Leaders, Followers, and Risk Dynamics in Industry Equilibrium

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Abstract

This paper identifies relationships between industry and individual firm risk that reflect the strategic interplay of option exercise by imperfectly competitive firms. We examine the risk dynamics of heterogeneous duopolistic firms that strategically manage options to expand and contract capacity. We characterize industries by the extent to which unexercised options exist (i.e. adolescent, juvenile and mature). Importantly, in all but 'mature' industries, i.e. industries where all real options have been exercised, the existence of a rival reduces risk due to the ability of a rival to *either* expand or contract. We identify the explicit relationship between the industry characterization and industry and own firm risk. We find that both own firm and industry characteristics such as beta, size and book to market have distinct and sometimes opposite relationships to firm risk and return.

Key Words: Growth options and industry risk, asset pricing and investment decisions, risk dynamics in oligopolistic industries

JEL-Classification: C23, C35

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1 Introduction

Traditional asset pricing largely ignores the influence of industry structure and competitors' reactions when analyzing the risk dynamics of a single firm. Nevertheless it seems obvious that investment decisions in an oligopolistic market, that cause the existing capacity of rivals to change, also determine the cash flows earned by the firms in the industry, and hence influence the risk dynamics. Therefore it seems natural to link industry structure with asset returns and risk.

In a recent paper Hou and Robinson (2006) relate industry concentration to the size of stock returns. They find that firms in more concentrated industries earn lower returns, even after controlling for size, book to market or other factors. Theoretically it is not clear why firms in a monopoly market should earn lower returns. A feasible explanation of these findings might be the lack of risky R&D investment in more concentrated industries thereby resulting in lower expected returns. While these theoretical arguments seem to be plausible, they lack a sound understanding of the linkages between industry structure, the level of competition and risk dynamics. In order to overcome this gap, an approach is needed that brings together research in asset pricing and industrial organizations.

The recent literature on corporate investment decisions using a real options framework helps to bridge the gap between asset pricing and industrial organizations. Grenadier (2002) introduces a generalization of McDonald and Siegel (1986) in which symmetric oligopolists face the option to expand capacity in an industry where demand is stochastic. Assuming that firms operate in a homogenous product market he derives a symmetric equilibrium with strategic exercise of options and shows that the value of the option to wait diminishes as the industry becomes more competitive (i.e. the number of firms increases). Aguerrevere (2003) takes up the model of Grenadier (2002) but allows for operating flexibility, i.e. firms need not produce with full capacity. He shows that firms might find it optimal to expand capacity even if current output levels do not fully use existing capacity and that endogenous output price fluctuations are increasing with the number of competitors. While both these papers analyze the link between industry competition and capacity expansion, they do not look at the risk implications of investment strategies.

The paper by Aguerrevere (2005) is the first that relates industry structure and risk dynamics in a model with N symmetric firms. He introduces two alternative settings. One in which the symmetric firms face fixed capacities that can only be changed by exercising a growth option, the other one in which firms have operating flexibility. He finds that in case of operating flexibility industry risk is higher the more competitive the industry is. Industry risks are an immediate consequence of operating leverage and irreversibility. While this is an interesting result that has empirical support found in the paper by Hou and Robinson (2006), it is not clear which one of the two effects dominates. To explore this, an approach is required that looks at asymmetric firms in an industry setup.

In a recent paper Novy-Marx (2006) analyzes investments of firms competing in oligopolistic product markets. He finds that equilibrium investment behavior explains two empirical regularities found in financial markets, investment-cash flow sensitivity and a counter-cyclical value premium.

There are several papers in the real options literature that study corporate investment decisions in an oligopolistic industry when firms face options to expand. Boyer, Lassere, Mariotti and Moreaux (2004) formulate a model with asymmetric firms that strategically exercise growth options. They concentrate on preemptive effects of capacity expansion and do not study asset price dynamics. Pawlina and Kort (2006) study an asymmetric duopoly model with a single option for each firm and look at different exercise equilibria.

