The Brazilian Fiscal System and the decision to invest in petroleum reserves: A Real Options Approach

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Abstract

The aim of this paper is to evaluate the impact of the Brazilian petroleum fiscal regime – royalties and special participation tax – on the decision to invest in the development of oil and gas reserves in Brazil. We use the Real Option approach, which considers the value of managerial flexibilities. The proposed model admits the option to wait and the option to abandon, resulting in a nonlinear system whose solution allows simulating the impact of different royalties and special participation taxes rates on the threshold price-cost ratios to development and to exit the industry. Through a Monte Carlo experiment, it is also estimated a path for both prices and costs with the purpose of assessing the expected revenues to the government under each tax rate. Results show that flexible rates, focusing on special participations, are preferable to a structure heavily based on royalties, due to the lower impact on the decision to invest as well as potentially higher revenues.

Keywords: real options, investment, taxes, oil industry.

1 Introduction

Like many other countries, Brazil's Constitution assigns to the Federal Union the ownership of all onshore and offshore natural resources. The goal of the

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law is to avoid possible common pool inefficiencies (Libecap and Wiggins, 1984). Thus, the government has the right to usufruct the resource rent and the main instrument to accomplish this aim is the creation of a special fiscal regime over natural resources upstream, including oil and gas. However, the uncertainty in exploratory activities is very peculiar, so the tax design affects directly the perception of risk (Mayo, 1979; Fraser, 1993) and, consequently, the decision to invest. On the other hand, conventional investment analyses, based on net present value, ignore the value of managerial flexibility in the project (Kulatilaka, 1995; Laughton, 1998).

The aim of this paper is to evaluate the impact of this special fiscal regime in the Brazilian oil and gas industry – called "Governmental Benefits" [1]- on the decision to invest in the development of oil and gas reserves. The paper contributes through the addition of two managerial flexibilities in the project: the option to postpone the development of the deposit and the option to exit the industry. The methodology, based on the Real Option approach (Dixit and Pindyck, 1994), consists in simulating the impact of some combinations of royalties and special tax rates on the price-cost ratios that trigger both the decision to develop the deposit and the decision to lock up the production. Real Option is a powerful tool that incorporates the value of managerial options embedded in an investment project.

The paper contains five further sections, besides this introduction: section 2 presents an overview on Real Options and on tax neutrality in extraction of nonrenewable resources; section 3 describes the Brazilian special fiscal regime on oil and gas upstream; section 4 presents a real option model of investment to support the simulations; section 5 brings the results that simulate the impact of Brazil's Governmental Benefits on the decision to invest as well as on the expected collected revenue. Section 6 concludes. Results show that an intermediary combination of rates – differently from what is practiced in the contemporary Brazilian system – impact less on the decision to invest.

[1] Brazilian Oil and Gas Law, number 9478/1997.

2 Real Option Approach and Taxation of Mineral Rents

Standard investment analyses are based on Net Present Value (NPV), which is given by the difference between the discounted value of the future cash flow and the initial investment. The goal is to find an objective rule to evaluate whether an investment is viable or not. According to this analysis, the project is accepted when $NPV > 0$; investments with negative NPV must be discarded.

Dixit and Pindyck (1994) point three main failures in the NPV method, despite its power and operational easiness: i) the lack of a suitable treatment for uncertainty, which is incorporated through the addition of a risk premium in the discount rate; ii) the misevaluation of the opportunity cost to invest, since it does not take into account that expenditures are sunk; iii) the non-incorporation of managerial flexibilities, like the ability of delaying the decision to invest. In reality, the agent is able to adjust the timing of an irreversible decision and this flexibility adds a value that cannot be measured under a conventional NPV analysis. Thus, the rejection of a negative NPV project may be a myopic decision, since the investor has the option to wait for further information before undertaking an irreversible expenditure.

The weakness of NPV as a decision rule has contributed to develop the Real Option Approach, a powerful tool that allows to incorporate the effect of irreversible expenditures, uncertainty and managerial flexibilities on the decision to invest[2]. The investment is treated as an American call whose underlying asset is the value of the project. The flow of (irreversible) investments is interpreted as the strike price.

The literature about Real Options has shown a strong growth in the last two decades, through the gradual addition of several stylized flexibilities in seminal contributions, like the option to shutdown (McDonald and Siegel,

[2] Besides Dixit and Pindyck (1994), good overviews can be found in Pindyck (1991), Trigeorgis (1995, 1996), Brennan and Trigeorgis (2000), Schwartz and Trigeorgis (2001), Minardi (2004). A pratical guide is supplied by Copeland and Antikarov (2002). For an overview about applications in oil and gas investments, see Dias (2001).

1985), the option to expand capacity (Pindyck, 1988), the option to invest (Paddock et. Al, 1988; Abel et. Al., 1996), the option to abandon (Myers and Majd, 1990), among others. The real option tools have spread toward applications in several areas (Schwartz and Trigeorgis, 2001), like land development (Capozza and Li, 1994), real estate (Grenadier, 1996), investment in inventories (Cortazar and Schwartz, 1993), railroads (Emery and McKenzie, 1996), among many others. More recently, the methodology of real options has been applied to evaluate electricity projects (e.g.: Laughton et. Al, 2000; Deb, 2004; Saphores et. Al, 2004).

Regarding the taxation of nonrenewable resources rent, the literature is also consolidated. As Garnaut and C. Ross (1983) argue, since the extractive sector has peculiarities related to the exploratory uncertainty, tax burdens affect directly the risk perception of investors and, consequently, the volume of investment. Researchers in this area have always tried to answer the following question: which fiscal regime over nonrenewable resources upstream minimizes the distortion on the decision to invest?

