Rivalry with Market and Efficiency Uncertainty in the Adoption of New Technologies

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Abstract

We derive a multi-factor pre-emption real options game model for a duopoly market where there is market and “efficiency after adoption” (“EAA”) uncertainty, and provide analytical or quasi-analytical solutions for the value functions and investment thresholds of the firms. We show that EAA uncertainty has an asymmetric effect on the firms’ investment behavior, delaying significantly the investment of the follower and only slightly the investment of the leader; a high positive correlation between “market revenue” and EAA delays slightly the investment of the leader and significantly the investment of the follower; and the size of the leader’s “first-mover market advantage” speeds up slightly the investment of the leader and delays significantly the investment of the follower.

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Key Words: Real Options, Pre-emption, Duopoly Game, Efficiency Uncertainty.

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1. Introduction

In some circumstances the efficiency (quality) of a new technology becomes apparent only after adoption. Consequently, the assumption that a new technology, after adoption, will perform, technically, as the developer/adopter predicts is not appropriate for some investment decisions. There are two main reasons for the existence of efficiency uncertainty in the adoption of a new technology. The first is due to the difficulty of fully testing some technologies before launch. The second, concerns the fact that the performance of some technologies after adoption may be dependent, at least to some extent, on the quality of the technical skills, human resources, organizational culture and management commitment of the adopting firm, characteristics that vary over time.

We relax the assumption often made in real option models which requires that once a technology is adopted its performance will be exactly as predicted (see Grenadier and Weiss, 1997, Huisman 2001, and Murto 2007). Events described in the press such as the delay in the construction of the new Airbus A380[3] and the more recent problem with one of its engines[4], and the appearance of cracks in the boiler pipes of the British Energy number 3 and number 4 nuclear power reactors[5], show that this assumption may not be realistic.

In this article we study the combined effect of market and efficiency (technical) uncertainty in the timing optimization of the adoption of a new technology. "Market uncertainty" represents the uncertainty of changes in demand, price and competition (for example, income, tastes, and the pricing decisions of competitors can change unpredictably, or a substitute product might arrive making the firm's product suddenly obsolete). "Efficiency uncertainty" is the uncertainty regarding the performance of a technology that persists after being adopted.

The efficiency of a technology after adoption can be quantified using the concept of "efficiency production frontier" ("EPF"), from the theory of industrial organization (see Aigner et al., 1997, and Coelli et al., 1998)[6]. This concept defines, for a current stage of technological development (state-of-the-art), an upper boundary for a firm production performance (efficiency). The EPF is only achieved in the ideal scenario where, after adoption, the technology operates without

[3] According to the information released, during the development of the Airbus A380 the delays in the project in late 2006 appear to be due to technical reasons (the great difficulty in integrating a huge number of new technologies).
[4] For details about the technical failure of the Rolls-Royce engine of the Quantas A380 superjumbo, and subsequent expected economic consequences for Rolls-Royce/Airbus, see The Economist, November 13, 2010, p. 80, and the Financial Times, December 5, 2010, p. 14.
[5] Technical problems with the British nuclear power reactors forced the company to take the power stations out of service for several months and, to avoid further cracks, the reactors were forced to operate in the future at a 70 percent load (see Financial Times, November 18, 2006, p. 1).
[6] Other methods can also be used, see for instance Slack and Lewis (2002) and Todinov, M. (2005).

technical imperfections and in a context of 100% human efficiency (if the technology needs human intervention to operate).

The "efficiency of the technology after adoption" ("EAA") is denoted here by $E(t)$, with $E(t) \in [0,1)$ and t continuous, where the lower limit represents a "catastrophic scenario", which occurs when after adoption the technology fails completely (operates with zero percent efficiency), and the upper limit represents a "perfect scenario", which occurs when after adoption the technology operates with100 percent efficiency. Between these two extreme scenarios there are, theoretically, an infinite number of feasible scenarios.[7]

The importance of each of the uncertainties above depends on the economic conditions underlying the investment decision and the type of technology involved. For instance, software programs and telecommunication technologies can be almost fully tested in a laboratory before launch and are, to some extent, independent of human intervention. Consequently, efficiency uncertainty is usually very low. Firms operating in the manufacturing, renewable energy, agriculture and mining sectors, for instance, are, usually, exposed to high technical uncertainty given that the efficiency of new equipments after adoption is, at least to some extent, human/natural resources-dependent and, therefore, cannot be fully tested before adoption. Our model is particularly useful for assessing investments on renewable energy technologies, such as waves and wind turbines and photovoltaic solar panel whose electricity production efficiency is dependent on the weather (wave/wind/sun) conditions.

Huisman (2001) considers market and technological uncertainty, but neglects technical uncertainty. Our model has also some similarities with that of Paxson and Pinto (2005), in the sense that both use two underlying variables following gBm processes; however, the underlying variables used, "price" and "quantity", relate only to market uncertainty. Smith (2005) studies the effect of revenues and investment cost uncertainty on the adoption of two complementary technologies, but neglects competition. Azevedo and Paxson (2011a), extends Smith's model to a duopoly market with a "first-mover market share advantage" ("FMA"). A good survey about the literature on new technology adoption models can be seen in Hoppe (2002). For an extensive literature review on real option game models see Azevedo and Paxson (2011b).

[7] In section 2, $E(t)$ is defined as following a geometric Brownian (gBm) process. Consequently, the domain $E(t) \in [0,1)$ above is inconsistent with a gBm. However, in reality, for most technologies/production processes, 100% (daily/monthly/annually) efficiency is rarely achieved.

This paper is organized as follows. In Section 2, we describe the model and derive the firm value functions and investment thresholds. In Section 3, we show some illustrative results and sensitivity analysis. In Section 4 we conclude.

