Real Options Models with Non-Linear Dynamics

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Abstract

The value of a real or financial option depends among other factors on the assumption of the underlying stochastic process. Linear and loglinear processes are most common, such as the arithmetic Brownian motion, the geometric Brownian motion and the Ornstein-Uhlenbeck process. In the time series literature, non-linear continuous time models have been developed. One such class of models is the threshold-autoregressive model, where the dynamic process changes character depending on whether the process is above or below a certain threshold. In this paper we investigate real option modelling when uncertainty can be described by a continuous time threshold autoregression. Closed form solutions to perpetual American options on such processes are derived. Various applications are studied, focusing on how uncertainty and non-linearity can affect option valuation and investment. This includes examples where uncertainty encourages investment, contrary to the result with most real options models.

JEL classification: D21; D81; D92.

Keywords: Investment; Uncertainty; Growth, Threshold Diffusions.

1 Introduction

In this paper we develop and apply a real options modeling framework where the underlying state variable can be described by continuous time threshold diffusions. More precisely, the dynamics of the state variable changes character at certain thresholds.

Threshold models have existed in the time series literature for a long period of time, with the contribution of Tong (1978) for the discrete time threshold autoregressive model a seminal paper. The book by Tong (1990) summarizes many models in the non-linear time series literature, and Franses and van Dijk (2000) review non-linear models applied in finance. The bulk of time series literature has focused on discrete time models, but continuous time versions of threshold models have been developed; see for instance Brockwell et al. (2006). Still, most applications of TAR-models in economics and finance have been cast in discrete time.

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One such application is Ng (1996). She investigates the classical inventory model in a commodity market. Essentially, in this model the commodity price is in one of two possible regimes, depending on whether inventories are held or not. She tests the model using a TAR-model and finds threshold behaviour in the commodity price series.

Threshold models have been used to investigate price linkages in spatially separated markets. The law of one price commands equal prices across spatial markets, but transaction costs and other market frictions allow for temporary departure from equilibrium. Goodwin and Piggot (2001) test a model for spatial market integration for several North American corn and soybean markets. They find that threshold models suggest much faster adjustments in response to deviation to equilibrium than is the case when thresholds are ignored.

Sødal et al. (2008) suggest a real option application of integrated shipping markets. They devolop a valuation model for a combination carrier (a ship that can carry both oil and iron ore) derived from the spread between freight rates in dry bulk and tanker markets. The valuation of the combination carrier depends directly on the dynamics of the spread between the two markets. However, no explicit threshold dynamics is assumed for the spread, rather it is modeled as a linear Ornstein-Uhlenbeck process.

Threshold models have been applied to the foreign exchange markets to model currency regimes. Krugman (1991) dicussed the dynamics of the foreign exchange rate in a target zone model. This model has been the focus of a lot of research, both theoretical modifications and empirical estimation. For instance, Lundbergh and Terasvirta (2003) estimate variations of the model using threshold autoregressions to series of daily observations of the Swedish and the Norwegian currency indices. Stegenborg and Sørensen (2003) use a continuous time model for the exchange rate in a target zone model, where the exchange rate is modeled by a Jacobi diffusion. This approach leads to fairly straight forward option pricing formulas. However, it is a restricted model in the sense that the exchange rate cannot depart from the targets set by monetary authorities.

The case for threshold dynamics can also be supported by partial equilibrium arguments in several real market settings with entry and exit by independent firms. Imagine a competitive industry with a large number of unit-sized firms, fixed entry and exit costs, and demand given by a fixed downward-sloped demand curve. Suppose, for example, that the demand curve is exposed to geometric Brownian shocks. Then the price process facing individual firms will also typically be geometric Brownian but with two reflecting barriers: an upper barrier caused by entry and a lower barrier caused by exit. Each firm will end up making the optimal investment timing decisions even if they ignore the existence of these barriers; that is, by assuming that the price is geometric Brownian without barriers. Profits will be affected by the existence of the barriers but the optimal thresholds at which to invest or disinvest will not; see Leahy (1993) for a discussion of equilibrium, and Mossin (1968) and Dixit (1989) for seminal earlier contributions in this field.

In practice, entry and exit as mentioned above more likely will create thresholds where the price process changes character rather than becoming perfectly reflecting barriers. Still the qualitative nature consists of some kind of reversion back from extreme values. In markets with economies of scale, the situation could be the opposite at least for significant periods of time. For examples, demand or prices can sometimes be expected to increase after some critical mass has been achieved in an industry. In a simplified model, such a critical mass can be reflected in a price threshold above which a positive price drift is expected. Examples include industries with positive network externalities, bandwagon effects etc. Our main example illustrates with urban agglomeration and growth, focusing on when to develop a piece of rural land that is located in the vicinity of an urban area. For completeness, we also present a brief example of a sales decision which does not yield the same kind of non-linear effects as the main example. The decision in the latter case will be like exercising an American put option as opposed to a call option in the first example. The second example is also illustrated with the real estate business, although several interpretations may be possible for both models.

The rest of the paper is organised as follows: In section 2 we set up a general non-linear model with two price domains that are separated by a exogenous price threshold. In section 3.1, the discount factor function under geometric Brownian price process in two such domains is derived. Then, in sections 3.2 and 3.3, this model is illustrated with the two examples mentioned above. Section 4 sums up and discusses briefly how the model can be extended in various directions.