According to Blake and Roberts (2006), petroleum fiscal system around the world can be classified in three general categories:

a) Taxes Royalty: it is the general denomination of wide range of systems that include income taxes with allowable deductions, royalties or ad valorem taxes. In Brazil, royalties are synonymous for ad valorem taxes.

b) Contractual Systems: in which the government and the firm set up a contract of concession, which foresees risk sharing and payments, according to the particular circumstances of each country. Production Sharing Contracts (PSC), which establishes the sharing of petroleum production (so-called Profit Oil) between local government and the investor, is a particular design of this class of fiscal arrangement. This system also embodies Service Agreements, in which the corporation (or a consortium) undertakes the project on behalf of the host government in exchange for a compensation.

c) Rate of Return: it consists in carrying forward all the (predicted and unpredicted) losses at some allowed rate of return to deduct them from the base of taxation. Rate of return fiscal system is seldom applied in real world, due to the complexity of required information. An example of this category is the Resource Rent Tax (Garnaut and C. Ross, 1983).

Under the traditional NPV approach, several works [e.g.: Leland (1978), Mayo (1979); Campbell and Lindner (1985), Garnaut and C. Ross (1983), Fraser (1993), Fraser and Kingwell (1997), Zhang (1997), Fraser (1998)] have concluded that a Tax Royalty fiscal system distorts more heavily the investment than a Rate of Return Tax. The Rate of Return Tax is an attempt to design a neutral tax the closest as possible to a Brown Tax[3], which is completely neutral since it does not change the rank of profitability among eligible projects. However, despite its theoretical attractiveness, the Brown tax is never applied in practice. As Blake and Roberts (2006, p. 96) argue, an intuitive description of a neutral tax is "*one which would maintain investors' before-tax ranking of possible investments in terms of attractiveness, even after the tax is applied*".

Under NPV, the neutrality of a fiscal regime is evaluated according to its ability to affect some dimension of the investment. However, when the interaction among irreversibility, uncertainty and flexibility is incorporated, nonlinearities emerge and the neutral tax under the NPV approach may present additional effects on the decision to invest.

The literature about neutrality of nonrenewable resources fiscal regimes is well-known, but few works incorporate uncertainty. Through the informational tree method, Bradley (1998) simulates how the value of a gas project is affected by a nonlinear royalty system. Lund (1992) uses a Contingent Claim Analysis to evaluate the distortions from petroleum fiscal system in Norway. In the same line, Blake and Roberts (2006) incorporates uncertainty by assuming a Geometric Brownian Motion for price. They analyze five upstream fiscal regimes under price uncertainty, investigating their neutrality concerning the impact on the value of the project. The distortion is measured by an index reflecting the deviation in the value due to the fiscal charge.

[3]Due to Brown (1948). A Brown Tax allows the deduction of all expenses as well as the offset of all losses. Under this system, the government assumes the same amount of risk as if it were a partner in the project. The tax rate is equivalent to the government's share in the venture. See also Garnaut and C. Ross (1983) for further analysis.

3 Petroleum fiscal regime in Brazil

Trying to solve common pool inefficiencies (Libecap and Wiggins, 1984), several countries, including Brazil, set up in their Constitutions that all onshore and offshore mineral resources belong to the people, who have the right to usufruct their rents, under the Hartwick's Rule[4] (Hartwick, 1977). Thus, as the resource owner, the government wishes to convert mineral rents in public investments, otherwise the welfare of future generations would be impaired.

This is the main rationale for a special fiscal regime over oil and gas upstream, that is, the government is interested in appropriating rent due to its ownership rights. However, since extractive sectors as a whole present special characteristics concerning the risk, the investment is particularly sensitive to fiscal burdens, because they affect the risk sharing between government and investors (Postali, 2002).

The Brazilian Petroleum Law, approved in 1997, defines four basic modalities of fiscal charges in oil and gas exploration: i) signature fee; ii) royalties rate; iii) special participations and iv) occupation fee. Royalties and occupation fee are obligatory in every lease contract. Special participations are applied only in highly productive fields.

Signature fee is the winner bid in the lease auction conducted by ANP – National Petroleum Agency. There are also other criteria for assigning concessions, like minimum investment programs and local purchase commitments. The tax must be paid at once at the beginning of the lease contract.

Royalty is a monthly 10%-ad valorem tax, applied over the gross revenue, priced according to an international average. The rule for natural gas is more complex, due to the absence of a developed international market, but the royalty rate is the same (10%). ANP can reduce the royalty rate to 5% if

[4]Hartwick (1977) approached theoretically the relationship between mineral rent, welfare and economic sustainability. He considered a stylized country wholly dependent on a nonrenewable resource, whose rent is the single source for investment funds. Hartwick showed that even such a limiting country is able to sustain indefinitely a constant per capita consumption, as long as it invests the mineral rent in physical and human capital. This result is known as Hartwick's Rule.

geological risks and poor conditions of production justify such a measure. The revenues collected with royalties are shared among Brazilian states, localities, National Treasure and public R & D funds.

Special participations are extra fiscal charges over highly productive projects. The tax is calculated in each lease according to a progressive system of rates over the net revenue, that is, the gross revenue minus royalties, exploratory investments, operational costs, depreciation and other legal taxes. The aim of the government is to appropriate a higher portion of rent from highly profitable projects. There are six rates: exemption, 10%, 20%, 30%, 35% and 40% of the net revenue according to a rule that considers the volume of extraction, the wells deep and the field's age. Resources collected are shared among states, producer localities and Federal Government.

Finally, the Occupation Fee is some sort of rent paid to the government by km^2 of retained area for exploration and production.

Among the modalities described above, royalties and special participations are the most important ones, since their revenues depend on production and prices. The signature fee, despite its potential to reduce investments (Postali, 2002) is not analyzed in this study[5].

We want to evaluate whether the Brazilian fiscal regime over oil production is neutral in the decision to invest but, instead of analyzing the influence of taxes on the value of investment, we study directly their impact on the decision to invest. Uncertainty is incorporated through a Real Option approach. In the next section, we present a model of investment in oil and gas, with the purpose of simulating how some combinations of tax rates impact on the decision to develop a reserve as well as on the decision to lock the production and exit the industry. The model is the base for simulations.

4 The Model

The model is based on the evaluation of oil and gas investments under a Real Option approach, following the methodologies developed by Paddock et Al.

[5]Moreover, the signature fee is not obligatory according to the law, and it can be replaced by other criteria of concession, like the investment program.