In this paper we study corporate investment decisions of asymmetric firms that compete in a duopolistic output market and analyze the risk dynamics in such a setup. In particular, we look at a market in which two firms supply a homogenous product. Demand in the market is stochastic and follows a Geometric Brownian Motion (GBM). Each firm in the industry starts out at a given capacity level with fixed costs but holds an option either to shrink or to expand capacity depending on the level of demand. After expansion (contraction) each firm operates with a higher (lower) level of capacity, output and fixed costs. Option exercise is costly. In case of an expansion investment costs include both adjustment costs and the price of the investment and in case of contraction firms get paid a salvage value for the downsized units.[1]

We assume that investment costs and salvage values are asymmetric so that there is a high cost and a low cost firm and a firm with a high and a low salvage value. In both cases option exercise is sequential with the low cost (high salvage value) firm expanding first (being the leader) and the high cost (low salvage value) firm expanding second (being the follower). This

[1] The introduction of both an expansion and contraction option is motivated by the assumption that firms have to produce at capacity levels. Allowing for both types of options can be seen as a substitute for operating flexibility.

sequential exercise gives rise to three different industry stages: a juvenile industry in which neither firm exercised any of the options, an adolescent industry in which the leader exercised his option but the follower did not, and a mature industry in which both firms exercised their options.

We find that the risk dynamics of the firms are driven by both operating leverage and irreversibility, two effects that we already know from the existing investment literature (see for example Carlson, Fisher, and Giammarino (2004)). In addition we identify a strategic (industry) risk factor that arises because of the imperfect product market. It turns out, for example, that in an adolescent industry where the follower did not exercise his option yet, the leader's risk is reduced by the follower's action. Hence, increasing competition results in a reduction of risk. The intuition for this result comes from a hedging effect that the leader can exploit together with the action of the follower. In case both firms operate with fixed capacity levels any profit uncertainty arises from the industry demand shock. Demand shocks directly translate into changes in the firms' cash flows. If, however, the leader who already exercised his growth option and faces fixed capacity from there on, experiences an increase in the capacity of the follower upon the follower's option exercise, demand shocks are hedged by an output increase. This hedge is larger the closer the follower comes to exercising his growth option. As a consequence, the leader's risk is reduced and is below the market risk normalized by 1. This is a novel result that adds to our understanding of the links between industry structure and risk dynamics. It applies to both firms in the industry. In a juvenile industry in which none of the firms exercised an option, each firm understands that option exercise of the rival causes the market price to change, and hence the firm is directly affected through this product market channel. As pointed out above, the risk reducing effect in any of the two cases (expansion and contraction) is driven by a hedging argument. The adjustment of industry output as a consequence of option exercise of the rival causes prices to change less widely so that firms face a risk reducing effect. As it turns out the industry risk effect has opposite risk implications in an expanding industry and same risk implications in a shrinking industry. This prediction has important implications for empirical analysis. If we add industry factors to firm's own risk characteristics we have to expect both, same and opposite risk implications.

Our findings are also able to disentangle the two driving forces behind risk dynamics of individual firms. As many recent papers on corporate investment decisions and asset price dynamics, such as Gomes, Kogan and Zhang (2003), Zhang (2005), and Cooper (2006), point out, the risk dynamics are governed by operating leverage and the degree of irreversibility.

While both effects are important, it is not clear how they can be separated and identified by observable variables. Our approach documents that operating leverage must be associated with the firm's book to market, while irreversibility can be captured by industry book to market. This implies a set of new testable hypotheses that can shed new light in the empirical asset pricing literature. It is important to point out, that only a framework with asymmetric firms is able to separate these two effects. In a symmetric equilibrium both effects occur simultaneously and therefore cannot be isolated.

Our paper is organized as follows. In Section 2 we present the model. Section 3 derives the results for a monopolistic industry while Section 4 looks at duopolistic competition. Section 5 concludes the paper.

2 The Model

Consider a duopolistic industry in which two firms produce a homogenous product. Output of firm i operating at capacity level k is given by Q_k^i. Industry output is denoted by $Q_{k,s} = Q_k^1 + Q_s^2$ where k, s denote different capacity levels. Demand and hence equilibrium price is stochastic and specified by an iso-elastic inverse demand curve

$$P_t = X_t Q_{k,s}^{\gamma-1}, \tag{1}$$

where X_t is an industry wide shock at time t, and $0 < \gamma < 1$ is a given parameter. The industry demand shock follows a geometric Brownian motion (GBM)

$$dX_t = gX_t dt + \sigma X_t dW_t, \tag{2}$$

where dW_t is the increment of a Wiener process, g is the constant drift, and σ the constant variance.[2]