2 A baseline model

Suppose that the revenue, P, from an irreversible investment, C, is given by the following continuous and autonomous Ito processes:

$$dP = \begin{cases} f_1(P)dt + g_1(P)dZ & (P < P_x) \\ f_2(P)dt + g_2(P)dZ & (P > P_x) \end{cases} \tag{1}$$

Here $f_i(P)$ and $g_i(P), i = 1, 2$ are usual drift and volatility functions, while P_x is the threshold separating the two price process domains. The investment cost is fixed, so the optimal decision will be to invest as soon as the price reaches some optimal level that is to be determined.

Assume that the current price is a fixed P_0 and that the trigger price for investment is some high value. As in Dixit et al. (1999), the expected net present value from investing at an arbitrary price $P \quad (> P_0)$ equals

$$V_0 = D(P_0, P)(P - C) \tag{2}$$

Here $D(P_0, P) = E[e^{-\rho T}]$ is the expected discount factor for a first-hitting time (T) up to P when starting from P_0 and discounting at rate ρ. The optimal price at which to invest, P^*, is found by maximizing V_0 with respect to P.

Alternatively, suppose that the option is a sales option where one can sell an asset for a fixed price K. The value of the asset fluctuates according to a process similar to (1). Then the expected net present value from selling when some $P < P_0$ is reached from above, equals

$$V_0 = D(P_0, P)(K - P) \tag{3}$$

where $D(P_0, P)$ is once again the expected discount factor for a first-hitting time from P_0 to P, but now in a downward direction.

The method for finding such discount factors presented in Dixit et al. (1999), combined with smooth pasting theory, implies that the discount factor function $D(P_0, P)$ for a motion from P_0 to P are found by solving the two differential equations

$$\frac{1}{2}g_i^2(P_0)\frac{\partial^2 D(P_0, P)}{\partial P_0^2} + f_i(P_0)\frac{\partial D(P_0, P)}{\partial P_0} - \rho D(P_0, P) = 0 \qquad (i = 1, 2) \tag{4}$$

Each of the two general solutions to (4) typically involve two constants. The discount factor function is derived by support from up to four boundary conditions. First, we must have $D \to 0$ as P_0 and P get far apart. Second, there is no discounting if the price is already at the destination, implying $D(P, P) = 1$. The process is continuous, so the threshold is irrelevant if $P_x \notin (P_0, P)$ because the price in such a case will stay in one domain all the time. Then the two first boundary conditions just described are sufficient to determine the discount factor function. In cases where $P_x \in (P_0, P)$, the value matching and smooth pasting conditions apply at the threshold between the price domains. This yields

$$D(P_x^-, P) = D(P_x^+, P) \tag{5}$$

$$\left[\frac{\partial D(P_0, P)}{\partial P_0}\right]_{P_0 = P_x^-} = \left[\frac{\partial D(P_0, P)}{\partial P_0}\right]_{P_0 = P_x^+} \tag{6}$$

The two superscript signs for P_x indicate whether the limit is taken from the upper or lower price domain.

3 Examples

3.1 Discount factors for geometric Brownian motions

If the processes in (1) are geometric Brownian motions with drift μ_i and volatility σ_i $(i = 1, 2)$, it is well known that equation (4) has the general solutions

$$D(P_0, P) = A_i P_0^{-\alpha_i} + B_i P_0^{\beta_i} \qquad (i = 1, 2) \tag{7}$$

where A_i and B_i are constants (which in our boundary problem generally depend on P). The variables $-\alpha_i$ and β_i are the negative and positive roots, respectively, of the following quadratic equations:

$$\frac{1}{2}\sigma_i^2 x(x-1) + \mu_i x - \rho = 0 \qquad (i = 1, 2) \tag{8}$$

Note that the α_i's and β_i's are defined as positive numbers. Moreover, one needs to distinguish between four different kinds of relationships between the variables that make up the discount factor function:

(1) $P_0 < P$ and $P_x \notin (P_0, P)$. This reflects an upward price change for a regular geometric Brownian motion with no non-linearities. The applicable constant A_i must be equal to zero since $P_0^{-\alpha_i} \to \infty$ as $P_0 \to 0$. The condition $D(P,P) = 1$ then yields the familiar discount factor function

$$D(P_0, P) = (P_0/P)^{\beta_i} \tag{9}$$

where the subscript i refers to the price domain (upper or lower) in question.

(2) $P_0 > P$ and $P_x \notin (P_0, P)$. This reflects a downward price change, also over a price interval with no threshold. Now we must have $B_i = 0$, and the following discount factors result:

$$D(P_0, P) = (P/P_0)^{\alpha_i} \tag{10}$$

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The two remaining cases include crossings over the threshold P_x in either direction. Value matching (5) and smooth pasting (6) applies at the border:

$$\begin{aligned} A_1 P_x^{-\alpha_1} + B_1 P_x^{\beta_1} &= A_2 P_x^{-\alpha_2} + B_2 P_x^{\beta_2} & (11)\\ -\alpha_1 A_1 P_x^{-\alpha_1 - 1} + \beta_1 B_1 P_x^{\beta_1 - 1} &= -\alpha_2 A_2 P_x^{-\alpha_2 - 1} + \beta_2 B_2 P_x^{\beta_2 - 1} & (12)\end{aligned}$$

Once again the expression for the discount factor depends on the direction of motion:

(3) $P_0 < P$ and $P_x < P$. For such an upward motion the boundary condition $D(P,P) = 1$ implies

$$A_2 P^{-\alpha_2} + B_2 P^{\beta_2} = 1 \tag{13}$$

In this case we also know from the discussion that $A_1 = 0$. Eq. (13) together with value matching (11) and (12) determines the constants A_2, B_1 and B_2. The following expression for the discount factor function is obtained:

$$D(P_0, P) = \frac{(\alpha_2 + \beta_1)(\frac{P_0}{P})^{\beta_2} + (\beta_2 - \beta_1)(\frac{P_x}{P})^{\beta_2}(\frac{P_x}{P_0})^{\alpha_2}}{\alpha_2 + \beta_1 + (\beta_2 - \beta_1)(\frac{P_x}{P})^{\alpha_2 + \beta_2}} \qquad (P_0 \geq P_x) \tag{14}$$

Note that eq. (14) only covers cases where $P_0 \geq P_x$. When $P_0 \leq P_x \leq P$ the discount factor can be found by using the fact that

$$D(P_0, P) = D(P_0, P_x) D(P_x, P) \tag{15}$$

for any P_x between P_0 and P; see Dixit et al. (1999).

(4) $P_0 > P$ and $P_x > P$: For this downward motion the boundary condition $D(P,P) = 1$ implies

$$A_1 P^{-\alpha_1} + B_1 P^{\beta_1} = 1 \tag{16}$$

We also have $B_2 = 0$, so the constants B_1, A_1 and A_2 are found from eqs. (16), (11) and (12). This yields

$$D(P_0, P) = \frac{(\alpha_2 + \beta_1)(\frac{P}{P_0})^{\alpha_1} + (\alpha_1 - \alpha_2)(\frac{P}{P_x})^{\alpha_1}(\frac{P_0}{P_x})^{\beta_1}}{\alpha_2 + \beta_1 + (\alpha_1 - \alpha_2)(\frac{P}{P_x})^{\alpha_1+\beta_1}} \qquad (P_0 \leq P_x) \qquad (17)$$

while cases for which $P_0 \geq P_x$ are handled by use of eq. (15).

The functional form made up by the expressions (9), (10), (14) and 17) completes the description of the discount factor for a non-linear stochastic process with two geometric Brownian motions separated by the exogenous threshold P_x. The discount factor simplifies as expected in limiting cases. For example, we have $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ if the two process are identical. Then eqs. (14) and (17) simplify to (9) and (10).

3.2 An investment (call) option: urban development

It is well known from economic theory of agglomeration and growth that a critical mass must sometimes be reached in order for an economic growth process to take off (Fujita et al. 1999). The price of land, which is the stochastic variable in our first application, can be expected to rise rapidly if the critical mass is reached. It is also reasonable to assume that the potential for growth will be reflected in the current price of land, P_0, in the sense that growth will be more likely to take off the higher the current price. For simplicity, we shall restrict to a setting with only two price regimes. We assume that a stochastic process with expectedly higher growth is spurred as soon as a certain price P_x is hit from below, in accordance with the specifications above.

The owner of a certain piece of rural land in the vicinity of an urban area is facing such a situation. The current price, P_0, is too low for immediate development of the property, but how long is it optimal to wait? One option consists of waiting for a normal return based on the rural price regime ($P < P_x$), but could it be wise to wait until agglomeration forces have spurred rapid growth in the region - i.e., wait for a price above P_x?

Fig. 1 plots the value function (2) for three sets of data. The numbers are chosen mainly in order to demonstrate some qualitative characteristics of such investment problems. Both price processes are geometric Brownian, so the applicable discount factor is found from either eq. (9) or eq. (14). The base case curve in the middle assumes $\mu_1 = 0$, $\sigma_1 = 0.10$, $\sigma_2 = 0.10$, $\mu_2 = 0.037$, $\rho = 0.05$, $P_x = 5$, $C = 3$, and $P_0 = 3$.[1] In this case the land owner is almost indifferent between investing before or after urbanization and growth forces set in. Ignoring the growth potential in the future, he would invest at the leftmost local maximum according to a McDonald-Siegel (1986) rule (at $P^* = \frac{\beta_1}{\beta_1 - 1}C$). Here this implies development at $P \simeq 4$. If waiting for growth to take off at $P = 5$, he should postpone development until $P \simeq 13$, where there is also a local

[1] Note that the optimal trigger price does not depend on P_0 as long as waiting applies. For simplicity we set $P_0 = C$, since it will never be optimal to invest before the gain from the investment exceeds the investment cost.

maximum. This could indeed take a long time, considering that the expected annual growth rate is only 3.7 percent. With the chosen numbers the two local maxima happen to be very close ($V_0 \simeq 0.35$).

< FIGURE 1>

The two outer curves in Fig. 1 are based on the same data as the middle curve except for the drift in the upper price domain ($P > P_x$). The lower assumption for this drift parameter ($\mu_2 = 0.030$) makes it less beneficial to wait than the base case value ($\mu_2 = 0.037$). With the higher drift rate ($\mu_2 = 0.040$) it is clearly optimal to wait for urbanization rather than ignoring this possibility. The rightmost local maximum (for $P \simeq 17$) is clearly higher than the leftmost maximum.

Fig. 1 demonstrates several results of interest. First, even a small change in a variable can create strong market shocks due to the non-linear character of the model. A marginal increase in the expected price drift after take-off could under some circumstances delay investment delay with several years. Second, the local maximum to the left has a higher second order derivative than the other one.[2] This implies that the exact timing of development is more critical for the owner under investment in the rural price domain. Third, it can be shown that the optimal trigger price for development after growth take-off is slightly below a McDonald-Siegel trigger price based exclusively on the upper price process ($P > P_x$). The reason is that even if growth takes off it may not be sustained due to uncertainty. If a recession later comes about with prices once again below P_x, one could be trapped there. The risk of such outcomes represents a cost that makes it optimal to invest earlier than in the McDonald-Siegel model. (It is extremely small for this data set.) Finally, if the leftmost local maximum were to be ignored, we recognize the familiar result from many models without non-linear effects, that increased price drift (μ_2) increases the optimal trigger price for investment.