2. The Model

In games of timing the adoption of new technologies, the potential advantage of being the first to adopt may introduce an incentive for pre-empting the rival, speeding up the investment. Reinganum (1981) develops a deterministic game-theoretic approach, where the adoption of one firm is assumed to have a negative effect on the profits of the other firm. The increase in profits due to the adoption is assumed to be greater for the leader than for the follower. Fudenberg and Tirole (1985) also study the adoption of a new technology and illustrate the effects of pre-emption in games of timing. We use the Fudenberg and Tirole (1985) principle of "rent equalization" in our derivations.

In a duopoly market, two idle firms are considering the adoption of a new technology in a context where there is uncertainty about both the "market revenues" and the EAA. The technology is available and the firm that invests first (the leader) gets a first-mover revenues "market share advantage". "Market uncertainty" is measured as the volatility of the market revenues, "efficiency uncertainty" as the volatility of the EAA. Due to spillover information firms are assumed to be symmetric regarding the EAA, i.e., the follower adopts the technology latter but from the instant it adopts onwards the EAA of both firms are the same. For the sake of mathematical tractability, without losing any insight, we assume that firms are not allowed to invest at the same time. To proceed with the new technology adoption both firms have to spend a sunk cost I .

Let $X(t)$ be the market revenues and $E(t)$ the EAA, with $X(t)$ expressed in monetary units and $E(t)$ dimensionless. Both $X(t)$ and $E(t)$ follow gBm processes given, respectively, by Equations (1) and (2): [8]

$$dX = \mu_X X dt + \sigma_X X dz_1 \tag{1}$$

$$dE = \mu_E E dt + \sigma_E E dz_2 \tag{2}$$

where, μ_X and μ_E are the instantaneous conditional expected percentage changes in X and E per unit of time, respectively; σ_X and σ_E are the instantaneous conditional standard deviation of X and E per unit of time, respectively; and dz_1 and dz_2 are the increment of a standard

[8] For simplicity of notation, hereafter we drop the *t*.

Wiener process for X and E, respectively. For convergence of the solution we assume $r-\mu_X-\mu_E>0$, where *r* is the riskless interest rate.

The firm's net revenue flow is given by:

$$\varphi de_{k_i k_j} \tag{3}$$

where, φ is the "Efficiency Weighted Revenues" ("EWR"), given by $\varphi=(X)(E)$; $de_{k_i k_j}$ is a deterministic competition factor that represents the proportion of the EWR assigned to each firm for each investment scenario, with $k=\{0,1\}$, where "0" and "1" means that the firm is idle and active, respectively; $i,j=\{L,F\}$, where L means "leader" and F "follower". [9]

The intuition underlying the first-mover "market share advantage" is the same as that used in Dixit and Pindyck (1994) following Smets (1993). Consequently, for the leader, inequality (4) holds:

$$de_{1_L 0_F} > de_{1_L 1_F} > de_{0_L 0_F} \tag{4}$$

Inequality (4) should be interpreted as follows: for the leader, the best investment scenario in terms of the market EWR share is when it operates with a new technology (denoted tech 1) alone ($de_{1_L 0_F}$); the second best investment scenario is when it adopts tech 1 and the follower does so later ($de_{1_L 1_F}$); the worst investment scenario is when it is idle with the follower ($de_{0_L 0_F}$).

The follower's market EWR share is a complement of the leader's, i.e., $de_{k_F k_L}=\left(1-de_{k_L k_F}\right)$, so for the follower inequality (5) holds:

$$de_{1_F 1_L} > de_{0_F 1_L} = de_{0_F 0_L} \tag{5}$$

[9] To get the intuition about how the multiplicative form $\varphi=(X)(E)$ works in practice, suppose that firm i adopts tech 1 first, becoming the leader, and firm j adopts tech 1 later, becoming the follower; the market revenues are 100 million ($X=100$), the EAA is 100 percent ($E=1$), and after the follower adoption the market shares of the leader and the follower, as a proportion of the market EWR, are 60 and 40 percent, respectively. In this case, the market EWR are $\varphi=(X)(E)=100(1)=100$ million, and the competition factors are $de_{1_L 1_F}=0.6$, for the leader, and $de_{1_F 1_L}=0.4$, for the follower. In terms of firms' EWR (Eq. 3) this leads to 60 million ($\varphi de_{1_L 1_F}=100(0.60)=60$), for the leader, and 40 million ($\varphi de_{1_F 1_L}=100(0.40)=40$), for the follower, a FMA of $60-40=20$ million.

In section 3, we use the following base inputs for the sensitivity analysis:

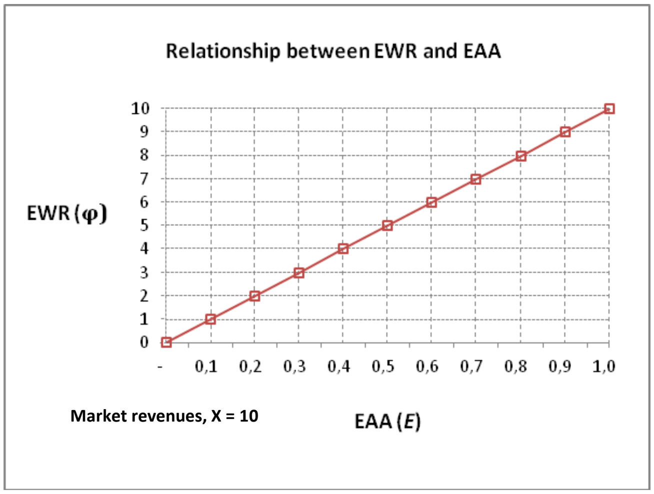
	Investment Scenario (Competition Factors: Proportion of the market EWR)		
	Both firms inactive	Leader active/Follower inactive	Both firms active
Leader's Market Share	$de_{0_L 0_F}$	$de_{1_L 0_F}$	$de_{1_L 1_F}$
	0.0	**1.0**	**0.60**
Follower's Market Share	$de_{0_F 0_L}$	$de_{0_F 1_L}$	$de_{1_F 1_L}$
	0.0	**0.0**	**0.40**
Total Market Share	**0.0**	**1.0**	**1.0**

Table 1: Deterministic Competition Factors

Inequalities (4) and (5), and the information in Table 1, ensure the following conditions: (i) when the leader is active alone it gets 100% of the market share; (ii) when both firms are active the leader's market share is higher than the follower's market share due to the FMA; (iii) at the instant the follower becomes active it gets 40% of the market share, i.e., the leader's market share drops from 100% to 60%; (iv) when both firms are active, the sum of their market shares is 100 percent; (v) when firms are inactive their market shares are null.