Each firm has an initial capacity level given by $K_0^i = K_0^j = K_0 > 0$. Current capacity is used to produce output. We assume that firms operate with technologies that are linear in the capital stock, the single variable factor of production. Hence, current output is proportional to current capacity, i.e.,

$$Q_k^i \equiv K_k^i.$$

In principle, firms could choose a level of output that does not make use of all available capacity, i.e., firms exploit operating flexibility. Here, we

[2]The specification of the demand shock as GBM implies that expected demand grows exponentially. Each firm can take advantage of this growth by adjusting its current level of capacity.

assume that firms cannot flexibly adjust output so that current capacity levels are identical to current output. In such a setting a capacity expansion (contraction) is identical to an increase (decrease) in output. To at least capture some features of operating flexibility we allow firms to do both, expand and contract.

Each firm has a single option either to expand capacity from the level K_0^i to K_1^i where $K_1^i > K_0^i$ holds or to contract capacity from the level K_0^i to K_{-1}^i with $K_0^i > K_{-1}^i$. Once a firm exercised its option, capacity cannot be removed, i.e., it is irreversible.

Firms are symmetric with respect to their capacity levels, $K_k^i = K_k^j = K_k,\ k = -1, 0, 1$, but investment costs and the salvage values differ. In case of expansion firm i faces investment costs given by IC_i where $IC_1 < IC_2$ holds. Hence, firm 1 is the low cost firm and firm 2 the high cost firm. Investment costs include the price of the investment and adjustment costs. In case of contraction firm i faces a salvage value given by SV_i where $SV_1 > SV_2$ holds.

At each capacity level firms face fixed costs but no variable production costs so that profits are given by

$$\pi^i(X_t) = X_t Q_{k,s}^{\gamma-1} Q_k^i - f_k^i, \text{ where } k, s = -1, 0, 1 \text{ and } i = 1, 2.$$

With the simplifying assumption that each firm only operates with one of three different capacity levels the profit functions can be rewritten as

$$\pi^i(X_t) = X_t R^i_{k=-1,0,1;s=-1,0,1} - f^i_{k=-1,0,1},$$

where $R^i_{k=-1,0,1;s=-1,0,1}$ denotes the deterministic part of revenues of firm i with $k = 0, s = 0$ referring to the cases that neither i nor j did exercise their options, $k, s = 1$ indicates capacity expansion, while $k, s = -1$ refers to capacity contraction. We assume that the fixed costs increase with the capacity levels, $f_{-1}^i < f_0^i < f_1^i$.

In the general case the deterministic part of the revenue function is defined as

$$R^i_{i=k,j=s} \equiv \left(Q_k^i + Q_s^j\right)^{\gamma-1} Q_k^i.$$

where the individual levels of output satisfy $Q_{-1}^i < Q_0^i < Q_1^i$. As a consequence of the iso-elastic demand function deterministic revenues satisfy the following properties.

$$
\begin{aligned}
R^i_{l,l} = \left(Q^i_l + Q^j_l\right)^{\gamma-1} Q^i_l &> R^i_{l-1,l-1} = \left(Q^i_{l-1} + Q^j_{l-1}\right)^{\gamma-1} Q^i_{l-1}, \\
R^i_{l,l-1} = \left(Q^i_l + Q^j_{l-1}\right)^{\gamma-1} Q^i_l &> R^i_{l-1,l-1} = \left(Q^i_{l-1} + Q^j_{l-1}\right)^{\gamma-1} Q^i_{l-1}, \\
R^i_{l,l} = \left(Q^i_l + Q^j_l\right)^{\gamma-1} Q^i_l &< R^i_{l,l-1} = \left(Q^i_l + Q^j_{l-1}\right)^{\gamma-1} Q^i_l \\
R^i_{l,l} = \left(Q^i_l + Q^j_l\right)^{\gamma-1} Q^i_l &> R^i_{l-1,l} = \left(Q^i_{l-1} + Q^j_l\right)^{\gamma-1} Q^i_{l-1}. \\
R^i_{l,l-1} = \left(Q^i_l + Q^j_{l-1}\right)^{\gamma-1} Q^i_l &> R^i_{l-1,l-1} = \left(Q^i_{l-1} + Q^j_{l-1}\right)^{\gamma-1} Q^i_{l-1}, \;\; l = 0, 1
\end{aligned}
$$

These inequalities characterize the different industry stages and are crucial for the derivation of the strategic effects.