Fig. 2 shows similar plots but now varying price uncertainty in the growth domain (σ_2) between 0.07 and 0.13. Uncertainty in the rural domain is kept constant for all curves ($\sigma_1 = 0.10$) . Therefore, the curve in the middle coincides with the middle curve in Fig. 1. Increased uncertainty could make waiting *less* profitable. With $\sigma_2 = 0.13$ it is better to invest before the growth take-off. The opposite is true for $\sigma_2 = 0.07$. This contrasts the results of most real options models, for which the value of an investment option and the value from waiting are increasing in uncertainty. The reason is once again the risk of growth failure. Uncertainty increases the value of waiting for higher prices but also the risk of a price decline and slow recovery from a potential recession. As the arrow in Fig. 1 indicates, however, uncertainty shifts the trigger price for investment marginally to the right when ignoring the leftmost local maximum. Then uncertainty would again encourage waiting. Hence, the effect of uncertainty on the value of waiting is generally ambiguous in this model.

[2] The curve is steeper up to the leftmost local maximum. This is also true when using logarithmic scale, so the sensitivity is also higher in percentage terms.

< FIGURE 2>

Fig. 3 plots similar results as in Fig. 2 expect that uncertainty in both regimes is varied simultaneously. The lower curve is based on $\sigma_1 = \sigma_2 = 0.07$ while the upper curve assumes $\sigma_1 = \sigma_2 = 0.13$. Now a marginal increase of uncertainty encourages waiting marginally in both price domains. It may also create a non-linear shift of investment timing. This happens because the option value created by uncertainty is generally higher in the upper price domain, where the expected growth rate is higher and the cost of waiting accordingly lower (due to discounting).

< FIGURE 3>

Fig. 4 illustrates the non-linear effects of the model by plotting the option value as a function of the initial price for two levels of uncertainty in the price growth domain ($\sigma_2 = 0.07$ and $\sigma_2 = 0.13$). Other parameters are as in Fig. 2 ($\mu_1 = 0$, $\sigma_1 = 0.10$, $\mu_2 = 0.037$, $\rho = 0.05$, $P_x = 5$, $C = 3$). The option value curve with low uncertainty ($\sigma_2 = 0.07$) is almost tangent to the value line ($P_0 - C$) at the point where investment is optimal with high uncertainty ($\sigma_2 = 0.07$), but the value of the growth option takes over soon enough to dominate.

< FIGURE 4>

The wider implications of the results from this example are difficult to infer without expanding to an equilibrium setting. For example, if growth creates positive welfare effects, policy makers are typically concerned with the question of whether it makes sense to pay for reducing risk by political means. The model showed that uncertainty could encourage early rather than late development, but the opposite could also be true. Moreover, does the timing of investment or political investment stimuli influence the likelihood of urbanization and growth? One the one hand, early investment could be expected to encourage self-reinforcing agglomeration forces if many land owners reacted similarly. In this respect early investment ought to encourage (endogenous) growth. On the other hand, with many land owners investing in this manner, prices will typically go down. It is evident that our example, with a single threshold separating two otherwise geometric stochastic processes, is not rich enough to reflect all such equilibrium effects.

3.3 A sales (put) option: Ownership or a perpetual lease?

Some 10-15 percent of privately owned homes in Norway are built on property that is rented under a perpetual lease contract. A typical contract leaves the land owner with few rights other than a flow of land rents. The rents as well as the price if the land owner were to sell a lot to the house owner, are typically regulated by law in ways which not fully reflect market prices.[3] This particular ownership structure has a long history, but it has also led to many conflicts as well as to some recent changes in laws and regulations. Leaving aside such complicating issues, the interesting fact in our context is that a shift of ownership from the land owner to the house owner could often create additional value, e.g. as it might enable the house owner to develop the property more efficiently than under the lease restrictions.

The decision facing the land owner might, in practice, have the form of a sales option. He would be able to sell one or more lots basically any time for a certain, typically fairly low price (K) under the prevailing restrictions. By doing so, however, he would give up a perpetual flow of rents. Suppose that the expected and discounted value of rents by selling today equals P_0, and that the similar cost of selling in the future is governed by a stochastic process of the usual kind (1). By exercising the option at some P ($< P_0$), the land owner's expected net present value will be given by eq. (3). His optimal decision can be found by maximization as above, and non-linearity be expressed by a threshold P_x and explained by similar arguments. For example, rents can be expected to increase in case of urban growth that is not reflected in the permitted sales price.

As in the previous example, we assume that both price processes are geometric Brownian motions, but the parameters values are quite different. We set $\sigma_1 = \sigma_2 = 0.2$, $\mu_2 = 0.1$ and $\rho = 0.1$. The threshold is set at $P_x = 5$, and we also set $K = 5$.

Fig. 5 plots the value function (3) for three different drift terms in the lower regime, assuming $P_0 = 5$. Not unexpectedly, we see that reducing $\mu_1 = 0.1$ to $\mu_1 = 0$ and $\mu_1 = -0.05$ increases the value of the option. It also becomes optimal to wait for a lower P before exercising the right to sell.

