergraduate failure (10%

vs. 7.5%). As shown, the value of the undergraduate option is zero for low levels of volatility as it is optimal to exercise the option in these cases. When the probability of undergraduate failure is 10%, the option takes on positive value (i.e. is not exercised) at a level of volatility approximately equal to 5.5% and declines rapidly as volatility increases. At a volatility level of approximately 18.7%, the option value is at a minimum and begins to increase for higher levels of volatility. It is highly nonlinear relationship between the probabilities of failure and the exercise boundary that induce this type of nonlinearity. For example, when the probability of undergraduate failure is lower at 7.5%, *ceteris paribus*, the option is exercised and therefore has no value for volatility levels less than about 20.9%. Volatility in excess of 20.9% suggests positive option values that are increasing with volatility.

Although no specific results are presented, option and earnings values are inversely related to the correlation between salary and tuition. Recall, the base values reported in Table 1 are consistent with zero correlation. This implies that positive (negative) correlation between salary and tuition would decrease (increase) the value of the option to obtain undergraduate and graduate degrees and earnings value. By contrast, option values are positively related to the risk free rate while earnings values are negatively related to the risk free rate. This implies that if the risk free rate increases there is an ambiguous effect on the overall value of earnings.

5 Summary and Conclusions

In this paper, an analytic model is developed to value an individual's earnings under alternative levels of education. The model is sensitive to the stochastic nature of earnings and the financial cost of obtaining a college education. In addition, a key feature of the model is the idea that capitalized earnings is insufficient for determining value because an individual with a high school diploma has the real option to attempt earning an undergraduate degree and the embedded real option of earning an advanced degree. Similarly, an individual with an undergraduate degree has the real option to get an advanced degree. The latter option is a compound option and if exercised, is done so in a sequential manner. The options are shown to have considerable value which implies that lifetime earnings estimates necessarily understate the value of higher education.

There are alternatives and refinements to the approach taken here that would likely capture some of the other realities facing individual's making educational decisions. For example, one could model the sequential options on a semester by semester basis. That is, successful completion of a semester of coursework gives an individual the right but not the obligation to undertake a subsequent semester. Alternatively, recognizing the fact that it takes time to complete a degree, one could model degree attainment with a stochastic completion time and therefore an uncertain cost until the degree is actually completed. In either case, the individual's option to abandon their studies could be expressly captured in such a model. In addition, a different salary specification may allow for a model that captures the switching option associated with changing majors.

Better data would also represent a significant improvement to the empirical analysis presented here. Institution specific data on tuition and fees, salaries for graduates of a specific degree program, and the probability of failure specific to a particular degree program and institution would greatly improve the empirical applicability of the approach. In addition, with such data, it may be possible to compare and rank colleges/majors in terms of the earnings valuation of their graduates.

Table 1. Present value of perpetual wage plus the value of undergraduate and graduate options, F_h.