(1988), McDonald and Siegel (1985) and Dixit and Pindyck (1994, chapters 6 and 7). The context of the model can be set after the end of the exploratory phase (when there is no further geological uncertainty) and the beginning of the development[6], when the agent declares his willingness to extract, once the resource's economic potential is confirmed. Based on Dixit (1989), we assume that once in operation, the firm evaluates the option to lock the extraction to leave the industry if economic conditions are unfavorable.

In real option terms, the investor faces two possible scenarios to undertake an irreversible decision: a) option to convert undeveloped reserves in developed ones (option to invest); b) option to convert operating reserves in inactive ones (option to exit).

Unit Value of Developed Reserve. We assume that the unit value of developed reserve, $V(P, C, t)$, is governed by two state variables: the resource price (P) and the operational cost of production (C). As in Costa Lima and Suslick (2006)[7], it is assumed that these variables evolve according to Geometric Brownian-Motions (GBM):

$$dP = \alpha P dt + \sigma_P P dZ_P \tag{1}$$

$$dC = \phi C dt + \sigma_C C dZ_C \tag{2}$$

in which α and ϕ are the expected growth rates of price and operational cost, respectively; Zp and Zc are Wiener processes such that $E(dZ_P dZ_C) = \rho dt$, $E(dZ_P) = E(dZ_C) = 0$, $E(dZ_P)^2 = E(dZ_C)^2 = dt$. ρ is the correlation coefficient between changes in P and C. σ_P and σ_C are instantaneous standard deviations from each process.

The ϕ-parameter is the expected growth rate of unit cost. It can also be understood as the stock decay rate, if one assumes that the physical stock is

[6] We ignore the time to build, that is, the development is concluded immediately after investment expenditure. According to Majd and Pindyck (1987), time to build does not impact the results qualitatively, but only the magnitude of the uncertainty's effect.

[7] Costa Lima and Suslick (2006) study the relationship between price/cost uncertainties and the project volatility. Here, it is not necessary to estimate the project volatility, since we are dealing directly with trigger values for investment.

inversely proportional to the unit operational cost[8]. In the extent that the deposit is depleted, the cost of production increases, due to the decreasing pressures in the wells (Jevons' Effect).

Differently from Blake and Roberts (2006), we derive the equation that governs the reserve value through dynamic programming. The disadvantage of this approach is that the discount rate is taken as exogenous in the objective function, while under a Contingent Claim Analysis, a risk-adjusted discount rate is derived under equilibrium conditions in capital markets (e.g.: CAPM). On the other hand, under dynamic programming, there is no need of a spanning asset to compose a portfolio that replicates the value of the project[9].

The investor wishes to maximize the unit value of the reserve and the profits provided by the resource. By Bellman's Principle[10] and assuming risk-neutrality, we have:

$$rV(P,C,t)dt = \Pi(P,C,t)dt + E_0(dV) \tag{3}$$

Equation (3) establishes the optimal condition for the problem, which requires that, in equilibrium, the reserve's return at the risk free rate, $rVdt$, equals the profits provided by the resource, Π, plus the expected change in the reserve value.

The unit profit function is given by:

$$\Pi(P,C) = (1-R)[(1-\tau)P - C] \tag{4}$$

in which τ is the royalty rate and R is the Brazilian special participation rate.

By Ito's Lemma and discarding $O(dt) \geq 3$, we have :

[8]If S is the stock and $C_0 S_0 = S_t C_t$, so $C_t = C_0 e^{\phi t}$ implies $S_t = S_0 e^{-\phi t}$. See Postali and Picchetti (2006b) for further details. For dynamic properties of cost functions in the production of oil, see Osmundsen (1998).

[9]Dixit and Pindyck (1994, p. 121)

[10]According to Dixit and Pindyck (1994, p. 100), the Bellman's Principle of Optimality can be stated as follows: "*An optimal policy has the property that, whatever the initial action, the remaining choices constitute an optimal policy with respect to the subproblem starting at the state that results from the initial actions*".

$$dV = V_P dP + V_C dC + \frac{1}{2}V_{PP}(dP)^2 + \frac{1}{2}V_{CC}(dC)^2 + V_{PC}dPdC + V_t dt \quad (5)$$

Since Z is a Wiener Process, $E_0(dZ_P) = E_0(dZ_C) = 0$, $E_0(dZ_P)^2 = E_0(dZ_C)^2 = dt$ and $E_0(dZ_P dZ_C) = \rho dt$. Therefore[11]:

$$\begin{aligned} E_0(dV) &= V_P \alpha P dt + V_C \phi C dt + \frac{1}{2}V_{PP}\sigma_P^2 P^2 dt + \frac{1}{2}V_{CC}\sigma_C^2 C^2 dt \\ &\quad + V_{PC}\sigma_P \sigma_C PC dt + V_t dt \end{aligned} \quad (6)$$

Replacing (6) in (3) and after some algebra:

$$\begin{gathered} \tfrac{1}{2}V_{PP}\sigma_P^2 P^2 + \tfrac{1}{2}V_{CC}\sigma_C^2 C^2 + V_P \alpha P + V_C \phi C + V_{PC}\sigma_P \sigma_C PC \\ + V_t + (1-R)[(1-\tau)P - C] - rV = 0 \end{gathered} \quad (7)$$

Expression (7) is a partial differential equation without analytical solution, due to the term V_t, which expresses the unit reserve value as time goes on. Following Brennan and Schwartz (1985), it is possible to work in real terms, thus we deflate the value of the reserve such that $V(P,C,t) = \widetilde{V}(P,C)e^{-\pi t}$, where π is the inflation rate. By definition, $\widetilde{V}_t = 0$. Thus, it is easy to see that $V_t = \pi \widetilde{V}(P,C)$. Replacing in (7), we find:

$$\begin{gathered} \tfrac{1}{2}\widetilde{V}_{PP}\sigma_P^2 P^2 + \tfrac{1}{2}\widetilde{V}_{CC}\sigma_C^2 C^2 + \widetilde{V}_P \alpha P + \widetilde{V}_C \phi C + \widetilde{V}_{PC}\sigma_P \sigma_C PC \\ + (1-R)[(1-\tau)P - C] - (r-\pi)\widetilde{V} = 0 \end{gathered}$$

In which $r - \pi$ is the real interest rate. Besides, GBM allows a further simplification: it is possible to eliminate one dimension of the problem and reduce it to a single state variable, since $\widetilde{V}$ is an homogeneous function. Defining $x \equiv P/C$, one takes $\widetilde{V}(P,C) = Cv(P/C) = Cv(x)$ and the aim of the problem becomes to find $v(x)$.

