Modeling Choices in the Valuation of Real Options: Reflections on Existing Models and Some New Ideas

Mikael Collan

University of Turku, School of Economics, Pori Unit,
PL 17, 28101 Pori, Finland
e-mail: mikael.collan@utu.fi

Abstract

This paper discusses option valuation logic and four selected methods for the valuation of real options in the light of their modeling choices. Two of the selected methods the Datar-Mathews method [16, 24] and the Fuzzy Pay-off Method [13] represent later developments in real option valuation and the Black & Scholes formula and the Binomial model for option pricing the more established methods used in real option valuation.

The goal of this paper is to understand the big picture of real option valuation models used today and to discuss modeling perspectives for the future.

Key words: Real Option Valuation, Option Valuation Models

1. Introduction

Real options are the different types possibilities found in connection with real investments that allow managers to capture the potential in the investment, these possibilities are often referred to as managerial flexibility. A real investment that does not have any managerial flexibility is a static creatur that can only take as given the changes in its environment. Such monolithic investments, or projects, are in actuality not often found, but when such investments exist they are often highly sensitive to changes and hence their profitability is vulnerable.

Investments with real options, that is investments that include managerial flexbility can in contrast to investments without real options, react to changes and adjust themselves to new circumstances. This allows such investments to incur smaller losses in case of adverse changes or to capture the potential and become more profitable, when changes are favourable. This means that investments with real options are more valuable, than the same investments without real options, ceteris paribus.

The term real options was coined, to the best of our knowledge, by Myers [30] in 1977 and is based on the observation that as managerial flexiblity found in real investments exhibits an analogy of construct [22] with financial options they can be called real options. The original idea with real option valuation is to extend the analogy further: as there is analogy in the the construct of real options with financial options (contracts), then the valuation methods for financial options can be used for the valuation of real options. After Black & Scholes 1973 article [3] on pricing of options an elegant model for option valuation was available and the road for application to real options was open. As new methods for option valuation were developed, for example the binomial option pricing model by Cox, Ross and Rubinstein [15] in 1979, it also was applied in the valuation of real options.

As real options are not a thing of fiction, but important real options are often available in real investments it is a matter of interest to managers and business / project owners to be able to understand the value of the real options connected to their investments. Real option value is not only interesting from the point of view of understanding the whole value of an investment (with real options) , but also, and perhaps especially in situations where comparisons between possible investment alternatives are made. Real option value is an additional measure in project selection that is able to capture the value of potential. Indeed, real options are an often used tool in, e.g., the valuation and analysis of Research and Development (R&D) investments that are known to exhibit large risks (in the financial sense), but often simultaneously offer high potential rewards, see e.g. [32, 23, 9, 35]. It is also to be noted that the most often used capital budgeting method, the Net Present Value method is not able to consider aspects of investment profitability that are covered by real options analysis [21]. Lately, criticism has been voiced about the application of the 1970's methods for real option valuation and their apparent lack of focus on real world relevance and usability for practitioners [4].

These reasons are sufficient to make real options valuation, models for the valuation of real options, and modeling the valuation of real options an interesting issue for research and relevant for both managers and the academia.

2. Real Option Valuation as a Modeling Problem

The option valuation problem, or the logic of option pricing is actually rather straight forward: the value of an option is the present value of the chance of occurrence weighted expected value of the outcomes of the distribution of the future option values, while mapping the negative values zero. The reason for considering the negative values of the future option value distribution as zero is that the holder of the option has the right, but not the obligation to exercise her option contract. She will not exercise if it would cause a loss, but exercises only if profit is created, thus making her downside zero at maximum. Figure 1 shows graphically the option valuation problem, with the major issues that must be accounted for when modeling the value of an option, or the value of a real option.

The three major components of modeling the value of a real option are:

a) the modeling of the future value distribution

b) the calculation of the expected value of the future value distribution while mapping negative values of the distribution zero, and

c) modeling the calculation of the present value of the expected value.

To be more precise: the modeling of the future value distribution can be interpreted as the modeling of how the future value distribution is created and the calculation of the expected value of the future distribution as the selection of the procedure that is used in the calculation of a single expected value (used as the expected value of the option price).