							$u_t = \gamma c_t$						
$h_t = \alpha_h g_t$	***$10,000***	***$20,000***	***$30,000***	***$40,000***	***$50,000***	***$60,000***	***$70,000***	***$80,000***	***$90,000***	***$100,000***	***$110,000***	***$120,000***	***$130,000***
$9,575	$385,820	$220,316	$219,375	$214,296	$211,663	$210,096	$209,075	$208,367	$207,852	$207,464	$207,162	$206,923	$206,729
$11,171	$451,790	$262,503	$260,408	$252,854	$248,945	$246,622	$245,112	$244,064	$243,303	$242,730	$242,285	$241,932	$241,646
$12,767	$517,760	$306,962	$289,996	$292,500	$286,993	$283,723	$281,599	$280,128	$279,059	$278,255	$277,631	$277,136	$276,735
$14,363	$583,730	$353,933	$330,474	$333,325	$325,866	$321,443	$318,572	$316,585	$315,144	$314,059	$313,218	$312,551	$312,011
$15,959	$649,700	$403,647	$372,301	$375,414	$365,625	$359,827	$356,065	$353,464	$351,578	$350,159	$349,061	$348,189	$347,484
$17,555	$715,670	$456,330	$415,588	$399,993	$406,327	$398,916	$394,111	$390,791	$388,385	$386,576	$385,175	$384,065	$383,167
$19,151	$781,640	$771,640	$460,444	$440,632	$448,027	$438,750	$432,741	$428,591	$425,585	$423,325	$421,577	$420,191	$419,071
$20,747	$847,610	$837,610	$506,974	$482,282	$470,087	$479,371	$471,986	$466,888	$463,197	$460,425	$458,280	$456,581	$455,207
$22,343	$913,580	$903,580	$555,282	$525,007	$510,054	$520,815	$511,875	$505,707	$501,243	$497,891	$495,298	$493,245	$491,586
$23,939	$979,550	$969,550	$605,470	$568,868	$550,790	$563,121	$552,438	$545,070	$539,740	$535,739	$532,646	$530,196	$528,218
$25,534	$1,045,520	$1,035,520	$657,638	$613,925	$592,335	$579,992	$593,702	$585,001	$578,708	$573,985	$570,335	$567,446	$565,113
$27,130	$1,111,490	$1,101,490	$711,884	$660,238	$634,730	$620,146	$635,696	$625,520	$618,163	$612,645	$608,381	$605,006	$602,282
$28,726	$1,177,460	$1,167,460	$1,157,460	$707,865	$678,014	$660,948	$650,212	$666,650	$658,126	$651,732	$646,794	$642,887	$639,733
$30,322	$1,243,430	$1,233,430	$1,223,430	$756,865	$722,227	$702,423	$689,967	$708,412	$698,612	$691,263	$685,588	$681,099	$677,476
$31,918	$1,309,400	$1,299,400	$1,289,400	$807,293	$767,406	$744,602	$730,257	$750,828	$739,638	$731,251	$724,775	$719,653	$715,521
$33,514	$1,375,370	$1,365,370	$1,355,370	$859,206	$813,590	$787,510	$771,106	$760,067	$781,223	$771,710	$764,367	$758,561	$753,877
$35,110	$1,441,340	$1,431,340	$1,421,340	$912,660	$860,816	$831,176	$812,532	$799,987	$823,381	$812,654	$804,376	$797,831	$792,552
$36,706	$1,507,309	$1,497,309	$1,487,309	$1,477,309	$909,122	$875,626	$854,557	$840,380	$830,350	$854,098	$844,813	$837,474	$831,556
$38,302	$1,573,279	$1,563,279	$1,553,279	$1,543,279	$958,543	$920,887	$897,201	$881,263	$869,987	$896,053	$885,690	$877,501	$870,898
$39,898	$1,639,249	$1,629,249	$1,619,249	$1,609,249	$1,009,116	$966,986	$940,485	$922,653	$910,037	$938,535	$927,019	$917,920	$910,585

Table 2. Present value of perpetual salary plus the value of the graduate option, F_b.