< FIGURE 5>

Fig. 6 plots the value of the put option for different initial values. We vary P_0 from 7 down to the optimal exercise price. This is done for each parameter scenario. No shockwise effects as in the first example results. Nonetheless, the curves in Fig. 6 indicate that the drift term in the lower regime have a fairly large and non-linear influence on the option price. It would be exciting if similar results were obtained with a well-founded data set of practical interest, e.g. from a financial market application.

< FIGURE 6>

[3] The exact specification of the contract as well as the government regulations of rents are of utmost importance in such contexts if such market prices were to be defined. Without rent regulations, any irreversible investment by the home owner on the property could otherwise leave the land owner with an option to extract the return by increasing the rent after the investment is made.

4 Conclusions

A framework for option valuation and optimal investment decision making with non-linear dynamics has been spelled out and applied. All numerical examples were based on geometric Brownian prices in two domains separated by a fixed threshold. We have seen that non-linearity can create discrete time shift of investment arising from marginal changes in exogenous variables. Non-linear dynamics also enabled a more complex and potentially more realistic modelling of risk. Uncertainty can create two opposing effects under such conditions. On the one hand, increased uncertainty increases the value of waiting and discourages investment as usual in real option models. On the other hand, it may also increase the risk of some failure or catastrophy in the future, thereby encouraging investment now. The net effect is generally ambiguous.

Multiple thresholds and price processes beyond geometric Brownian motions could be applied to increase realism. As discussed in the introduction, one cannot expect extreme outcomes like processes with reflecting barriers in most markets of practical interest. Firms differ in size and costs, and demand and technology characteristics change continuously. This turns otherwise clear-cut barriers for price movements in the markets into softer pillows or elastic bands. More likely, a price process may have mean-reverting properties but not necessarily towards a fixed target. An Ornstein-Uhlenbeck process and similar processes have such a unique long-term mean value, implying that the price drifts downward as soon as the mean is exceeded while drifting upwards in the opposite case. The drift is stronger the farther away the current value is from the mean. Equilibrium considerations indicates that the mean reversion characteristics could be more complicated. With fully or partly irreversible entry and exit, the mean-reversion drift may not be towards a certain value but towards a price band caused by uncertainty and irreversibility. Inside such a band of in-action, the price could be a non-drifting geometric Brownian motion or a similar diffusion. Outside the band, the mean reversion force may be stronger the more homogenous the firms and the more competitive the industry. The exact character may well depend on whether the price is above or below the normal band for price variations. In order to model such markets properly one needs at least three different processes: one for the motion inside the normal price band and one for the motion on each side of the band.

The modelling framework of this paper can handle such non-linear cases, but stochastic processes other than the geometric Brownian motion easily lead to technical problems as far as closed-form solutions are concerned. Such problems could be circumvented by allowing for many thresholds but by letting all sub-processes be geometric Brownian. The number of discount factor domains and expressions for the discount factor function grow accordingly, but the discount factor is still found from a linear system. For example, a process with three domains (two thresholds) requires two extra sets of boundary conditions similar to (5) and (6), and solving 3 x 3 linear equation systems.

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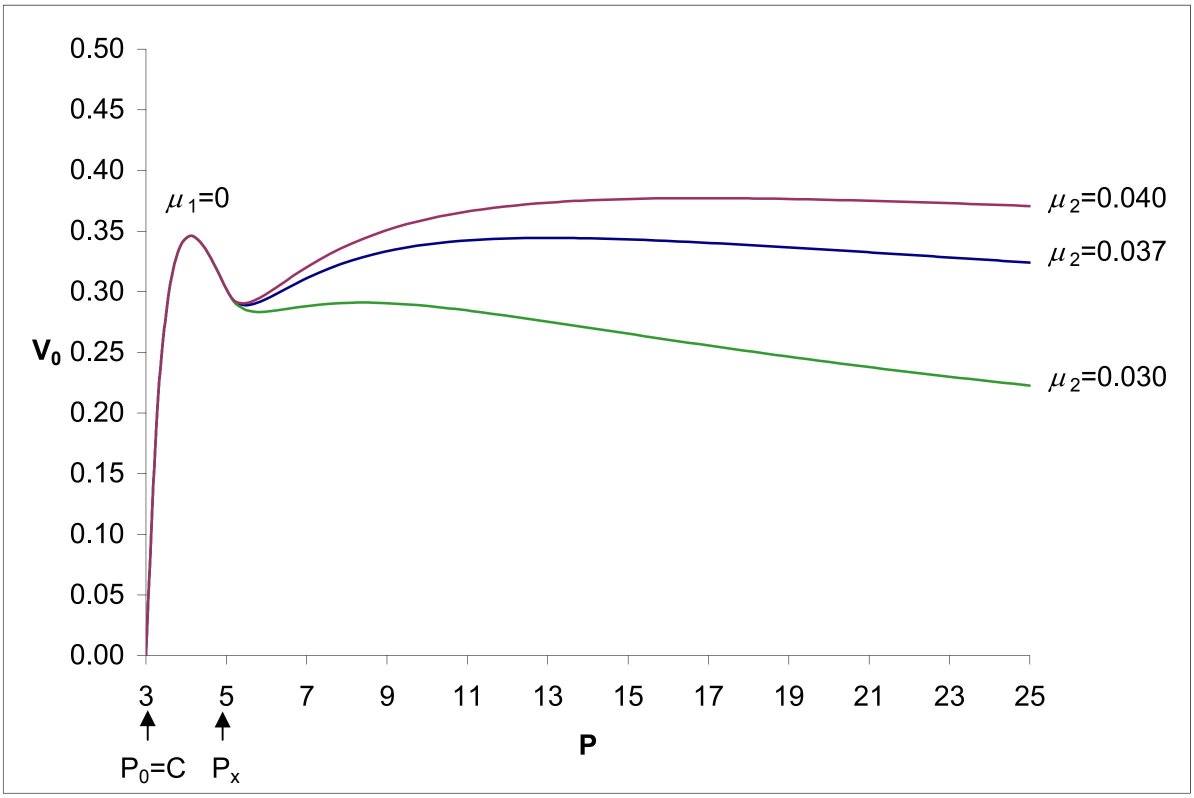