The previous relationships between $\widetilde{V}(P,C)$ and $v(x)$ generate the following expressions[12]:

[11] Notation: $V_K = dV/dK$ and $V_{KK} = d^2V/dK^2$.

[12] The notations $v'(.)$ and $v''(.)$ mean, respectively, the first and the second derivative with respect to x.

$$
\begin{aligned}
\widetilde{V}_P &= v'(x) \\
\widetilde{V}_C &= v(x) - xv'(x) \\
\widetilde{V}_{PP} &= v''(x)/C \\
\widetilde{V}_{PC} &= -xv''(x)/C \\
\widetilde{V}_{CC} &= x^2 v''(x)/C
\end{aligned}
$$

Replacing the relationships above in (7) and dividing both sides by C, we have[13]:

$$
\begin{gathered}
\tfrac{1}{2}\left(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2\right) x^2 v''(x) + (\alpha - \phi) x v'(x) - (r - \pi) v(x) + \\
+ (1 - R)\left[(1 - \tau)x - 1\right] = 0
\end{gathered} \tag{8}
$$

Expression (8) is an ordinary differential equation. The particular solution is given by:

$$
v^* = (1 - R)\left[\frac{(1 - \tau)x}{r - \alpha + \phi - \pi} - \frac{1}{r - \pi}\right] \tag{9}
$$

which represents the fundamental unit reserve unit value.

The homogeneous solution has the form $v_H = Ax^\lambda$. Replacing in (8), λ solves the following characteristic equation:

$$
\frac{1}{2}\left(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2\right)\lambda^2 + \left[\alpha - \phi - \frac{1}{2}\left(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2\right)\right]\lambda - (r - \pi) = 0 \tag{10}
$$

Equation (10) admits two solutions, $\lambda_1 > 1$ and $\lambda_2 < 0$, so the homogeneous solution for the differential equation is:

[13] The property of homogeneity is valid only when both state variables follow a GBM. It is not possible to perform the same transformation when at least one variable follows a mean reverting (or an Ornstein-Uhlenbeck) process, since the drift depends on the level of the state variable. See Postali and Picchetti (2006a) for an overview on properties of some stochastic processes used to evaluate oil and gas investments.

$$v_H = B_1 x^{\lambda_1} + B_2 x^{\lambda_2} \tag{11}$$

Thus, the general solution is $v = v^* + v_H$. B_1 and B_2 are both constants determined by boundary conditions, which depend on the flexibilities present in the project. We assume the producer can lock the production, at a cost E, whenever economic conditions are unfavorable, e.g., when the price falls too much or when there is a depression. In this sense, the profit function (4) becomes:

$$\Pi^O = Max\left\{-E, \Pi\right\}$$

in which Π is giving according to (4). The differential equation representing the reserve value depends on the profit function. Since the reserve is developed and production is on, the solution for (8) is given by:

$$v(x) = B_1 x^{\lambda_1} + B_2 x^{\lambda_2} + (1 - R)\left[\frac{(1-\tau)\,x}{r - \alpha + \phi} - \frac{1}{r}\right] \tag{12}$$

Boundary conditions are required to determine the constants. In the extent that the deposit is producing, the unit value of the reserve is the option to exit[14] and when $x \to \infty$, the investor never exerts it. Therefore, since $\lambda_1 > 0$, one must have $B_1 = 0$.

In summary, in the presence of the option to exit, the unit value of the developed reserve is given by:

$$v(x) = B_2 x^{\lambda_2} + (1 - R)\left[\frac{(1-\tau)\,x}{r - \alpha + \phi} - \frac{1}{r}\right] \tag{13}$$

Unit value of undeveloped reserve. Let $F(P, C, t)$ represent the unit value of undeveloped reserve. Again, it is assumed that this value is governed by price and cost, according to GBM (1) and (2). The unit value of undeveloped reserve is an American call whose underlying asset is the developed reserve and the strike price is the investment – I.

[14] This interpretation presumes value additivity, discussed by Trigeorgis (1993).

According to the Bellman's Principle[15]:

$$rF(P, C, t)dt = E_0(dF) \tag{14}$$

Following the same procedures than before (including the Brennan and Schwartz's (1985) device to eliminate the time dependence of value), we have:

$$\frac{1}{2}\left(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2\right)x^2 f''(x) + (\alpha - \phi)xf'(x) - (r - \pi)f(x) = 0 \tag{15}$$

Equation (15) admits the following analytical solution:

$$f(x) = A_1 x^{\lambda_1} + A_2 x^{\lambda_2} \tag{16}$$

where $\lambda_1 > 1$ and $\lambda_2 < 0$ are the roots of the characteristic quadratic polynomial equation and A_1 and A_2 are both constants. Since for $x \longrightarrow 0$ it is unlikely that the option to develop is exerted, $f(0) = 0$, so $A_2 = 0$.

Two boundary conditions must also be satisfied. The *value-matching* and the *smooth-pasting* conditions. They are given, respectively, by[16]:

$$\widetilde{F}(P, C) = \widetilde{V}(P, C) - I \implies f(x) = v(x) - \frac{I}{C} \tag{17}$$

or

$$f'(x) = v'(x) \tag{18}$$

The value-matching condition establishes the optimal rule for exerting the option to invest, which is the equality between the value of undeveloped reserve and the value of developed reserve net of the irreversible investment. If $\widetilde{F}(P, C) > \widetilde{V}(P, C) - I$, the best decision is to wait. Otherwise, it is optimal to develop immediately. Therefore, the value-matching condition

[15] To be rigorous, one should define F(V,t). However, the diffusion process for V exhibits a very complicated expression and the differential equation linking F to V is too hard to solve. An alternative and simpler approach is to find the value of undeveloped reserve as a function of $x \equiv P/C$, using the solution for V as the boundary condition. See Dixit and Pindyck (1994, p 182) for details.