The relation between the competition factors above, inequalities (4) and (5) and the information in Table 1, should not be seen as static applying to all duopoly markets, technologies and industries, but as expected ex-post leader/follower market EWR share relations that need to be defined for each particular investment decision.

Figure 1 illustrates the linear relationship between EWR (φ) and EAA (E). The solid line is plotted by setting annual "market revenues" (X) equal to 10 million and changing the firms' annual E from 0% to 100%. Notice that X and E are assumed to be stochastic, so φ is also stochastic. For an annual market revenue of 10 million, if the E is null the φ are null; if the E is 100% the φ are 10 million (maximum). Between these two extremes, as E increases, φ increases linearly.



X	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**
E	**0**	**0.1**	**0.20**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**1**
φ	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0

Figure 1 – Relationship between EWR and EAA

In the current real options literature it assumed that the adopter knows, ex-ante, how the technology will perform once in place. Embedded in the adoption of a technology and its EAA is a certain level of output production (quantity) which once sold generates revenues. So the existence of uncertainty about the EAA of a technology introduces extra difficulties in the timing optimization of an investment decision in the sense that it is more difficult to plan the fitting of the production capacity to be installed with the market demand.

For an empirical illustration about how the variable EAA works in practice see Appendix A, sections 1a,b.

Figure 2 illustrates the investment thresholds for both firms:

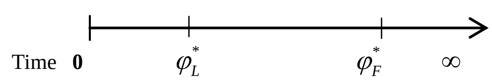


Figure 2 –Firms' investment thresholds

where, $\varphi_{1_L}^*$ and $\varphi_{1_F}^*$ are, respectively, the leader and the follower investment thresholds to adopt tech 1.

2.1 Follower

Let $F_F(X,E)$ be the follower's option value to adopt tech 1 for a context where the leader is active with tech 1. Setting the returns on the option equal to the expected capital gain on the option and using Ito's lemma, we obtain the partial differential equation (6) for the value function of the follower for the region where it waits to adopt tech 1.

$$\frac{1}{2}\frac{\partial^2 F_F}{\partial X^2}\sigma_x{}^2X^2+\frac{1}{2}\frac{\partial^2 F_F}{\partial E^2}\sigma_E{}^2E^2+\frac{\partial^2 F_F}{\partial X\partial E}XE\sigma_x\sigma_E\rho_{XE}+\frac{\partial F_F}{\partial X}\mu_x X+\frac{\partial F_F}{\partial E}\mu_E E-rF_F=0 \tag{6}$$

Using similarity methods [10] we can find an explicit closed-form solution for X and E, using the following change in the variables: $\varphi=(X)(E)$. Doing the respective substitutions in Equation (6) we get the ("Ordinary Differential Equation") ("ODE") (7), which describes the follower's option value as a function of φ:

$$\frac{1}{2}\varphi^2\sigma_m{}^2\frac{\partial^2 F_F(\varphi)}{\partial\varphi^2}+\varphi\left(\sigma_X\sigma_E\rho_{XE}+\mu_X+\mu_E\right)\frac{\partial F_F(\varphi)}{\partial\varphi}-rF_{F_{22}}(\varphi)=0 \tag{7}$$

where, $\sigma_m^2=\sigma_X{}^2+\sigma_E{}^2+2\rho_{XE}\sigma_X\sigma_E$.

See full derivation in Appendix B, section 1. Notice that the use of this technique implies that EAA is exogenous to firms.[11]

The ODE (7) has an analytical solution, whose general form is given by:

$$F_F(\varphi)=A\varphi^{\beta_1}+B\varphi^{\beta_2} \tag{8}$$

where, A and B are constants to be determined, using the boundary conditions ("value-matching" and "smooth-pasting" conditions), and β_1 and β_2 are the roots of a characteristic quadratic function of an Euler type ODE given by:

$$\frac{1}{2}\sigma_m^2\beta(\beta-1)+(\rho\sigma_X\sigma_E+\mu_X+\mu_E)\beta-r=0 \tag{9}$$

Solving the Equation (9) for β we get two roots, one positive, β_1, and one negative, β_2 given by:

$$\beta_{1(2)}=\frac{0.5\sigma_m^2-(\rho_{XE}\sigma_X\sigma_E+\mu_X+\mu_E)\pm\sqrt{\left(-0.5\sigma_m^2+\rho_{XE}\sigma_X\sigma_E+\mu_X+\mu_E\right)^2+2r\sigma_m^2}}{\sigma_m^2} \tag{10}$$

In order to find the follower's value, F_F, and investment threshold, φ_F^*, the following boundary conditions apply to Equation (8):

$$F_F(\varphi_F^*)=\frac{\varphi_F^* de_{1_F1_L}}{r-\mu_X-\mu_E}-I \tag{11}$$

[10] For a detailed discussion about similarity methods see Bluman and Cole (1974).