							c_t						
$b_t = \alpha_b g_t$	***$5,000***	***$10,000***	***$15,000***	***$20,000***	***$25,000***	***$30,000***	***$35,000***	***$40,000***	***$45,000***	***$50,000***	***$55,000***	***$60,000***	***$65,000***
$23,054	$510,079	$505,079	$500,762	$498,625	$497,446	$496,710	$496,213	$495,856	$495,590	$495,385	$495,223	$495,091	$494,983
$26,896	$595,926	$590,926	$586,003	$582,947	$581,260	$580,207	$579,495	$578,986	$578,605	$578,312	$578,079	$577,891	$577,736
$30,739	$681,772	$676,772	$671,772	$667,682	$665,382	$663,946	$662,976	$662,281	$661,762	$661,361	$661,044	$660,787	$660,576
$34,581	$767,619	$762,619	$757,619	$752,849	$749,825	$747,938	$746,662	$745,748	$745,066	$744,539	$744,122	$743,785	$743,507
$38,423	$853,465	$848,465	$843,465	$838,465	$834,603	$832,192	$830,562	$829,395	$828,523	$827,851	$827,318	$826,887	$826,532
$42,266	$939,312	$934,312	$929,312	$924,312	$919,726	$916,718	$914,683	$913,227	$912,140	$911,300	$910,636	$910,098	$909,655
$46,108	$1,025,158	$1,020,158	$1,015,158	$1,010,158	$1,005,205	$1,001,523	$999,033	$997,251	$995,920	$994,893	$994,079	$993,421	$992,879
$49,950	$1,111,005	$1,106,005	$1,101,005	$1,096,005	$1,091,005	$1,086,617	$1,083,618	$1,081,471	$1,079,868	$1,078,631	$1,077,652	$1,076,859	$1,076,206
$53,792	$1,196,852	$1,191,852	$1,186,852	$1,181,852	$1,176,852	$1,172,006	$1,168,444	$1,165,894	$1,163,990	$1,162,520	$1,161,357	$1,160,415	$1,159,639
$57,635	$1,282,698	$1,277,698	$1,272,698	$1,267,698	$1,262,698	$1,257,698	$1,253,517	$1,250,523	$1,248,288	$1,246,563	$1,245,197	$1,244,092	$1,243,182
$61,477	$1,368,545	$1,363,545	$1,358,545	$1,353,545	$1,348,545	$1,343,545	$1,338,842	$1,335,364	$1,332,768	$1,330,764	$1,329,177	$1,327,893	$1,326,835
$65,319	$1,454,391	$1,449,391	$1,444,391	$1,439,391	$1,434,391	$1,429,391	$1,424,425	$1,420,422	$1,417,432	$1,415,125	$1,413,298	$1,411,820	$1,410,603
$69,162	$1,540,238	$1,535,238	$1,530,238	$1,525,238	$1,520,238	$1,515,238	$1,510,238	$1,505,699	$1,502,285	$1,499,651	$1,497,564	$1,495,876	$1,494,486
$73,004	$1,626,084	$1,621,084	$1,616,084	$1,611,084	$1,606,084	$1,601,084	$1,596,084	$1,591,201	$1,587,330	$1,584,343	$1,581,978	$1,580,063	$1,578,487
$76,846	$1,711,931	$1,706,931	$1,701,931	$1,696,931	$1,691,931	$1,686,931	$1,681,931	$1,676,931	$1,672,570	$1,669,206	$1,666,541	$1,664,384	$1,662,608
$80,689	$1,797,777	$1,792,777	$1,787,777	$1,782,777	$1,777,777	$1,772,777	$1,767,777	$1,762,777	$1,758,009	$1,754,241	$1,751,256	$1,748,841	$1,746,852
$84,531	$1,883,624	$1,878,624	$1,873,624	$1,868,624	$1,863,624	$1,858,624	$1,853,624	$1,848,624	$1,843,650	$1,839,452	$1,836,126	$1,833,435	$1,831,219
$88,373	$1,969,470	$1,964,470	$1,959,470	$1,954,470	$1,949,470	$1,944,470	$1,939,470	$1,934,470	$1,929,470	$1,924,841	$1,921,153	$1,918,170	$1,915,713
$92,216	$2,055,317	$2,050,317	$2,045,317	$2,040,317	$2,035,317	$2,030,317	$2,025,317	$2,020,317	$2,015,317	$2,010,410	$2,006,340	$2,003,047	$2,000,334
$96,058	$2,141,164	$2,136,164	$2,131,164	$2,126,164	$2,121,164	$2,116,164	$2,111,164	$2,106,164	$2,101,164	$2,096,164	$2,091,688	$2,088,067	$2,085,085

Table 3. The option value of an undergraduate degree, V_b.