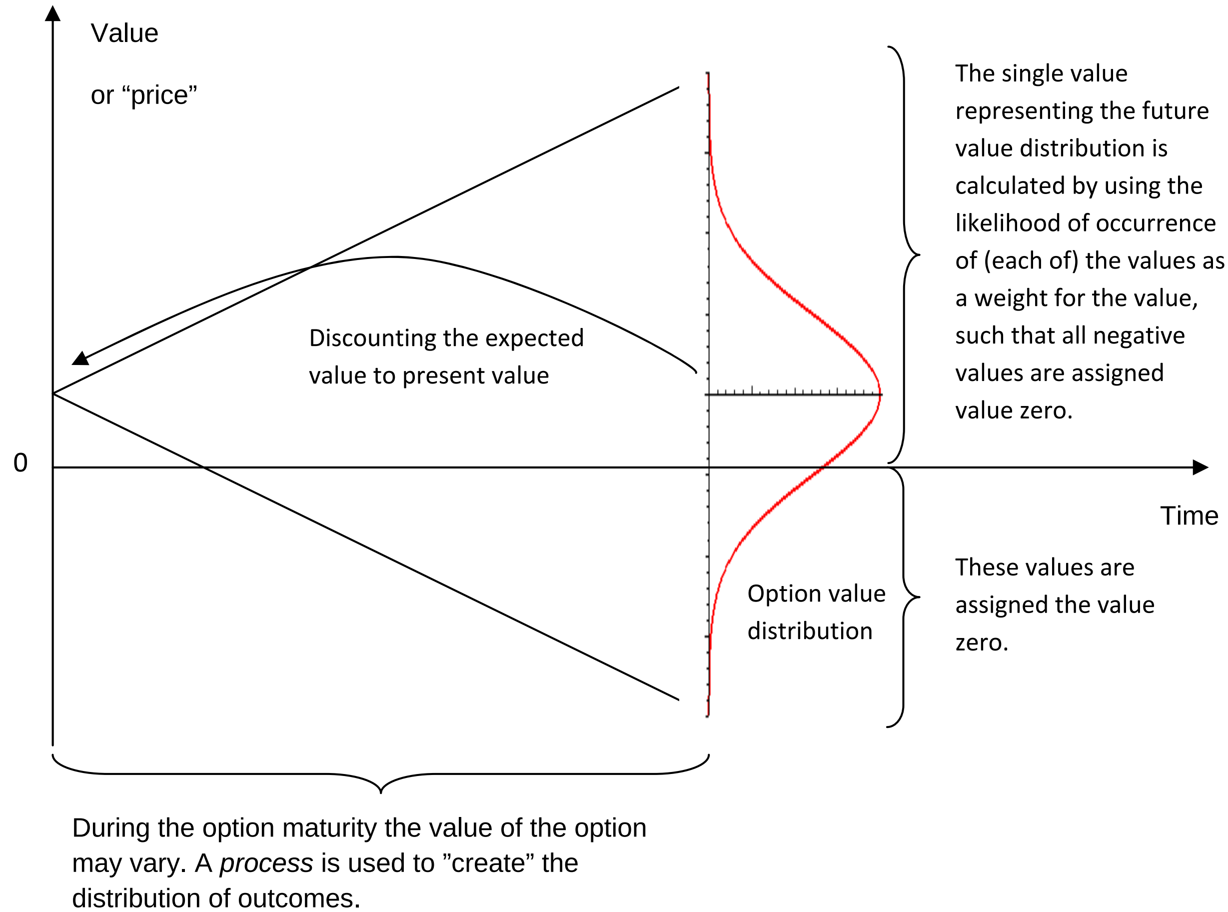

Figure 1. A simple sketch of the real option valuation logic

Also the discounting of the expected value requires modeling decisions, that is, determining the discount rates used and derivation of the discount rates and selecting the compounding interval are normative choices open to the modeler. All of the choices made with respect to these three major issues and within the issues themselves contribute to how the option valuation problem is modeled and have an effect on what the output from the model is.
The reality in which real options exist is that of real investments, id est the projects and the investments are most often non-tradeable in any established market and the existence of arbitrage is commonplace. Very large industrial real investments may also exhibit special characteristics, such as the ability to steer the markets in which they are active [20, 10] making them price makers rather than price takers (a common assumption when financial securities are discussed). The managers of real investments most often also have the power to steer their investments according to changes in the environments of their investment, thus being able to „pull the option levers“ [22] a possibility that is deemed impossible under complete and perfect markets. The reality within which real options reside is the reality the modeler of real option value should take into consideration.

Interestingly, looking into the whole scale of different possible (new) ways to mathematically model the three major components of option valuation, has by far not been exploited by academics. Indeed, most of the work on option valuation models has been done based on the setup, assumptions, and modeling choices, laid by the 1970’s models.

In the following section a short review of how four selected models for (real) option valuation, two from the 1970's and two from after the millenium change, have solved the problem of modeling the above mentioned three components.