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Fig. 1

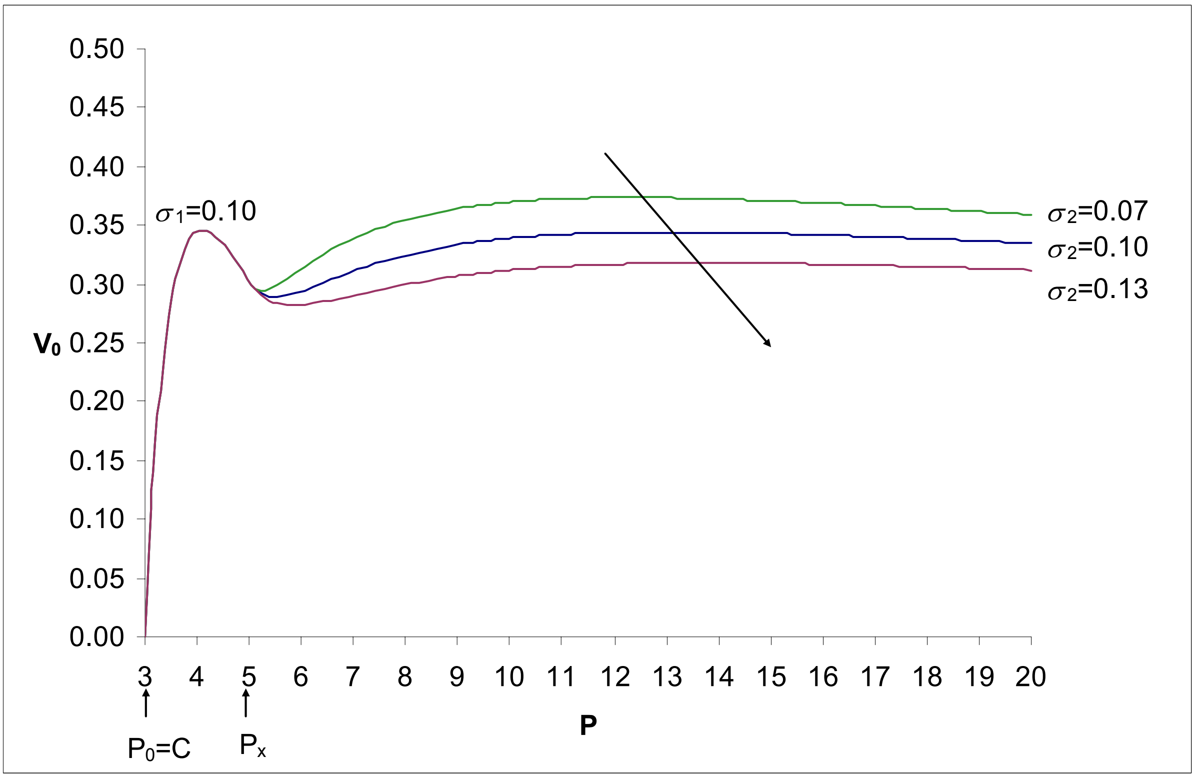

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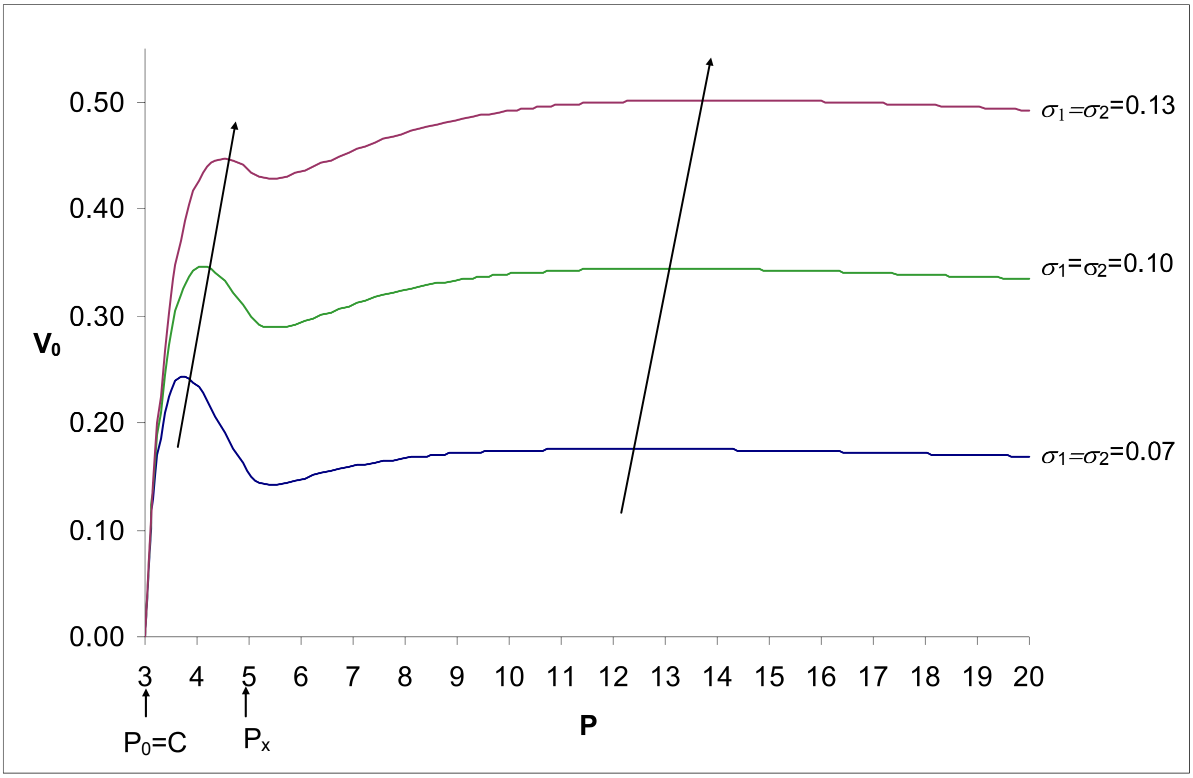
Fig. 2