[16] See Dixit and Pindyck (1994, ch.4).

imposes continuity in v and f [17]. The smooth pasting condition guarantees the continuity in the slope as well, preventing breaks at the optimum.

The decision to invest depends on the critical ratio $x \equiv P/C$. Let x^E and x^S represent the trigger price-cost ratios that induce, respectively, the development and the abandonment of the deposit. The optimal decision depends both on the level of x and on the operation mode: if the reserve is undeveloped, the decision variable is x^E, that is, the trigger ratio that determines the exercise of the option to develop[18]. On the other hand, if the deposit is developed, the relevant decision is whether it is optimal to exit or not, which is triggered when x falls below x^S.

Table 1 summarizes optimal decisions according to the level of x.

Table 1: Trigger ratios and optimal decisions

$x \equiv P/C$	*Undeveloped Reserve*	*Developed Reserve*
$x < x^S$	to remain	to abandon
$x = x^S$	to remain	indifferent
$x^S < x < x^E$	to remain	to operate
$x = x^E$	indifferent	to operate
$x > x^E$	to develop	to operate

It is important to notice how x^S and x^E are different each other and all $x \in (x^S, x^E)$ represent an *hysteresis* phenomenon (Dixit, 1992): despite x^E is the trigger price-cost ratio that induces the development, the firm only exerts an option to exit if this ratio falls below x^S. Likewise, the ratio x^S is not enough to trigger the development, which only happens when the price-cost ratio reaches the minimum level x^E.

Whether the firm invests or not depends on the relative values of the undeveloped and developed reserves.

Trigger ratios x^E and x^S are determined by the boundary conditions (value-

[17] $f(x)$ can also be interpreted as the opportunity cost of development (Dixit and Pindyck, 1994), which consists in waiting for more information before the investment.

[18] The assumption is that it does not make sense to develop a deposit to keep it off.

matching and smooth pasting) in each mode of operation. Firstly, consider the firm facing the decision to develop or not the deposit. Replacing (13) and (16) into (17) and (18), we have:

$$-A_1(x^E)^{\lambda_1} + B_2(x^E)^{\lambda_2} + (1-R)\left[\frac{(1-\tau)x^E}{r-\alpha+\phi-\pi} - \frac{1}{r-\pi}\right] - i = 0 \quad (19)$$

$$-A_1\lambda_1(x^E)^{\lambda_1 - 1} + B_2\lambda_2(x^E)^{\lambda_2 - 1} + \frac{(1-R)(1-\tau)}{r-\alpha+\phi-\pi} = 0 \quad (20)$$

in which $i \equiv I/C$, that is, the investment-cost ratio (in real terms).

Secondly, an operative firm must consider the decision to close the deposit if the economic scenario worsens, which means that the price-cost ratio should fall below x^S. The boundary conditions become:

$$v(x^S) = f(x^S) - \varepsilon$$
$$v'(x) = f'(x)$$

in which $\varepsilon \equiv E/C$ and E is the cost to exit. Following the same procedure with the solutions in the boundary conditions, we have:

$$-A_1(x^S)^{\lambda_1} + B_2(x^S)^{\lambda_2} + (1-R)\left[\frac{(1-\tau)x^S}{r-\alpha+\phi-\pi} - \frac{1}{r}\right] + \varepsilon = 0 \quad (21)$$

$$-A_1\lambda_1(x^S)^{\lambda_1 - 1} + B_2\lambda_2(x^S)^{\lambda_2 - 1} + \frac{(1-R)(1-\tau)}{r-\alpha+\phi-\pi} = 0 \quad (22)$$

Equations (19), (20), (21) and (21) represent a nonlinear system with four equations and four unknowns: A_1, B_2, x^E and x^S. The system has not analytical solution, requiring numerical methods to be solved[19]. A_1 and B_2 are option values, so they must be positive.

The following section presents the data used to parameterize the model as well as its results.

[19] We used Newton-Raphson.

5 Results

5.1 Impacts on investment

The exercise of simulation aims at evaluating the impact of different combinations of royalties and special participations rates on the trigger ratios x^E and x^S. The distortion on the decision to invest is measured by the difference between these ratios under some combination of rates and their values without any tax. Table 2 summarizes the reference values to calibrate the model.

Table 2: Reference values (initial) for parameters

Parameter	*Value*	*Source*
P_0	\$50	(assumed)
C_0- high quality	\$3.70	Blake and Roberts (2006)
C_0- low quality	\$5.08	Blake and Roberts (2006)
Initial stock − high quality	300 million barrels	Blake and Roberts (2006)
Initial stock − low quality	75 million barrels	Blake and Roberts (2006)
r	10%	(assumed)
π	3%	(assumed)
α	6.2%	WTI Crude Oil – 1986-2005
ϕ	10%	Dixit & Pindyck (1994)
σ_P	21.18%	WTI Crude Oil – 1986-2005
σ_C	10%	(assumed)
ρ	+0.9	Adelman et Al. (1989)
$i \equiv I/C$	1	(assumed)
$\varepsilon \equiv I/C$	1	(assumed)
τ	10%	Brazilian Law (ceiling-rate)
R	0%	Brazilian Law (minimum)

The discount rate was taken as 10% a year; both drift and standard deviation of oil prices were estimated from annual data of WTI Crude Oil, from 1986 to 2005[20]. We assume the standard deviation of operational cost as around one half of the standard deviation of oil price, since the producer has more information regarding his costs than the set of factors that affects

[20]Source: EIA/US Department of Energy.

oil price. Variable i is the investment-cost ratio to develop a barrel of oil. The higher i, the lower the ore grade[21]. We assume a reference value of 1 for this variable, as well as for the exit-operational cost ratio, ε.

The instantaneous correlation coefficient between price and cost, ρ, is also a relevant parameter. It was calibrated considering the well-known strong positive relationship between price and costs (Adelman et Al, 1989). The idea behind it is that when oil prices are high, marginal producers enter the market, pressuring input prices, wages and operational costs due to the higher demand for machines and equipments. Trigger ratios x^E and x^S were calculated with $\rho = .9$ (Adelman et Al, 1989; Dias, 1996).