							$u_t = \gamma c_t$						
$b_t = \alpha_b g_t$	***$10,000***	***$20,000***	***$30,000***	***$40,000***	***$50,000***	***$60,000***	***$70,000***	***$80,000***	***$90,000***	***$100,000***	***$110,000***	***$120,000***	***$130,000***
$23,054	$0	$15,128	$7,439	$4,495	$3,041	$2,210	$1,687	$1,336	$1,087	$904	$765	$657	$571
$26,896	$0	$23,118	$11,367	$6,869	$4,648	$3,378	$2,579	$2,041	$1,661	$1,381	$1,169	$1,004	$872
$30,739	$0	$33,379	$16,412	$9,918	$6,710	$4,877	$3,723	$2,947	$2,398	$1,994	$1,687	$1,449	$1,260
$34,581	$0	$46,151	$22,692	$13,713	$9,278	$6,743	$5,148	$4,075	$3,315	$2,757	$2,333	$2,003	$1,741
$38,423	$0	$61,667	$30,322	$18,323	$12,398	$9,010	$6,878	$5,444	$4,430	$3,684	$3,118	$2,677	$2,327
$42,266	$0	$80,153	$39,411	$23,816	$16,114	$11,710	$8,940	$7,077	$5,758	$4,788	$4,052	$3,480	$3,025
$46,108	$0	$0	$50,068	$30,257	$20,471	$14,877	$11,358	$8,990	$7,315	$6,083	$5,148	$4,420	$3,842
$49,950	$0	$0	$62,400	$37,709	$25,514	$18,541	$14,156	$11,205	$9,117	$7,581	$6,416	$5,509	$4,789
$53,792	$0	$0	$76,511	$46,236	$31,283	$22,734	$17,357	$13,738	$11,178	$9,295	$7,867	$6,755	$5,872
$57,635	$0	$0	$92,501	$55,899	$37,821	$27,485	$20,984	$16,609	$13,514	$11,238	$9,511	$8,167	$7,099
$61,477	$0	$0	$110,471	$66,758	$45,168	$32,825	$25,060	$19,836	$16,140	$13,421	$11,358	$9,753	$8,478
$65,319	$0	$0	$130,519	$78,873	$53,365	$38,782	$29,608	$23,436	$19,069	$15,857	$13,420	$11,523	$10,016
$69,162	$0	$0	$0	$92,303	$62,451	$45,385	$34,650	$27,426	$22,316	$18,556	$15,704	$13,485	$11,722
$73,004	$0	$0	$0	$107,104	$72,466	$52,663	$40,206	$31,824	$25,894	$21,532	$18,223	$15,648	$13,602
$76,846	$0	$0	$0	$123,334	$83,447	$60,643	$46,299	$36,647	$29,818	$24,795	$20,984	$18,019	$15,663
$80,689	$0	$0	$0	$141,050	$95,433	$69,354	$52,949	$41,911	$34,101	$28,356	$23,998	$20,607	$17,913
$84,531	$0	$0	$0	$160,305	$108,462	$78,821	$60,177	$47,632	$38,756	$32,228	$27,275	$23,420	$20,358
$88,373	$0	$0	$0	$0	$122,569	$89,074	$68,005	$53,828	$43,797	$36,419	$30,822	$26,467	$23,006
$92,216	$0	$0	$0	$0	$137,793	$100,137	$76,451	$60,513	$49,237	$40,943	$34,650	$29,754	$25,863
$96,058	$0	$0	$0	$0	$154,168	$112,037	$85,536	$67,705	$55,088	$45,808	$38,768	$33,290	$28,937

Table 4. The option value of a graduate degree, V_g.