3. Modeling Choices of Four Selected Models Used in Real Option Valuation

We have selected four models that are used the valuation of real options for a closer look at the modeling choices that they are based on. The models selected are:

i) the famous, 1997 "Nobel Prize"-winning, Black & Scholes option pricing formula from 1973 that is an often used model for the valuation of real options
ii) the Binomial Option Pricing model by Cox, Ross & Rubinstein from 1979, also a very used model in real option valuation
iii) the Datar – Mathews model for real option valuation from 2004 [16], specifically built for the valuation of real options to be used in a corporate investment decision-making setting
iv) and finally the Fuzzy Pay-off Method for real option valuation from 2009 [13], a method built based on using fuzzy numbers to represent the future distribution of expected option value and applying fuzzy mathematics to reach the option value

The main selection criterion for the selection of the above four models is that each one of them represents a different modeling approach to option valuation:

i) differential equation solutions (Black & Scholes formula and variants)
ii) discrete event and decision models (Binomial tree model)
iii) simulation based methods for option valuation (Datar – Mathews model)
iv) fuzzy logic based methods (Fuzzy Pay-off Method)

Secondary selection criterion for these particular method variants is that the basic Black & Scholes and binomial models are the starting points for the myriad of extensions and expansions of these models available and used for real option valuation. The Datar – Mathews model is a particularly interesting new example of a simulation based real option valuation method, and for the purposes of this research it acts also as a placeholder for other (real) option valuation methods [5] that have selected to use simulation as a part of their construct. The Fuzzy Pay-off Method for real option valuation represents the rather recent developments in building real option valuation models that are based on using fuzzy logic.

3.1. Black & Scholes Option Pricing Formula 1973

The original Black & Scholes formula [3] is designed to value a European call options contract based on the price of an underlying stock. It is assumed that the stock is a freely traded asset in complete and efficient markets (perfect markets), where there are no transaction costs and any price changes are continuous, random and are distributed log-normally with known variance. Further it is assumed that an investor in the markets is able to lend and borrow at the risk free rate of return and buy any fraction of any security. Implicitly the borrowing / lending assumption means that the discount rate used for discounting cash-flows can also be the risk free rate. In the original formula no dividends payments were assumed, this was modified later by Merton [26].

Perfect markets with the given assumptions effectively mean that the future development of any traded asset is a random walk allowing for the use of stochastic processes, in this case the Geometric Brownian Motion (GBM). Using the GBM is a modeling choice that acts almost as a corollary to assuming perfect markets with the above assumptions, and covers also the issue of modeling of the future value distribution of the option, because the GBM effectively defines the resulting distribution.

„The replication argument“ is a brilliant observation that is behind the construct of the Black & Scholes formula: any two assets with the same cash-flows and the same risk must be have the same price under perfect markets. Thus, assuming perfect markets with the above assumptions, any combination of securities that are traded in these markets and that delivers exactly the same cash-flow as an option contract must be worth exactly as much as the option contract. Black & Scholes observed that a cash-flow identical to the „option cash-flow“ can be reached by a constructing a combination of borrowing money and buying the underlying (stock) in the amount determined by the option delta.

$$C = SN(d_1) - Xe^{-r(T-t)}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + (r + ½\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Where:

C	is the European Call option price
S	is the price of the underlying asset
X	is the exercise price
T-t	is the time to maturity
r	is the risk-free rate of return
σ	is the volatility
N	is the cumulative normal distribution function

The construct of the Black & Scholes method visible in equation (1) is very clever, as the choice of assumptions and modeling and the replication argument are such that allow for a closed form solution that returns the call option value as a single number.

The modeling choices of the Black & Scholes model in light of the three major components of modeling the real option value are the following:

The modeling of the future value distribution is done by using a stochastic process for the option value (actually the value of the underlying), namely the geometric Brownian motion, a process already used in 1900 [2, 14] by Louis Bachelier, perhaps already earlier, and resulting in a continuous log-normal distribution (for the expected value).

The calculation of the expected value of the future value distribution while mapping negative values of the distribution zero, and the calculation of the present value of the expected value are embedded in the closed form solution; the replication argument results in the very elegant way of considering the calculation of the expected value. The discounting back of the „future expected value“ is done by using a continuously compounding risk-free rate of return as the rate of discount (in essence for both, the revenue- and the cost-side of the option).

Many extensions of the original Black & Scholes model exist, also hybrid models that utilize the original model framework but use fuzzy numbers, see e.g. [7],[11] and [37].

3.2. Binomial Option Pricing Model 1979

The binomial option pricing model [15] is based on the use of a „discrete-time“ binomial tree or latice for modeling the price variation of the underlying asset. In other words the creation of the expected value distribution is done by creating a binomial lattice by using a binomial process for stock price changes. The binomial process used allows for two possible directions for the underlying asset value at each time step, up or down, with connected probabilities q and 1-q.