Finally, both royalties and special participations rates were defined according to the Brazilian Law. Royalties' rates vary between 5% and 10% of gross production of oil, depending on the risk and on profitability conditions, but almost all projects are charged with the maximum rate of 10%, which is set as parameter. Special payments are progressive rates from 0 to 40% according to the volume extracted. The rate is applied over the net revenue, from which all costs and royalties already paid are deducted. Only highly profitable projects are charged with this tax and a typical (average size) oil project in Brazil is exempt. As a starting reference, the model is calibrated with a zero-rate of special participation[22]. Further, we simulate non-zero rates in combination with royalties to investigate the impact on the trigger ratios.

Calibrating the model with the values in table 2 gives the reference ratios displayed in table 3. To investigate the fiscal impacts on the decision to invest, we evaluate the effects in x^E and x^S due to simulated combinations of royalties and special participation rates (τ, R). The exercises were performed

[21] Grade is a concept linked to ores in general. According to Costa Lima and Suslick (2006), grade is defined as the ratio of useful mass of metal to the total mass of rock. For fluid resources, loosely speaking, grade is a measure of quality. Other things being equal, a lower grade means a higher average cost of extraction.

[22] We do not include usual Brazilian income taxes and other legal taxes (IRPJ and CSLL) which, together, represent a rate of 34% on the net revenue of every project. Thus, $R = 0$ means that only regular taxes charge the project. The only effect of these taxes would be a linear and homogeneous increase in all simulated values, without impairing qualitatively the analysis.

in *Matlab*® *7.0*.

Table 3: Reference results p* = P/C

Model	x^E	x^S
Net Present Value	1.834	-
Real Option -without option to exit	2.152	-
Real Option – With costless option to exit	1.584	.967
Real Option – With option to exit at costs ($\varepsilon = 1$)	1.716	.889

The development trigger ratio grows as the option to invest is incorporated to NPV, due to the irreversibility effect. However, when there is the option to abandon, this threshold value is reduced and the investment becomes more attractive. Managerial flexibilities add values to the project, making easier the decision to invest (Kulatilaka, 1995). When there are abandonment costs, the trigger ratio to invest increases while the trigger ratio to exit decreases, generating a higher hysteresis effect.

Table 4 displays the simulated trigger ratios to develop (x^E) in the absence of the option to abandon, for some combinations of tax-rate (τ, R). In this case, investors must decide an irreversible development[23]. Values in parenthesis are the distortion degree, which is the percent-change in x^E due to the fiscal charge. In the absence of the option to exit, royalties produce a heavier distortion on the decision to invest than the special participation, confirming results from the traditional literature, based on NPV.

[23] According to the Brazilian law, the concessionaire cannot abandon or shit the lease without ANP's permission.

Table 4: Trigger ratios without option to exit

$x^E \left(\frac{\Delta x^E}{x^E}\%\right)$		$\tau = 0\%$	$\tau = 5\%$	$\tau = 10\%$
	0	1.936	2.038	2.152
		-	(5.26%)	(11.11%)
	10	1.951	2.053	2.167
		(.72%)	(6.02%)	(11.91%)
R(%)	*20*	1.968	2.072	2.187
		(1.63%)	(6.98%)	(12.92%)
	30	1.991	2.096	2.212
		(2.80%)	(8.21%)	(14.22%)
	40	2.021	2.127	2.245
		(4.36%)	(9.85%)	(15.95%)

If there is the option to exit the industry, the conclusions about distortions change. Now, special participations are able to affect more deeply the investment, as we can see in the simulated values for x^E, x^S and for the degree of distortion, in table 5. For example, both combinations of $(\tau, R) = (0, 40\%)$ and $(\tau, R) = (10\%, 0)$ cause a similar distortion in x^E. However, special participations contributes to reduce x^S, that is, the decision to exit becomes less likely. Besides, while the royalty rate and the exit price (x^S) vary both in the same direction, an increase in the special payment rate (R) lowers x^S. The explanation for this effect is the risk sharing profile of a resource rent tax (Mayo, 1979), that is, it reduces both the average and the standard deviation of the investment's value.

Table 5: Distortion Degree , with the option to exit

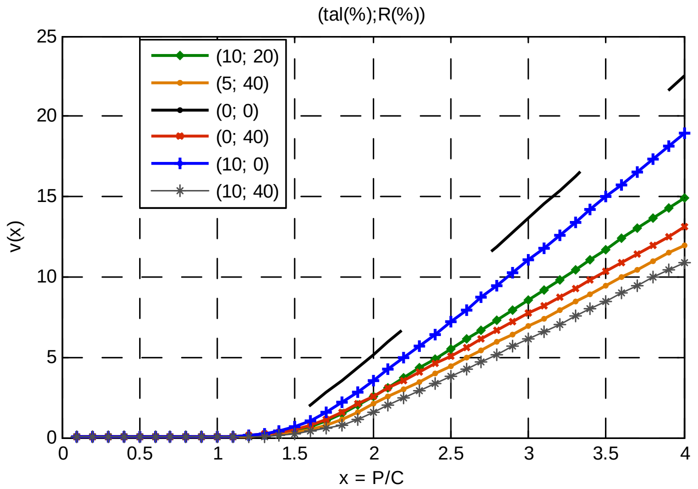
$x\ (\frac{\Delta x}{x}\%)$		$\tau = 0\%$		$\tau = 5\%$		$\tau = 10\%$	
		x^E	x^S	x^E	x^S	x^E	x^S
	0	1.544	.800	1.626	.842	1.716	.889
		-	-	(5.26%)	(5.26%)	(11.11%)	(11.10%)
	10	1.575	.792	1.658	.834	1.750	.880
		(1.99%)	(-.96%)	(7.36%)	(4.24%)	(13.32%)	(10.03%)
R(%)	*20*	1.612	.783	1.697	.824	1.791	.870
		(4.38%)	(-2.14%)	(9.87%)	(3.00%)	(15.97%)	(8.73%)
	30	1.657	.771	1.744	.812	1.841	.857
		(7.30%)	(-3.61%)	(12.95%)	(1.46%)	(19.22%)	(7.09%)
	40	1.714	.756	1.804	.795	1.905	.840
		(10.99%)	(-5.52%)	(16.84%)	(-.54%)	(23.32%)	(4.97%)

Figure 1 plots the unit value of developed reserve, $v(x)$, as a function of x-ratio for selected combinations of fiscal rates (τ, R). High special participation rates distort more the investment, as it reduces the reserve value and increases the trigger ratio for development.