							c_t						
g_t	***$5,000***	***$10,000***	***$15,000***	***$20,000***	***$25,000***	***$30,000***	***$35,000***	***$40,000***	***$45,000***	***$50,000***	***$55,000***	***$60,000***	***$65,000***
$30,000	$0	$0	$6,749	$4,613	$3,434	$2,698	$2,200	$1,844	$1,578	$1,373	$1,210	$1,078	$970
$35,000	$0	$0	$9,655	$6,599	$4,912	$3,859	$3,147	$2,638	$2,257	$1,964	$1,731	$1,543	$1,388
$40,000	$0	$0	$0	$8,999	$6,699	$5,263	$4,292	$3,597	$3,078	$2,678	$2,360	$2,104	$1,892
$45,000	$0	$0	$0	$11,831	$8,807	$6,919	$5,643	$4,729	$4,047	$3,520	$3,103	$2,766	$2,488
$50,000	$0	$0	$0	$15,111	$11,248	$8,838	$7,207	$6,040	$5,169	$4,496	$3,964	$3,533	$3,178
$55,000	$0	$0	$0	$0	$14,036	$11,028	$8,994	$7,537	$6,450	$5,611	$4,946	$4,408	$3,965
$60,000	$0	$0	$0	$0	$17,180	$13,498	$11,008	$9,226	$7,895	$6,867	$6,054	$5,396	$4,854
$65,000	$0	$0	$0	$0	$0	$16,256	$13,257	$11,111	$9,508	$8,271	$7,291	$6,498	$5,845
$70,000	$0	$0	$0	$0	$0	$19,310	$15,748	$13,198	$11,294	$9,824	$8,661	$7,719	$6,943
$75,000	$0	$0	$0	$0	$0	$22,667	$18,485	$15,492	$13,257	$11,532	$10,166	$9,061	$8,150
$80,000	$0	$0	$0	$0	$0	$0	$21,475	$17,998	$15,401	$13,397	$11,810	$10,526	$9,468
$85,000	$0	$0	$0	$0	$0	$0	$24,722	$20,719	$17,730	$15,423	$13,596	$12,118	$10,900
$90,000	$0	$0	$0	$0	$0	$0	$0	$23,661	$20,247	$17,613	$15,527	$13,838	$12,448
$95,000	$0	$0	$0	$0	$0	$0	$0	$26,828	$22,957	$19,970	$17,604	$15,690	$14,114
$100,000	$0	$0	$0	$0	$0	$0	$0	$30,222	$25,862	$22,497	$19,832	$17,676	$15,900
$105,000	$0	$0	$0	$0	$0	$0	$0	$0	$28,965	$25,197	$22,212	$19,797	$17,808
$110,000	$0	$0	$0	$0	$0	$0	$0	$0	$32,271	$28,072	$24,747	$22,056	$19,840
$115,000	$0	$0	$0	$0	$0	$0	$0	$0	$0	$31,126	$27,439	$24,455	$21,998
$120,000	$0	$0	$0	$0	$0	$0	$0	$0	$0	$34,360	$30,290	$26,996	$24,284
$125,000	$0	$0	$0	$0	$0	$0	$0	$0	$0	$37,778	$33,303	$29,682	$26,699