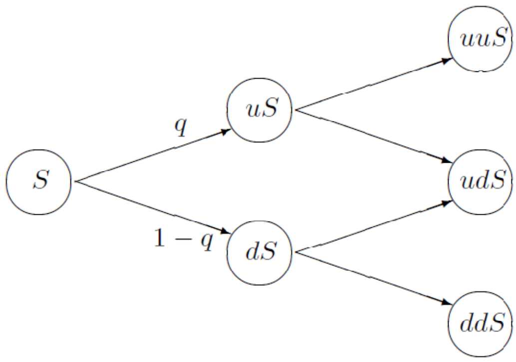

Figure 2. Binomial process for asset price with two steps

With these assumptions, the price of the European call option Vi(k) at time ti, up to which the stock price process has experienced k steps, follows the the equations [15]

$$\begin{aligned} V_i(k) &= \frac{1}{1+R}[q_u V_{i+1}(k+1) + q_d V_{i+1}(k)], \\ V_N(k) &= \Phi(Su^k d^{N-k}), \\ q_u &= \frac{1+R-d}{u-d}, \\ q_d &= \frac{u-(1+R)}{u-d}, \end{aligned}$$

Where:

R is the discount rate for each time step

N is the number of time steps

Option pricing using the binomial pricing model is a three-stage process: first the binomial tree is constructed, then the option value at each final node (end of maturity) is calculated, and finally the option value for all earlier nodes is calculated by iterating backwards from the final nodes. The process is a simple and a usable one, especially if one considers the existence of American options.

The assumptions underlying the original binomial pricing model are similar to the assumptions made for the Black & Scholes model and discussed above. This means that the binomial pricing model is a discrete time *approximation* of the continuous process underlying the Black & Scholes model: for European options with a large number of time-steps the result from the binomial pricing model converges with the result from Black & Scholes model. In other words, this means that the binomial distribution for the expected values approaches the normal distribution.

The modeling choices of the binomial option pricing model in light of the three major components of modeling the real option value are the following:

The modeling of the future value distribution is done by using a binomial process for the underlying asset price that results in a discontinuous quasi-log-normal distribution that approaches the continuous distribution that is the result of the GBM process used in the Black & Scholes model.

The calculation of the real option value is done by starting from the „end“ or final values of the binomial lattice, created by the binomial process as described above. From the final values the earlier node values are calculated, all the way back to the first node that is the real option value at time zero. In the process a compounding risk-free rate of risk is used as the discount rate is used and the rate of compounding is the number of nodes per year. Cost and revenue side of the real option are not separated.

Extensions of the binomial option valuation model include a number of different modifications. These include, e.g., changes to the original binomial process to include jumps, considering other types of processes to replace the binomial process [6], and combining fuzzy logic with the binomial model [27, 29, 28].

3.3. Datar – Mathews Method for Real Option Valuation 2004

The Datar-Mathews method [16, 24, 17] is a simulation based valuation algorithm that has been specifically constructed for the purpose of real option valuation. The method relies on cash-flow scenarios for the operational cash-flows of an investment project that is the real option. The cash-flow scenarios are created by managers and experts in charge of the project. The cash-flow scenarios are used an input into a Monte Carlo simulation that is used to create a probability disttribution of the expected net present value for the project under analysis, i.e. the real option. This distribution is also known as the pay-off distribution. The present value distribution is calculated by using (allowing the use of) separate discount rates for the the revenues and the costs. Real option value is calculated from the pay-off distribution by finding the probability weighted mean while mapping the negative pay-off distribution values zero.

The intuition of the Datar-Mathews method in a nut-shell can be expressed as [17]:

Real Option Value = Risk Adjusted Success Probability * (Benefits – Costs)

Translating the Datar-Mathews method intutition into a spread-sheet expression [17] looks like:

$$\text{Real option value} = \\ \text{Average}[\text{MAX}\,(\overline{\text{operating profits}} - \text{launch cost},\, 0)]\ .$$

From the point of view of the three major components of modeling the real option value the Datar-Mathews method differs from the Black & Scholes pricing model and the Binomial model.

The the Datar-Mathews method does not rely on a selection and the use of a predetermined process for the modeling (forecasting) of the future distribution for the underlying asset value. The modeling of the future value distribution is done by using (normative) managerial cash-flow information, given in the form of cash-flow scenarios, as the basis for a Monte-Carlo simulation that is used to create a probability distribution of the expected net present value of the project in question, i.e. the real option.

As the method „exists“ as a spread-sheet application the discounting of the future value distribution is automated and the use of many possible discounting options is possible, i.e., the compounding interval can be freely selected (continuous / discrete) and the discount rates can be different for revenues and for costs. Using different separate discount rates can even allowed for each revenue and cost „post“ according to their individual risk. This means that the Datar-Mathews method can reflect the (often observed) fact that revenues and costs are not equally risky. Due to the flexible construct of the method also non-lognormal cash-flow distributions can be quite easily accomodated, this is better in line with the reality of real options [17].

The calculation of the real option value is done by calculating the probability weighted mean of the pay-off distribution while mapping negative values of the pay-off distribution zero.

The method is „mathematically equivalent to Black-Scholes formula under certain assumptions“ [17] – the Datar-Mathews method result converges with the Black & Scholes result when the Monte Carlo is run enough times. The method has been patented [25] and special software can be bought on-line.