Figure 2 plots the trigger ratios to develop the reserve (x^E) as a function of royalty rates, considering the special participation rate fixed at 20%. Confirming the usual insights, the presence of option to abandon reduces the threshold ratio in the extent that adds value to the investment, lowering the value of waiting. Figure 3 plots the trigger ratio for development according to the special participation rate for a royalty rate of 5%. The special participation regime distorts more deeply the decision to invest (relative to full exemption) when the abandon is possible, but trigger ratios, both with and without option to exit, converge as R approaches to 1.

Figures 4 and 5 plot the trigger ratio to exit (x^S) varying, respectively, the royalty rate and the special participation rate. While higher royalty rates increase x^S - which eases the decision to exit - special participation decreases it, that is, the likelihood of exiting is lower. A possible explanation for this phenomenon is that the Brazilian special participation tax exhibits a risk sharing profile, since it searches to charge only the rent. According to Mayo (1979) a resource rent tax reduces both the average and the standard deviation of profits.

Figure 1: Unit value of reserve



Results show that there are more attractive fiscal designs (in terms royalties and special participation rates) for investments. For example, a combination of $(\tau, R) = (5\%, 20\%)$ makes the investor wait less to develop the reserve than a single 10% royalty-rate, which charges a typical oil and gas project in Brazil, since the most part of projects is exempt from special participation. Moreover, this combination $(5\%, 20\%)$ becomes the investor less willing to close under bad economic conditions, due to a lower x^S. Thus, the focus on intermediate combinations of rates, increasing the use of special participation, should be considered as a mechanism to attract investments in oil and gas production.

5.2 Expected revenues

If the aim of the government is to attract new investments in oil extraction, results above suggest there are less distorter combinations of fiscal rates than an uniform 10%-royalty rate. However, the distortion on the investment should not be the single variable to guide the decision to choose the fiscal rates, since it is also necessary to evaluate the potential fiscal revenues. The collection of rents, on the other hand, depends on the expected price and

Figure 2: Development trigger ratios x^E) and royalties.