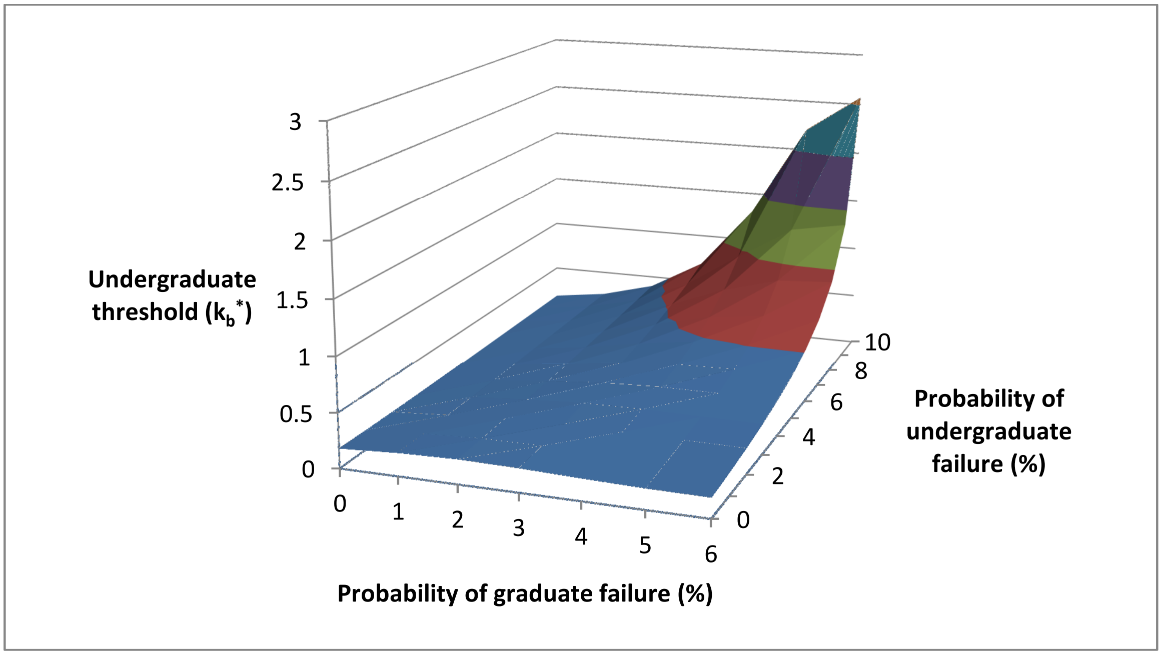

Figure 1. Effect of probabilities of failure on undergraduate exercise boundary.

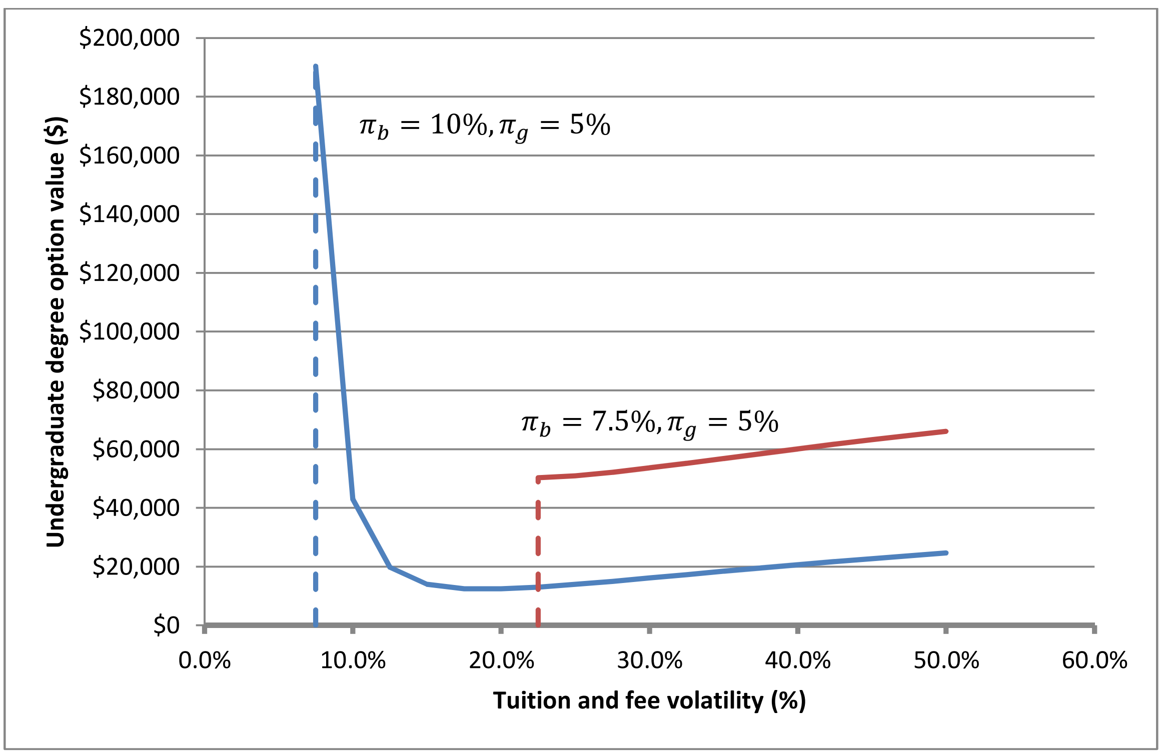

Figure 2. Effect of tuition and fee volatility on undergraduate option value.

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