[11] This restricts the model to economic contexts where after adoption there is no learning. For instance, renewable energy production technologies, such as wind and wave turbines or photovoltaic solar panels, are technologies where EAA is exosgenous, i.e., there is not learning from operating with the technology: before adoption there is uncertainty about the quantity of wind, wave or sun; after adoption, the higher the quantity of wind, waves or sun per unit of time, the higher is the EAA, but firms can not influence the evolution of the EAA. There are other investments where similar assumption applies such as those in technologies used in some activities in the agriculture or mining sectors.

$$\frac{\partial}{\partial \varphi} F_F(\varphi_F^*) = \frac{de_{1_F 1_L}}{r - \mu_X - \mu_E} \qquad (12)$$

$$F_F(0) = 0 \qquad (13)$$

Conditions (11) and (12) are the "value-matching" and "smooth-pasting" conditions, respectively, and ensure continuity and differentiability of the value function at the investment threshold. Condition (13) ensures that the option value is worthless at the absorbing barrier $\varphi = 0$. Consequently, in equation (8) $B = 0$. Solving together Equations (8) and (11)-(13) after some algebraic manipulation yields the follower's investment threshold and value function, given by Equation (14) and Expression (15), respectively.

$$\varphi_F^* = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu_X - \mu_E)}{de_{1_F 1_L}} I \qquad (14)$$

with β_1 given by expression (10). The follower's value function is given by,

$$F_F(\varphi) = \begin{cases} A\varphi^{\beta_1} & \varphi < \varphi_F^* \\ \dfrac{\varphi de_{1_F 1_L}}{r - \mu_X - \mu_E} - I & \varphi \geq \varphi_F^* \end{cases} \qquad (15)$$

with

$$A = \frac{de_{1_F 1_L}}{r - \mu_X - \mu_E} \frac{\varphi_F^{*(1-\beta_1)}}{\beta_1} \qquad (16)$$

2.2 Leader

Assuming that the follower adopts tech 1 as soon as φ_F^* is reached, at the instant (τ) the leader adopts, its payoff is given by,

$$E\left[\int_{t=\tau}^{T_{1_F}} \varphi de_{1_L 0_F} e^{-r\tau} d\tau - I + \int_{T_{1_F}}^{\infty} \varphi_F^* de_{1_L 1_F} e^{-r\tau} d\tau\right] \qquad (17)$$

The first integral represents the leader's EWR for the period where it is active alone; the second integral represents the leader's EWR for the period where both firms are active with tech 1; I is the investment cost. Applying the methodology used in Dixit and Pindyck (1994), pp. 309-315, we get the expression (18) for the leader's value function.

$$F_L(\varphi) = \begin{cases} \dfrac{\varphi de_{1_L 0_F}}{r - \mu_X - \mu_E} - I + \dfrac{\varphi(de_{1_L 1_F} - de_{1_L 0_F})}{r - \mu_X - \mu_E}\left(\dfrac{\varphi}{\varphi_F^*}\right)^{\beta_1} & \varphi < \varphi_F^* \\ \dfrac{\varphi de_{1_L 1_F}}{r - \mu_X - \mu_E} & \varphi \geq \varphi_F^* \end{cases} \qquad (18)$$

where, $\frac{\varphi de_{1_L 0_F}}{r - \mu_X - \mu_E} - I$ is the leader's payoff at the instant it invests if it operates alone forever; $\frac{\varphi(de_{1_L 1_F} - de_{1_L 0_F})}{r - \mu_X - \mu_E}\left(\frac{\varphi}{\varphi_F^*}\right)^{\beta_1}$ is derived using the continuity condition of $F_L(\varphi)$ at φ_F^*. It is negative given that $(de_{1_L1_F} - ds_{1_L0_F}) < 0$ (see inequality 4) and corresponds to the correction factor that incorporates the fact that in the future if φ_F^* is reached the follower will adopt tech 1 and the leader's payoff will be reduced [12]. $\frac{\varphi de_{1_L1_F}}{r-\mu_X-\mu_E}$ is the leader's payoff when active with the follower both with tech 1, from φ_F^* until infinity.

This is a pre-emption game where the Fudenberg and Tirole (1985) principle of rent equalization holds. Therefore, the leader adopts tech 1 at the point where the value functions of both firms cross, the point of rent equalization. Hence, equalizing Equations (15) and (18), for $\varphi < \varphi_F^*$, we get Equation (19).

$$\frac{\varphi de_{1_L0_F}}{r-\mu_X-\mu_E} - I + \frac{\varphi(de_{1_L1_F} - de_{1_L0_F})}{r-\mu_X-\mu_E}\left(\frac{\varphi}{\varphi_F^*}\right)^{\beta_1} - A\varphi^{\beta_1} = 0 \tag{19}$$

Replacing in (19) φ by φ_L^* and solving in order to φ_L^* we get the leader's investment threshold (φ_L^*). We use standard numerical methods to solve for φ_L^*.

3. Sensitivity Analysis

In this section we study the effect of changing some important parameters values on the leader's and the follower's investment thresholds. In our illustrative results we use the following base (annualized) parameters: [13,14]

$X(t)$	$E(t)$	$\varphi(t)$	I	σ_X	σ_E	μ_X	μ_E	r	ρ_{XE_k}
10	0.85	85.0	100	0.20	0.238	0.02	0.00	0.10	0.0

Table 2 – Market variables

$de_{0_L0_F}$	$de_{1_L0_F}$	$de_{1_L1_F}$
0.0	1.0	0.60

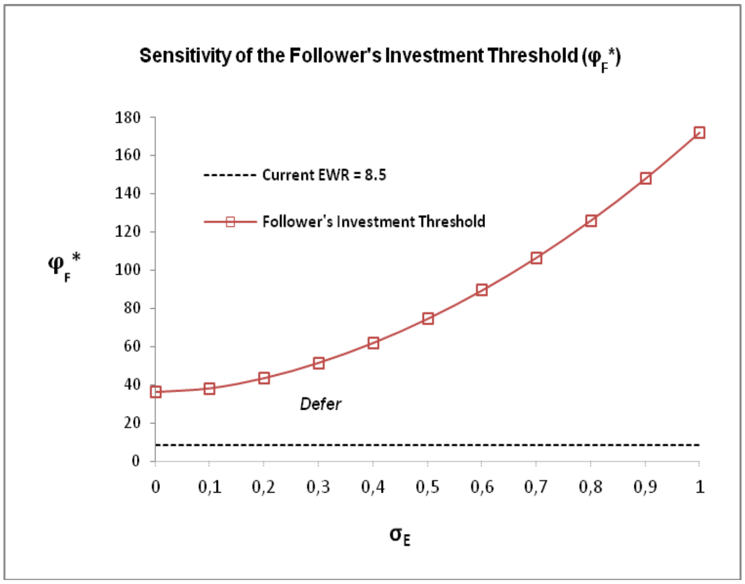
Table 3 – Competition factors (leader' market share)