3.4. Fuzzy Pay-off Method for Real Option Valuation 2009

Fuzzy Pay-off method (FPOM) [13, 12] is the latest addition of the four to the real option valuation method arsenal. The method is based on a similar construct as the Datar-Mathews method, it relies on cash-flow scenarios as a basis for creation of a net present value distribution for the real option under analysis. The difference with the other presented methods is that the method uses the cash-flow scenarios in the creation of a pay-off distribution that is treated as a fuzzy number (a special case of a possibility distribution) and does not treat the distribution as a probability distribution. The method is applicable to any shape of pay-off distributions, but simple triangular or trapezoidal distributions are most straight-forward to use: managers are most often asked to estimate three (or four) cash-flow scenarios for a real option, a best estimate-, a minimum possible-, and a maximum possible scenario. Net present value is calculated for each one of these scenarios. The discount rates can be selected separately for costs and for revenues and the selection of the rate of compounding is left to the analyst. The pay-off distribution in the three-scenario-case is created from the NPV’s of the scenarios by:

i) observing that the best estimate scenario is the most likely one and assigning its‘ NPV full degree membership in the set of expected NPV outcomes.
ii) deciding that the maximum possible and the minimum possible scenarios are the upper and lower bounds of the distribution – also making the simplifying assumption: values higher than the optimistic scenario and lower than the pessimistic scenario are not considered
iii) assuming that the shape of the pay-off distribution is triangular.

This procedure results in a triangular fuzzy number that is the pay-off distribution for the real option under analysis and that is used as real option the future value distribution. The fuzzy pay-off method calculates the real option value from the pay-off distribution (fuzzy NPV) as follows:

$$ROV = \frac{\int_0^\infty A(x)dx}{\int_{-\infty}^\infty A(x)dx} \times E(A_+)$$

Where A stands for the pay-off distribution (fuzzy NPV), $E(A_+)$ denotes the possibilistic mean value of the positive side of the pay-off distribution and $\int_{-\infty}^{\infty} A(x)dx$ computes the area below the whole pay-off distribution, and $\int_0^{\infty} A(x)dx$ computes the area below the positive part of the pay-off distribution.

To calculate the real option value from the pay-off distribution by using the fuzzy pay-off method, the possibilistic mean [8] of the part of the pay-off distribution resting on the value axis above values larger or equal to zero is calculated first. For the triangular case there are four possible situations for the positive side of the pay-off distribution, and hence four different possible cases where the possibilistic mean must be calculated. The calculation for all four cases is done as follows:

$$E(A_+) = \begin{cases} a - \alpha > 0 \; then \; E(A_+) = a + \dfrac{\beta - \alpha}{6} \\ a > 0 > a - \alpha \; then \; E(A_+) = \dfrac{(\alpha - a)^3}{6\alpha^2} + a + \dfrac{\beta - \alpha}{6} \\ 0 > a \; then \; E(A_+) = \dfrac{(a + \beta)^3}{6\beta^2} \\ a + \beta < 0 \; then \; E(A_+) = 0 \end{cases}$$

Then the possibilistic mean, $E(A_+)$, is multiplied by the area of the positive side of the pay-off distribution over the whole area of the pay-off distribution. This is the procedure that is the pay-off method for real option valuation.

The structure of the pay-off method is in line with the option valuation logic of the classical option valuation methods, and especially with the Datar – Mathews method. The method uses managerial information as a starting point for the creation of cash-flow scenarios, for which the net present value is then calculated. Costs and revenues can be discounted by using separate discount rates for each cost and the selection of compounding interval can be selected by the user. The expected value of the value distribution is calculated by using the possibilistic mean on the after-the-discounting pay-off distribution.

3.5. Summary of the selected models

The four models presented all solve the „real option valuation problem“ in different ways, below we present a short written summary of each one of the models and use table 1 to topically summarize the three main modeling choices of each one of the models.

The analytical solution of the Black & Scholes model is a mathematically very elegant construct that relies strongly on strict assumptions about the world in which the model works, namely a world with efficient complete capital markets. The model is built by using a stochastic process, the geometric Brownian motion for creating the future distribution of the (real) option value. The replication logic the model uses earned the namesake creators the The National Bank of Sweden prize in economics in the memory of Alfred Nobel. The model was not constructed originally for real option valuation use.

The binomial option pricing model is able to produce the same result as the Black & Scholes model by using a quite different approach for modelling the process that leads to the „same“ future value distribution, the binomial process. The model is uses a discrete event, stepwise process that creates, with a large number of time-steps a quasi log-normal distribution for the future value of the (real) option. Binomial model shares the same strict Black & Scholes assumptions regarding the world in which it is applicable. The model was not originally constructed for real option valuation use.