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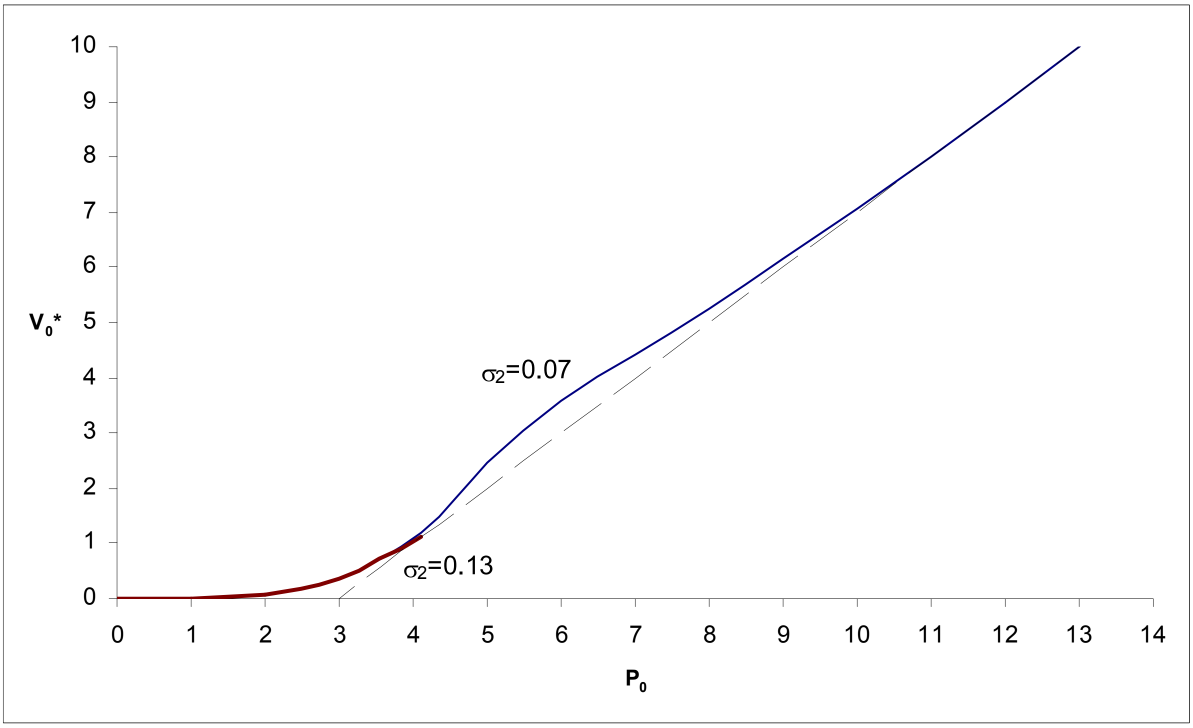
σ1=σ2=0.07

Fig. 3



10
9
8
7
6
5
4
3
2
1
0
V0*
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
P0
σ2=0.07
σ2=0.13