[12] This term equals the leader's loss discounted back from the (random) time at which the follower adopt tech 1. The term $(\varphi/\varphi_F^*)^{\beta_1}$ is interpreted as a stochastic discount factor which is equal to the present value of $1 received when the variable φ hits φ_F^* (see Pawlina and Kort, 2006, p. 10).

[13] To rationalize the inputs above (and our results) consider the empirical case (textile technology) described in Appendix A, sections a,b, and suppose that the leader while alone in the market gets 100% of the market share and a net profit margin of $1 per output (i.e., per meter of textile fabric), with 250 working days per annum. The maximum net profit per annum is (28,000m/day)(250working days)=$7,000,000. If, initially, it produces at 85% efficiency the profit is ($7,000,000)(0.85)=$5,950,000. For an investment of $100,000,000 the annual return is 5.95%. Suppose that when the follower enters the leader's market share drops to 60%, so annual net profit is reduce to ($5,950,000)(0.6)=$3,570,000, adjusted for the leader's efficiency at the time of the follower adoption.

[14] The input for the efficiency volatility (0.238) is from the dataset described in Appendix A, Figure A1.

Figure 3 shows our illustrative results for the (ceteris paribus) sensitivity of the follower's investment threshold, φ_F^*, to changes in the volatility of the EAA, σ_E.

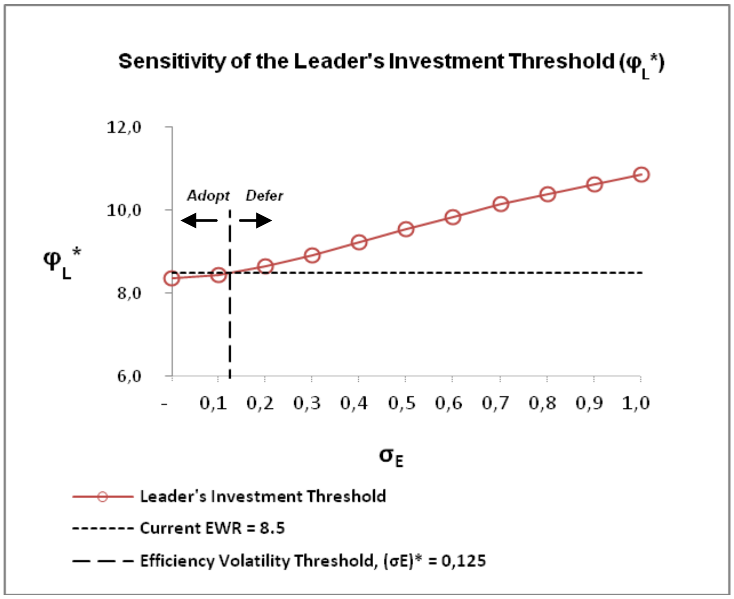


EWR $(\varphi) = 8.5$	σ_E	**0**	**0.10**	**0.20**	**0.30**	**0.4**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**1.00**
	φ_F^*	36.18	38.14	43.51	51.55	61.93	74.54	89.41	106.56	126.03	147.87	172.09

Figure 3 - Senstitivity of the Follower's Investment Threshold to changes in the Volatility of the EAA (σ_E)

The results show that φ_F^* is very sensitive to changes in σ_E. Ceteris paribus, the higher the σ_E the later is the adoption of the technology. The follower should defer the adoption for the all range of σ_E values used. The efficiency volatility underlying the textile technology described in Appendix A (Figure 1) is 23.8%, for which $\varphi < \varphi_F^*$, i.e., the follower should defer the investment.

Figure 4 shows our illustrative results for the (ceteris paribus) sensitivity of the leader's investment threshold, φ_L^*, to changes in the volatility of the EAA, σ_E.

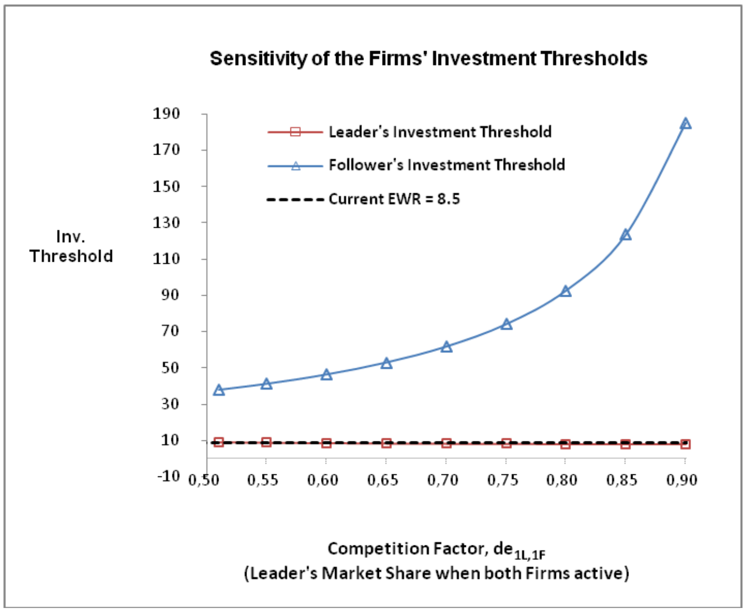


EWR $(\varphi)=8.5$	σ_E	**0**	**0.10**	**0.20**	**0.30**	**0.4**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**1.00**
	φ_L^*	8.37	8.45	8.65	8.92	9.23	9.54	9.85	10.14	10.64	10.64	10.86