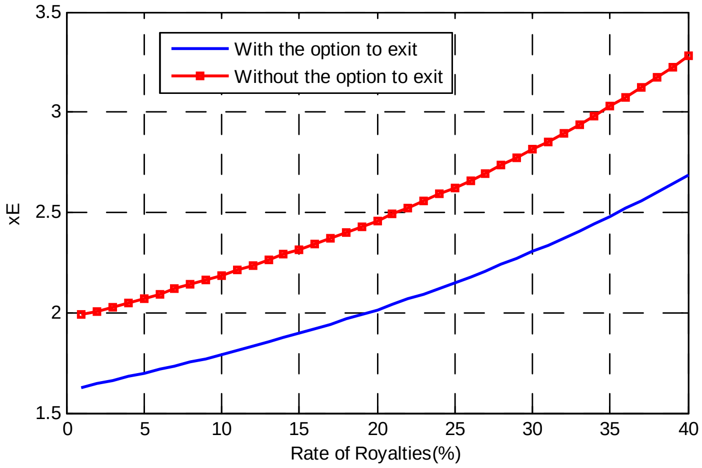


Figure 3: Development trigger ratios and the special participation

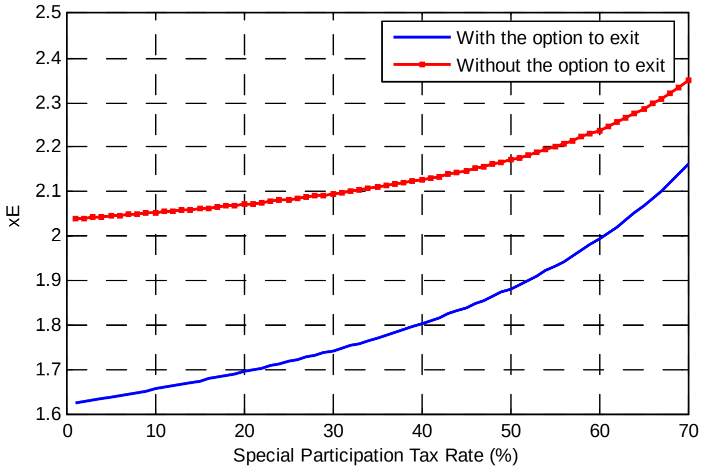


Figure 4: Trigger ratios to exit (x^S) and royalties

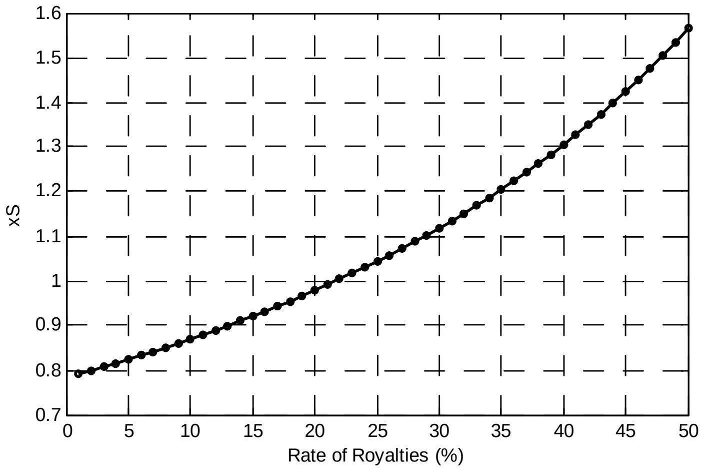


Figure 5: Trigger ratio to exit (x^S) and special participation

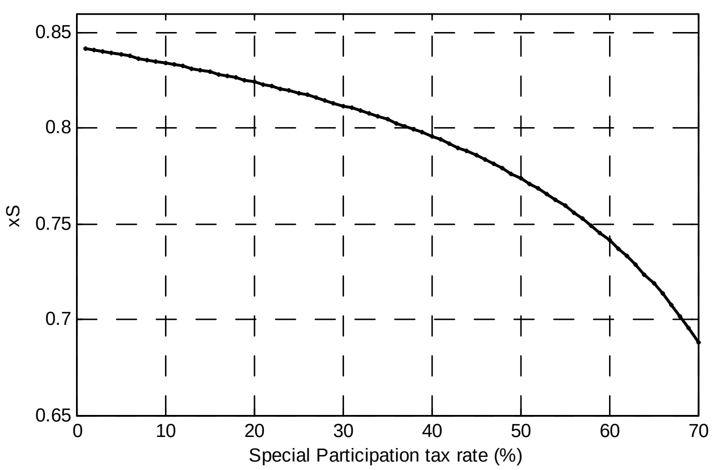
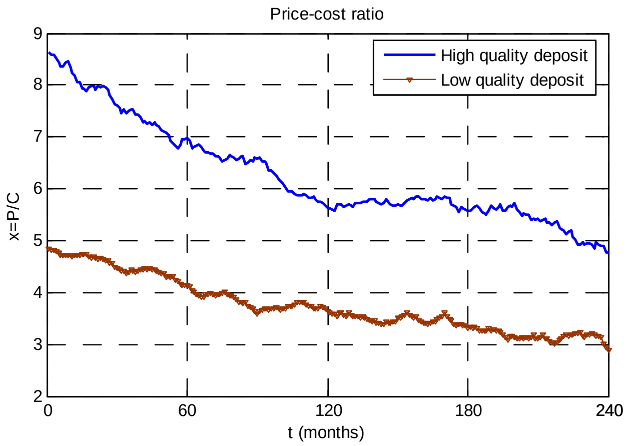


Figure 6: Expected path for $x \equiv P/C$ (sample=100)



cost paths.

With the purpose of evaluating the potential collection provided by each rate structure, it is performed a Monte Carlo exercise to simulate paths for price and cost in the next twenty years. The experiment was calibrated with data from table 2, through a sample of one thousand replications (that is, it was generated 1000 possible paths for P and C). Figure 6 reports the corresponding expected path for $x \equiv P/C$, assuming a initial oil price of \$ 50 a barrel. It was also assumed a reserve decaying rate of $\phi = 10\%$[24], that is, if S_0 is the initial stock level, than the stock in instant t is given by $S_t = S_0(1-\phi)^t$. Table 6 reports the present value of the collected revenue, according to the chosen combination of royalties and special participations rates for a high quality reserve, with initial cost of \$ 3.70[25] and 300 millions of barrels. Table 7 reports the same results for a low quality deposit of 75 millions of barrel, in which the initial cost is \$ 5.08. Both results consider the threshold ratios of exit (x^S) estimated in table 5.

[24] See footnote 7

[25] Based on a World Bank study, Blake and Roberts (2006) present data regarding the relationship between the field size and the total cost per barrel. A field of 300 millions of barrels has an estimated operational cost of \$ 3.70 per barrel while a field of 75 millions of barrel presents a cost of \$ 5.08. Based on Lund (1992) they assume an effort function that establishes a relationship between the investment in exploration and the size of the recoverable reserves.

Table 6: Expected present value of revenue, in millions of dollars – high quality deposit - 300 millions of barrels.

Revenue	$\tau = 0$	$\tau = 5\%$	$\tau = 10\%$
R(%)			
0	0	66.671	133.343
10	123.185	183.189	243.194
20	246.371	299.708	353.045
30	369.556	416.226	462.896
40	492.742	532.744	572.747

Table 7: Expected present value of revenue, in millions of dollars – low quality deposit, 75 millions of barrels.

Revenue	$\tau = 0$	$\tau = 5\%$	$\tau = 10\%$
R(%)			
0	0	16.676	33.353
10	29.860	44.869	59.877
20	59.719	73.061	86.402
30	89.579	101.253	112.927
40	119.439	129.446	139.451

Results suggest that a fiscal system based on special participations can be more effective in collecting a higher portion of resource's rent, mainly in high grade deposits. A combination of $(\tau, R) = (5\%, 20\%)$, for example, is able to collect more rent at the same time than distorts less the decision to invest (tables 4 and 5). Results clearly suggest there are combinations of rates that generate a lower distortion on the investment than a single 10% royalty-rate over the value of each barrel (as a typical project in Brazil is charged), with the advantage of producing higher expected revenues.

An important conclusion derived from simulations above is that since projects are qualitatively heterogeneous and present different managerial flexibilities, a more flexible fiscal rate, applying different tax burdens on different projects, can be an important mechanism to attract investments in oil and gas sector.

6 Conclusion

The aim of this paper was to evaluate the distorter effect of the Brazilian fiscal regime on petroleum upstream – royalties and special participations – on the decision to invest in the development of oil and gas reserves, in light of a real options analysis, which allows the incorporation of managerial flexibilities in the value of the project.

Conventional NPV-based literature shows that royalties are capable to introduce more distortions in the investment than the resource rent tax, because royalties can both become negative a NPV and alter the rank of profitability among projects.

Our results intend to contribute to discussions about changes in the Brazilian fiscal system on oil extraction. Simulations show there are better combinations of royalties and special participations than a single 10% royalty - rate under either criteria (impact on investment and expected collected revenue). Brazilian government may be underestimating the potential of special participation in the extent that, today, this modality is restricted to high profitable projects. Therefore, regulatory authorities would be able to increase investments in the development of oil and gas reserves without losing rents through the adoption of flexible rates, which would fit better in low quality deposits.

This work still has some limitations and several extensions are possible: more complexes stochastic processes to describe the state variables governing the value of reserve (e.g.: mean reverting process, jumps, more stochastic factors, etc.). We performed a risk free evaluation (the discount rate was assumed exogenous) but one could introduce risk aversion (through CAPM, for example). Finally, further managerial flexibilities can be considered beside option to invest and option to exit.

Nevertheless, such results are sufficient to show the importance of considering different fiscal structures for different reserves and flexible rates can be an instrument for attracting more investments. By adopting a single 10%-royalty rate with little or no emphasis on special participation, the Brazilian government may be wasting potential investment opportunities.

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