Model	Process used to create future value distribution	Distribution type	Discounting of the expected value	Other
Black & Scholes (1973)	Geometric Brownian Motion	Continuous, log-normal probability distr.	Continuous discounting with rf	Closed form solution, replication
Binomial (1979)	Binomial tree process	Quasi log-normal probability distr.	Compound discounting with rf	Backwards iteration to solve value, approaches B&S
Datar-Mathews (2004)	Cash-flow scenarios + MC-simulation	Probability distr. of various shapes	Flexible, user selectable	Practitioner oriented
Fuzzy Pay-off Method (2009)	Cash-flow scenarios => Creation of fuzzy number	Fuzzy number	Flexible, user selectable	Simplistic, uses fuzzy logic

Table 1. Summary of the selected (real)option valuation models

The Datar-Mathews model is built for real option valuation. It depends on managerial cash-flow scenarios as input into a Monte Carlo simulation that is used to create a probability distribution of the net present value of the real option (project). The distribution is infact the pay-off distribution from the real option. The method is very flexible and allows for any interval of compounding and free selection of discount rates used, this is due to the fact that it is applicable in the spread-sheet software domain. The model does not have strict underlying assumptions.

The fuzzy pay-off model for real option valuation uses the net present value of best estimate, minimum possible, and maximum possible manager given cash-flow scenarios to create a fuzzy pay-off distribution for the real option. The fuzzy pay-off (net present value) distribution is treated as a fuzzy number (fuzzy NPV) and the expected value of the real option is calculated from the distribution by using possibilistic mean value for the positive side of the fuzzy NPV. The model has been from the start designed to be used by practitioners for the valuation and analysis of real options and real investments. Any discount rates and compound intervals can be used and the model is free of any restrictive assumptions, but if a small number of cash-flow scenarios (2-4) are used the shape of the pay-off distribution will be a simplification of reality.

4. Discussion, New Ideas, and Conclusions

The four types of models for the valuation of real options all offer different insights into the variety of methods that can be used in framing the same problem. Three of the four models use a probability distribution in expressing the future distribution of the real option value one uses a fuzzy number. In broader terms the use of the probability distribution is greatly more widespread, perhaps due to the more widespread knowledge and experience on using probability distributions.

The process used in creating the future value distribution of the real options is different in each of the models, Black & Scholes model uses a closed form formula that uses a strictly pre-defined process that allows for no adjustment the Binomial model is in effect a discrete time version of the Black & Scholes model. Many models based on the Black & Scholes model have done changes / enhancements to the GBM used in the model, for example to allow skewed distributions and autocorrelation (see for example [33] and other works by the same authors). There are works that utilize fuzzy numbers together with the Black & Scholes original model, hybrids that mainly use the fuzzy numbers to tackle the inaccuracies in estimating the values for the revenues and costs [36, 37, 11]. Criticism of the Black & Scholes formula [19] reveals that the method has not been fully appreciated by all the practitioners in the financial industry, similar voices have been heard from the real options community on the modeling choices of real option valuation in general [4].

Going deeply into the question of selecting the process that creates the future value distribution may be relevant with regards to the criticism and important in our attempt to understand and create a picture about the modeling choices for framing the problem of real option valuation. This means also that we need to keep in mind that someone, namely the modeler, has selected a process that he/she has thought is good for „the job“ has also been a normative decision. The choice of the modeller is always putting the reality and the used model component for modeling that reality at a confrontation. Now this does not mean that the Black & Scholes model is wrong if it does not reflect reality of financial options or that of real options, because from the get-go the authors of the model assume a theoretical perfect markets world. But it needs to be said: the Black & Scholes modeling choice of using GBM is not very good choice for realistic modeling of real option value. This means logically that Binomial model may not be the perfect choice either.

The Datar-Mathews model and the Fuzzy pay-off model use expert generated cash-flow scenarios as an input into real option valuation. This allows for the process of future value distribution creation to include information that is outside of the „flexibility“ offered by a pre-determined process, for example information about hedging strategies used for taking out risks or similar. This allows for these methods to result „any form“ of distribution for the future value of the real option and therefore offer a more realistic fit to the reality of real option valuation, answering also to some of the the critical comments about real option valuation methods [4].

Discounting of the expected value, or the whole future value distribution for that matter, to the present time value is done by using the same logic. The difference in the four presented methods is that the Datar-Mathews method and the fuzzy pay-off method allow for flexiblity in choosing the discount rate for the components that make the total cost and revenue cash-flows separately and individually and in the selection of the compounding interval, while the Black & Scholes model and the Binomial tree method use the risk-free rate of return for the discounting for both the costs and the revenues and have predetermined intervals for compounding. In essence to reach a „similar“ result from the Black & Scholes & Binomial tree models and the Datar-Mathews model the revenue and cost cash-flows need to be preprocessed or „pre-discounted“ to risk neutrality so that the risk-free rate of return can be used as a discount rate. This means that in an „analysis well-done“ with both of these two methods there is in actuality the need to use separate discount rates for costs and for revenues to reach risk neutral revenue and cost numbers for real options analysis; this is not usually presented and is sometimes not considered.