Fig. 4

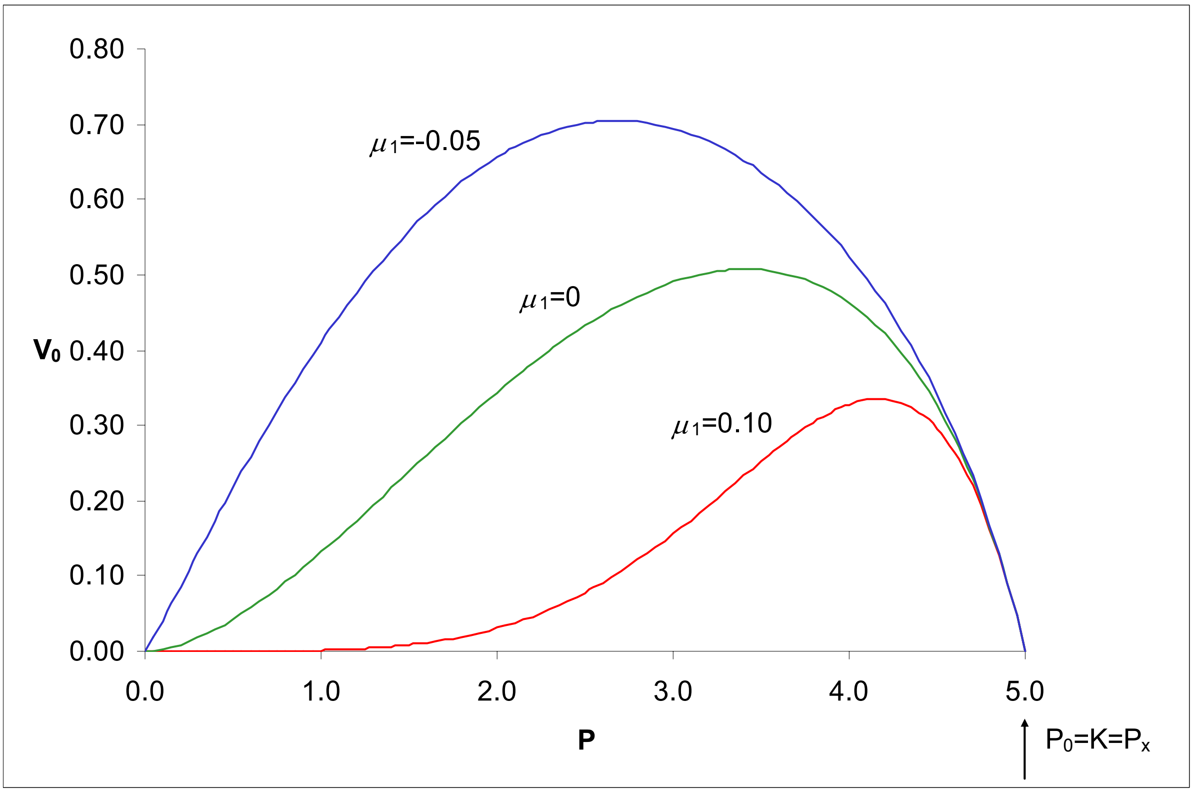

0.80
0.70
0.60
0.50
V0 0.40
0.30
0.20
0.10
0.00
0.0
1.0
2.0
3.0
4.0
5.0
P
μ1=-0.05
μ1=0
μ1=0.10
P0=K=Px

Fig. 5

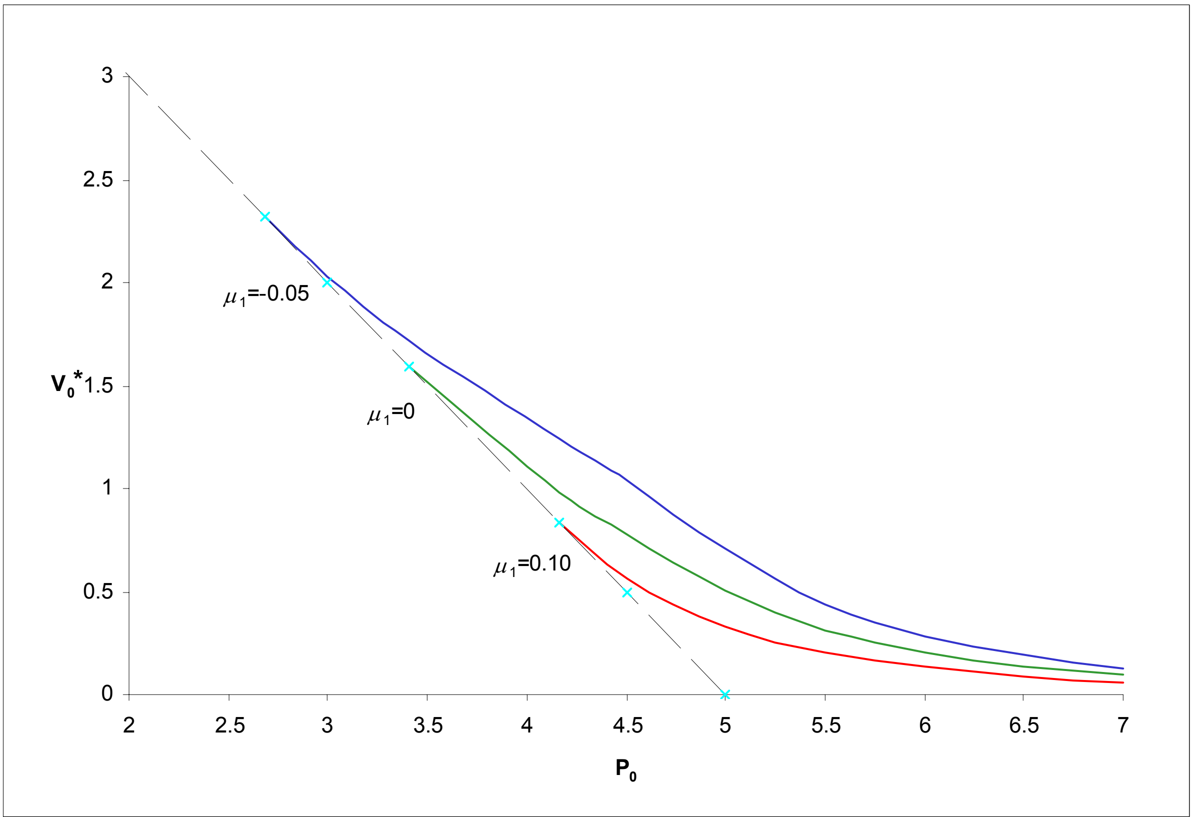

3
2.5
2
V0*1.5
1
0.5
0
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
P0
μ1=-0.05
μ1=0
μ1=0.10

Fig. 6