Figure 4 – Senstitivity of the Leader's Investment Threshold to changes in the Volatility of the EAA (σ_E)

The results show that the leader investment threshold is sensitive to changes in the volatility of the EAA (σ_E), although less sensitive than the follower. The higher the σ_E the later is the adoption of the technology. More especifically, for $\sigma_E < 0.125$ the curent EWR (φ) is above the follower's investment threshold (φ_F^*), i.e., $\varphi > \varphi_F^*$, so the leader should adopt the technology. For $\sigma_E > 0.125$, the the curent EWR (φ) is below the follower's investment threshold, i.e., $\varphi < \varphi_F^*$, so the follower should defer the investment. $(\sigma_E)^* = 0.125$ is the efficiency volatility threshold where if it decreases, the leader should adopt the technology, if it increases, the leader should defer the investment.

In Figure 5 are our illustrative results for the sensitivity of the investment thresholds of the leader and the follower to changes in the first-mover market share advantage. The results are computed using the base inputs and $de_{1_L 1_F} \in \{0.51, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95\}$. Notice that in a duopoly, the market share of the follower is a complement of the market share of the leader, i.e., $de_{1_F 1_L} = 1 - de_{1_L 1_F}$. Hence, for each $de_{1_L 1_F}$ above we adjust $de_{1_F 1_L}$ accordingly. For the inputs used we compute the respective "market share advantage" as follow: suppose (i) $de_{1_L 1_F} = 0.51$, as $de_{1_F 1_L} = 1 - de_{1_L 1_F}$ so $de_{1_F 1_L} = 0.49$ (i.e., when both firms are active the leader gets 51% of the market revenues and the follower the remaining 49%), leading to a first-mover market share advantage of $de_{1_L1_F} - de_{1_F1_L} = 0.51 - 0.49 = 0.02$ (2%); (ii) $de_{1_L1_F} = 0.80$, as $de_{1_F1_L} = 1 - de_{1_L1_F}$ so $de_{1_F1_L} = 0.20$ (i.e., when both firms are active the leader gets 80% of the market revenues and the follower the remaining 20%), leading to a first-mover market share advantage of $de_{1_L1_F} - de_{1_F1_L} = 0.80 - 0.20 = 0.60$ (60%). Using Equation (3) we convert the "first-mover market share advantage" into "first-mover revenues advantage", see footnote 9.

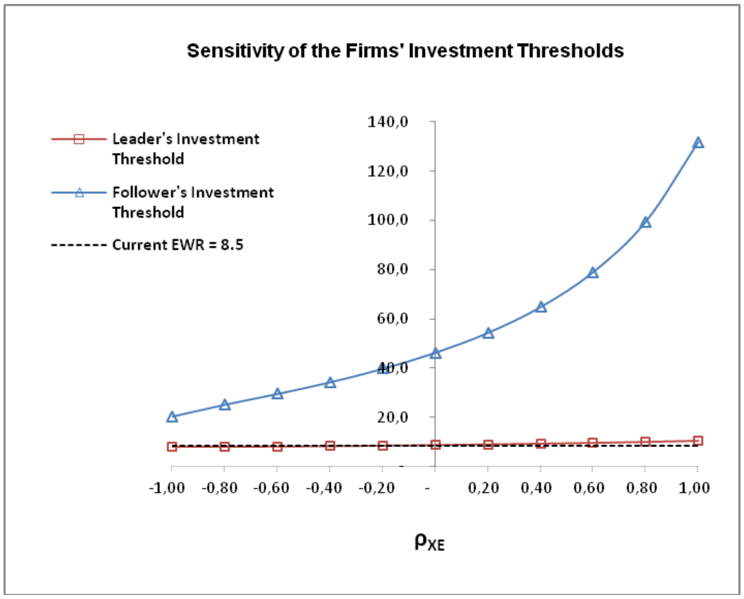


EWR $(\varphi) = 8.5$	$de_{1_L1_F}$	**0.51**	**0.55**	**0.60**	**0.65**	**0.70**	**0.75**	**0.80**	**0.85**	**0.90**
	$de_{1_F1_L}$	**0.49**	**0.45**	**0.40**	**0.35**	**0.30**	**0.25**	**0.20**	**0.15**	**0.10**
	FMA (%)	*2%*	*10%*	*20%*	*30%*	*40%*	*50%*	*60%*	*70%*	*80%*
	φ_L^*	9.27	9.00	8.74	8.54	8.39	8.26	8.17	8.10	8.05
	φ_F^*	37.28	41.13	46.28	52.89	61.70	74.04	92.55	123.40	185.10

Figure 5 – Sensitivity of the Firms' Investment Thresholds to changes in the Leader's Market Share when both Firms active ($de_{1_L1_F}$)

The results show that the follower's investment threshold is very sensitive to changes in the FMA, particularly when the asymmetry between the ex-post market share of the leader and the follower is very high. Ceteris paribus, the higher the first-mover market share advantage the later is the follower adoption. The leader's investment threshold is not very sensitive to changes in the FMA (reacts slightly in the opposite direction to that of the follower, i.e., the higher the FMA the slightly earlier is the adoption). Notice that in a leader/follower duopoly (one-shot) game, as soon as the leader invests the follower is in a "monopoly-like" regarding its "option to invest", therefore, the investment behavior above for the follower can be seen as a proxy of the investment behavior of monopolistic firms. For the base inputs and the all range of the leader/follower market shares above, the leader should adopt the technology (since $\varphi \geq \varphi_L^*$) and the follower should defer the investment (since $\varphi < \varphi_F^*$).