Above presented issues illustrate the fact that there are many ways to model real option valuation, but lately the requirement of the models seems to have been set on a new level: real option valuation models should be able to cope with the requirements of the real world that is to have tolerance for the many imperfections of real asset reality in comparison to the theoretical complete and efficient markets. This gives a possibility and motivation to think about some new ways of modeling the real option valuation problem or at least to look at some of the components used in real option valuation with an open mind.

4.1 Some new Ideas

From a mathematical modeling perspective it is interesting to think about new approaches for framing the option valuation problem. Indeed there are many possible feasible alternatives for example, for the creation of the future option value distribution. As the number of possible new avenues is great the presentation here is limited to four shortly presented ideas applicable to real option valuation models in the future.

Taking into consideration the limitations of the available information is a task that real option modeling should look at, apart from using stochastic models and fuzzy logic that have both been already tested we could also look at ***subjective probability*** [1, 31] and ***credibility*** [18] as a frame for our thought when we look at the real option valuation problem. These are „other“ ways to define uncertainty – other than probability and fuzzy logic and we should try them out!

Using simulation in creating a distribution for the future value of a real option is an established methodology, but has anyone used a manager to „draw“ the distribution as she sees it reflecting reality and, then running a ***curve fitting algorithm*** that would treat the distribution as a curve and find an „as close as possible“ fitting defined curve or a function to fully define the distribution. To the best of my knowledge such an attempt has not been reported.

Real option valuators should put more emphasis on the intuitive and understandable ***presentation of results*** and perhaps show more than “just the real option value“. The whole process of real option valuation itself contains information about the real option and is likely to often be of interest to the dececision-makers – yet they are most often shut out of that information. Presenting real option valuation results as if they were coming from a black box is not only an inferior way to use the obtained results, but is also prone to cause the rejection of the method by managers, who often want to understand the tools used in the evaluation. This IS also the modelers‘ problem even if the modeler may not be the analyst using and presenting the model. I am not alone in my want or craving for good presentation of data, scholars that have risen to become almost household names like Hans Rosling[i] have devoted their time to push presentation of data and results to a new level. The results of their presentation of results speak for themselves. Some in the RO community are voicing similar thoughts, since a long time ago „It makes little sense to use a numerical technique to calculate the option price accurate to 1% or 2% when the underlying asset price is only known to an accuracy of 10%, as in real options.” [34]

4.2 Conclusions

The bar for future modeling of real option valuation has been set higher, future models for practical use must be constructed to reflect that fact and to reflect the realities of real world real options. Problems like non-existing markets for the assets underlying the real options, lack of available information or imprecise information, and information assymmetry coupled with the fact that holders of real options are able to sometimes steer the value of their real options (pull the option levers) should be taken into consideration and integrated into the models as well as possible. The source of good information for real option valuation is more often than not experts, as existence of any historical data sets or collected data for similar previous endeavours is not available, it is the human knowledge about our real options that we must seek. Finding ways to correctly represent human knowledge, perhaps given in linguistic terms, not in numbers and perhaps conflicted by other human experts are formidable challenges that face real option valuation.

While the reality bites, in academia exploration for new ways to model remains however free of the constraints of reality and it should be encouraged. It is per se valuable to seek new and elegant constructs for modeling issues, but I encourage the modellers, you, to look outside the box and to „try to please“ the practitioner. It is not without meaning we should do this work of ours, but to serve the practitioner.

It is with this challenge for scholars and managers interested in mathematical modeling of real options that I close this paper: „I dare you to create new models for option valuation that challenge old assumptions and perhaps offer a better fit to the reality of real options“.

---

[1] Anscombe F J and Aumann R J 1963 A definition of subjective probability *The Annals of Mathematical Statistics* **34** 199-205

[2] Bachelier L 1900 Théorie de la spéculation. *Annales Scientifiques de l'École Normale Superieure* **17** 21-86

[3] Black F and Scholes M 1973 The Pricing of Options and Corporate Liabilities *Journal of Political Economy* **81** 637-59

[4] Borison A 2005 Real Options Analysis: Where Are the Emperor's Clothes? *Journal of Applied Corporate Finance* **17** 17-31

[5] Boyle P 1977 Options: A Monte Carlo approach *Journal of Financial Economics* **4** 323-8

[6] Boyle P 1986 Option Valuation Using a Three-Jump Process *International Options Journal* **3** 7-12

[7] Buckley J and Esfandiar E 2008 Pricing Stock Options Using Black-Scholes and Fuzzy Sets *New Mathematics and Natural Computation* **4** 165–76