In Figure 6 are opur illustrative results for the sensitivity of the firms' investment threshold to changes in the correlation between "market revenues" and "EAA".



EWR $(\varphi) = 8.5$	ρ_{XE}	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
	φ_L^*	8.00	8.02	8.13	8.30	8.51	8.74	9.01	9.31	9.64	10.02	10.47
	φ_F^*	20.48	25.26	29.63	34.35	39.78	46.28	54.35	64.78	78.92	99.33	131.56

Figure 6 – Sensitivity of the Firms' Investment Thresholds to changes in the Correlation between "market Revenues" and "EAA" (r_{XE})

The correlation between the "market revenues" and the "EAA" (ρ_{XE}) has a negligeable effect on the leader's threshold and a significant effect on the follower's threshold specially for high positive correlation values.

4. Conclusion and Further Research

Our model challenges the view underlying the current real option literature, which assumes that a technology once adopted will perform exactly as predicted by the adopter/developer. We provide a real option model based on more realistic assumptions. Our results show that efficiency (technical) uncertainty has an asymmetric effect on the leader's and the follower's investment behavior, delaying significantly the investment of the follower and only slightly the investment of the leader. These differences are due to the so called effect of the “fear of pre-emption” which affects the leader (turning efficiency uncertainty a less relevant variable), and does not affect the follower. Notice that as soon as the leader invests the follower is in a “monopoly-like” regarding its “option to invest”, hence our results for the follower can be taken as a proxy of monopolistic investment behaviors.

We found that the size of the leader’s FMA, speeds up slightly the investment of the leader and delays significantly the investment of the follower and that a high positive correlation between “market revenue” and EAA delays slightly the investment of the leader and significantly the investment of the follower.

In our model firms are not allowed to improve EAA due to learning. EAA is assumed to be an exogenous variable, i.e., firms are ex-ante/ex-post symmetric in their capability to operate with the technology and after the follower adoption the EAA is uncertain but the same for both firms. It would be interesting, however, to extend our model to the case where the evolution of EAA is firm-specific, allowing for ex-post efficiency asymmetries between firms. In addition, our model is based on two stochastic underlying variables (market revenue and EAA), where there is a FMA (pre-emption game). It would be interesting to add a third underlying variable, “technological uncertainty”, studying the simultaneous effect of “market”, “technical” and “technological uncertainty” on the timing optimization of the adoption of a new technology, and to consider the case where there is a “second-mover advantage” (attrition game). Finally, we use a competition framework where the FMA is based on deterministic competition factors defined as proportions of the market EWR. Although mathematically challenging, it would be interesting to refine this assumption using dynamic market share for both firms.

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Appendix A

1. Efficiency: the case of a weaving technology

Figure A1 shows empirical data about the "daily EAA" of a weaving technology over 262 days.

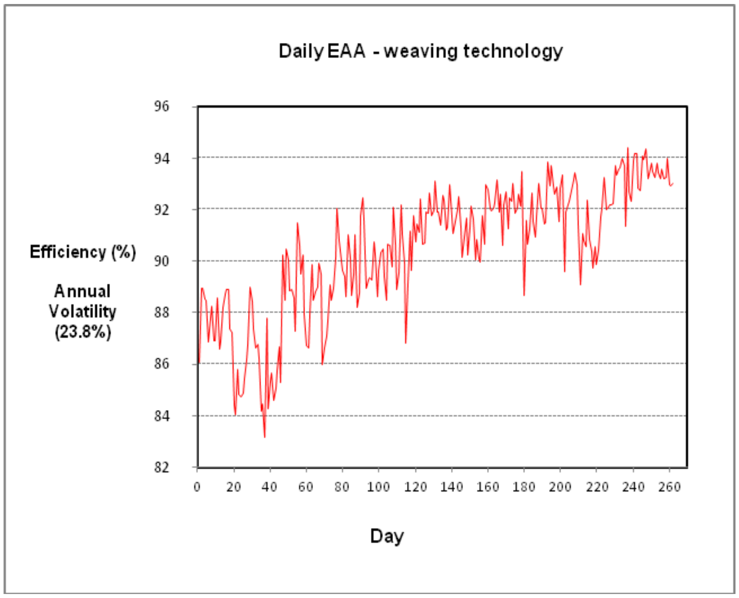

Figure A1

To rationalize the usefulness of our real option model, suppose that there is a weaving mill where machines and equipments need to be replaced and that a new version (not yet tested in the market) of the weaving technology currently in place has arrived in the market. Suppose that the supplier advertises that the new technology after being adopted will operate with 96% efficiency and that such estimation seems very optimistic to the adopter. Consequently, as a precautionary method, the adopter decides to assess the investment assuming that the EAA of the (not yet tested) weaving technology will be more or less like that of the current weaving technology in place (Figure A1), where we can see that, in the first 45 days, the efficiency is between 83 and 89 percent, in the last 30 days, the efficiency is between 92 and 95 percent, and between these two periods, the efficiency improves gradually. In the last months the volatility of the EAA of the technology decreases.

a. Measuring Efficiency: the case of a weaving technology

A weaving technology produces textile fabrics (output). Its output production is measured in "meters/day" (m/day) and depends on the technical specificities of the fabric (or mix of fabrics) that is(are) being produced, more specifically the "quantity of yarns per linear meter of fabric". For the same EAA, the higher the "quantity of yarns per linear meter of fabric" the lower is the output production (m/day). In Figure A2, the solid line describes the relationship between the "daily EAA" and the "daily output production", for a particular weaving technology/output. There is a linear relationship between the "daily EAA" and the "daily production". When the "daily EAA" is zero, the "daily production" is zero, when the "daily EAA" is 100%, the "daily production" is 28,000 meters of fabric (maximum). For a given weaving technology, the slope of the solid line depends on the technicalities of the textile fabric(s) that is being produced (output-specific) - the more time-consuming the production of one output unit (meter) the lower is the slope of the solid line.[15]

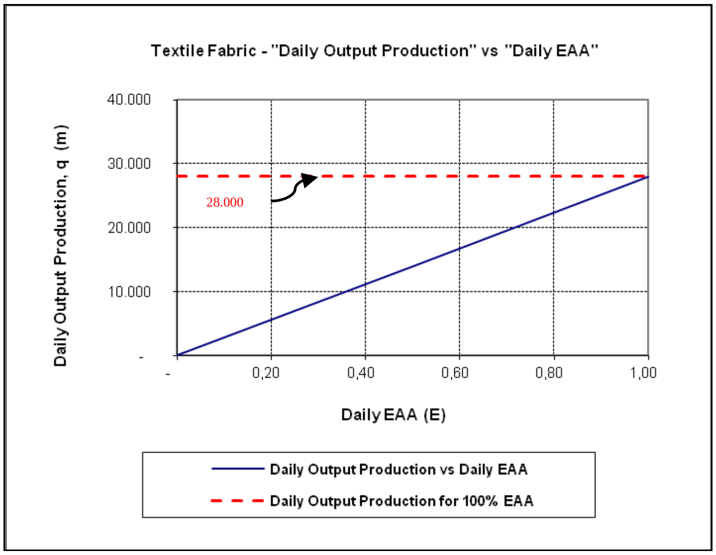

Figure A2

b. The Variable Efficiency (E_t)

The variable EAA (E_t) is defined in its most general form as:

$$E_t = \frac{[Actual\ output]_t}{[Effective\ capacity]_t} \quad \text{(A1)}$$

where, E_t is the EAA per unit of time *t*; $[Actual\ output]_t$ is the output produced per unit of time *t*; and $[Effective\ capacity]_t$ is the production capacity per unit of time *t*, for the scenario where the technology operates after adoption with 100% efficiency. Defining *t* as "day" (A1) becomes,

$$E_{day} = \frac{[Actual\ output]_{day}}{[Effective\ capacity]_{day}} \quad \text{(A2)}$$

The slope of the solid line in Figure A2 is given by:

[15] Our empirical data is about the efficiency of a weaving technology over 262 days where there were no changes in the mix of outputs produced. Notice that the efficiency/production of two weaving mills can only be compared if they produce the same type of output or different outputs but with similar technical (time-technology-resources) requirements.

$$C = \frac{\Delta q}{\Delta E} \quad \text{(A3)}$$

where $\Delta q = q_2 - q_1$, $\Delta E = E_2 - E_1$, with (q_1, E_1) and (q_2, E_2) defining any two points on the solid line.

For the output (i) /weaving technology underlying our data, the slope of the solid line is given by:

$$C_i = \frac{\Delta q}{\Delta E} = 28,000 \quad \text{(A4)}$$

Defining the time unit as "day", the relationship between "EAA per unit of time" (%) and "output production per unit of time" (meters) is given by:

$$q_{day} = C_i E_{day} \quad \text{(A5)}$$

For our data $C_i = 28.000$. Hence, knowing q_{day}, we determine E_{day}, using (A5), and vice-versa. For instance, if for a particular "day" the weaving technology produces 24.500 meters of fabrics ($q_{day} = 24.500 \text{ m}$), using (A5), we determine that the weaving technology operated on that day with an efficiency of $E_{day} = 24.500 / 28.000 = 0.875$ (87.5%).

Appendix B

1. Derivation - Ordinary Differential Equation (7)

Rewrite Equation (6) as,

$$\frac{1}{2}\frac{\partial^2 F_F}{\partial X^2}\sigma_x{}^2 X^2 + \frac{1}{2}\frac{\partial^2 F_F}{\partial E^2}\sigma_E{}^2 E^2 + \frac{\partial^2 F_F}{\partial E \partial X} X E \sigma_x \sigma_E \rho + \frac{\partial F_F}{\partial X}\mu_x X + \frac{\partial F_F}{\partial E}\mu_E E - rF_F = 0 \quad \text{(B1)}$$

In order to reduce the homogeneity of degree two in the underlying variables to homogeneity of degree one similarity methods can be used. Let $\varphi = (X)(E)$, so:

$$F_F(X,E) = F_F(\varphi)$$
$$\frac{\partial F_F(X,E)}{\partial E} = \frac{\partial F_F(\varphi)}{\partial \varphi} X$$
$$\frac{\partial F_F(X,E)}{\partial X} = \frac{\partial F_F(\varphi)}{\partial \varphi} E$$
$$\frac{\partial^2 F_F(X,E)}{\partial E^2} = \frac{\partial^2 F_F(\phi_2)}{(\partial \varphi)^2} X^2$$
$$\frac{\partial^2 F_F(X,E)}{\partial X^2} = \frac{\partial^2 F_F(\varphi)}{(\partial \varphi)^2} E^2$$
$$\frac{\partial^2 F_F(X,E)}{\partial X\ \partial E} = \frac{\partial^2 F_F(\varphi)}{(\partial \varphi)^2} XE + \frac{\partial F_F(\varphi)}{\partial \varphi}$$

Substituting back to Equation (B1) we obtain Equation (7), rewritten here as:

$$\frac{1}{2}\varphi^2 \sigma_m{}^2 \frac{\partial^2 F_F(\varphi)}{\partial \varphi^2} + \varphi\left(\sigma_X \sigma_E \rho_{XE} + \mu_X + \mu_E\right)\frac{\partial F_F(\varphi)}{\partial \varphi} - rF_F(\varphi) = 0 \quad \text{(B2)}$$

where, $\sigma_m^2 = \sigma_X{}^2 + \sigma_E{}^2 + 2\rho_{XE}\sigma_X\sigma_E$.