[8] Carlsson C and Fullér R 2001 On possibilistic mean value and variance of fuzzy numbers *Fuzzy Sets and Systems* **122** 315-26

[9] Carlsson C, Fuller R, Heikkilä M and Majlender P 2007 A fuzzy approach to R&D project portfolio selection *International Journal of Approximate Reasoning* **44** 93-105
[10] Collan M 2004 Giga-Investments: Modelling the Valuation of Very Large Industrial Real Investments. In: *Department of Information Systems,* (Turku: Abo Akademi University)
[11] Collan M, Carlsson C and Majlender P 2003 Fuzzy Black and Scholes Real Option Pricing *Journal of Decision Systems* **12** 391-416
[12] Collan M, Fullér R and Mezei J 2009 A Fuzzy Pay-Off Method for Real Option Valuation. In: *IEEE International Conference on Business Intelligence and Financial Engineering,* (Beijing, PRC: IEEE) pp 165-9
[13] Collan M, Fullér R and Mézei J 2009 Fuzzy Pay-off Method for Real Option Valuation *Journal of Applied Mathematics and Decision Systems* **2009**
[14] Courtault J-M 2000 Louis bachelier - On the centenary of théorie de la spéculation *Mathematical Finance* **10** 341-53
[15] Cox J, Ross S and Rubinstein M 1979 Option Pricing: A Simplified Approach *Journal of Financial Economics* **7** 229-63
[16] Datar V and Mathews S 2004 European Real Options: An Intuitive Algorithm for the Black Scholes Formula *Journal of Applied Finance* **14** 7-13
[17] Datar V, Mathews S and Johnson B 2007 A Practical Method for Valuing Real Options: The Boeing Approach *Journal of Applied Corporate Finance* **19** 95-104
[18] Dorweiler P 1934 A survey on risk credibility in experience rating *Proceedings of the Casualty Actuarial Society* **21** 1-25
[19] Haug E and Taleb N 2009 Why have we never used the Black-Scholes-Merton Option Pricing Formula. In: *Available at SSRN: http://ssrn.com/abstract=101207,*
[20] Keppo J and Lu H 2003 Real options and large producer: the case of electricity markets *Energy Economics* **25** 459-72
[21] Kulatilaka N and Marcus A 1992 Project valuation under uncertainty: When does DCF fail? *Journal of Applied Corporate Finance* **5** 92-100
[22] Leslie K and Michaels M 1997 The real power of real options *The McKinsey Quarterly* 5-22
[23] Lint O and Pennings E 1998 R&D as an option on market introduction *R&D and Management* **28** 279-87
[24] Mathews S and Datar V 2007 A Practical Method for Valuing Real Options: The Boeing Approach *Journal of Applied Corporate Finance* **19** 95-104
[25] Mathews S, Datar V, Nakamoto K and Forgie C 2006 System, method and computer program product for determining a minimum asset value for exercising a contingent claim of an option ed T B Company" (USA
[26] Merton R 1973 Theory of Rational Option Pricing *Bell Journal of Economics and Management Science* **4** 141-83
[27] Muzzioli S and Reynaerts H 2008 American optionnext term pricing with imprecise risk-neutral probabilities *International Journal of Approximate Reasoning* **49** 140-7
[28] Muzzioli S and Torricelli C 2000 Combining the Theory of Evidence with Fuzzy Sets for Binomial Option Pricing. In: *Materiale di Discussione* Dipartimento di Economia Politica, Universita degli Studi di Modena e Reggio Emilia)
[29] Muzzioli S and Torricelli C 2000 A model for pricing an option with a fuzzy payoff *Fuzzy Economics Review* **6**
[30] Myers S 1977 Determinants of corporate borrowing *Journal of Financial Economics* **5** 146-75
[31] Nau R 2002 De Finetti was right: probability does not exist *Theory and Decision* **51** 89-124

[32] Newton D and Pearson A 1994 Application of option pricing theory to R&D *R&D Management* **24** 83-9

[33] Perello J, Masoliver J and Napoleon A 2004 A comparison between several correlated stochastic volatility models *Physica A* **344** 134-7

[34] Sick G 1990 *Capital Budgeting with Real Options* (New York: Stern School of Business, New York University)

[35] Tolga C and Kahraman C 2008 Fuzzy Multiattribute Evaluation of R&D Projects Using a Real Options Valuation Model *International Journal of Intelligent Systems* **23** 1153-76

[36] Yoshida Y 2003 The valuation of European options in uncertain environment *European Journal of Operational Research* **145** 221-9

[37] Zmeskal Z 2001 Application of the fuzzy-stochastic methodology to appraising the firm value as a European call option *European Journal of Operational Research* **135** 303-10

---

[i] Hans Rosling at TED:
http://www.ted.com/talks/lang/eng/hans_rosling_shows_the_best_stats_you_ve_ever_seen.html