Based on the sequencing of the investment strategies of the firms we distinguish three different industry stages:

- **Mature Industry:** Both firms already exercised their growth options (contraction options) and each produces with a capacity level given by K_1 (K_{-1}).
- **Adolescent Industry:** One firm (the leader) exercised the growth option (contraction option), while the other firm (the follower) produces at the initial output level.[3]
- **Juvenile Industry:** None of the firms exercised their growth option and each produces at the capacity level K_0.

Based on the industry structure and sequential exercise of options we can distinguish the following time line of events in case of capacity expansion. At time τ_L the leader exercises the growth option; at time τ_F the follower exercises his growth option. Based on this exercise sequence the three industry scenarios are given as follows. From $t = 0 \rightarrow t = \tau_L$ the industry is a juvenile industry, from $t = \tau_L \rightarrow t = \tau_F$ it is an adolescent industry, and for $t > \tau_F$ it is a mature industry (see Figure 1 below). The same applies to the case of the contraction options with the leader exercising first and the follower exercising second.

[3] In this paper we focus on an equilibrium with sequential exercise of options. In section 4.4 we present a set of sufficient conditions for which sequential exercise corresponds to equilibrium behavior. Sequential exercise implies that the low cost firm exercises first (i.e. becomes the leader) while the high cost firm acts as the follower.

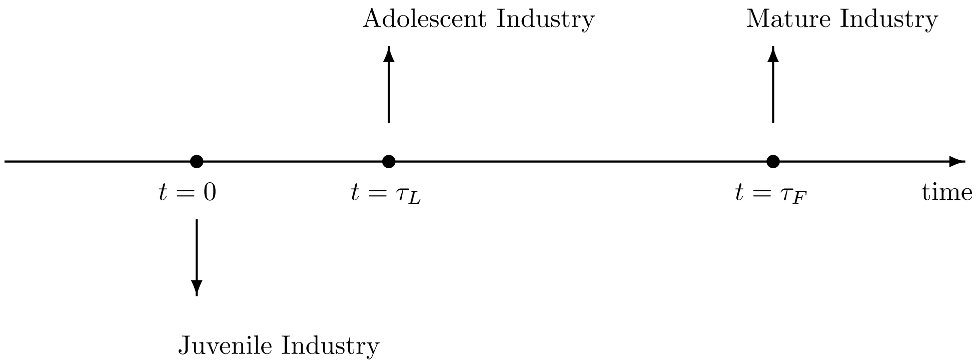

Figure 1

Although the duopolistic market structure is our primary interest we use a monopoly market, i.e. a market in which a monopoly runs two divisions, as the benchmark case. In case of capacity expansion the profit function of the monopolist becomes,

$$\begin{aligned}
\pi_0^M(t) &= X_t \left(Q_0^1 + Q_0^2\right)^{\gamma-1} (Q_0^1 + Q_0^2) - f_0^1 - f_0^2 = X_t Q_0^\gamma - F_0, \\
\pi_1^M(t) &= X_t \left(Q_1^1 + Q_0^2\right)^{\gamma-1} (Q_1^1 + Q_0^2) - f_1^1 + f_0^2 = X_t Q_1^\gamma - F_1, \\
\pi_2^M(t) &= X_t \left(Q_1^1 + Q_1^2\right)^{\gamma-1} (Q_1^1 + Q_1^2) - f_1^1 - f_1^2 = X_t Q_2^\gamma - F_2.
\end{aligned}$$

with $F_0 \equiv f_0^1 + f_0^2$, $F_1 \equiv f_1^1 + f_0^2$, and $F_2 \equiv f_1^1 + f_1^2$. Every time we deal with the monopoly market the notation Q_0 refers to the case of neither division having exercised the growth option, Q_1 to the case one option has been exercised and Q_2 to the case both options have been exercised.

The monopoly model presented here is identical to the one used in Carlson, Fisher, and Giammarino (2004). In the present paper we use this model to be able to explore the implications of alternative industry structures on equilibrium returns.

In order to derive the option values included in the firms' valuation problems we assume that there exist two traded assets that can be used to hedge industry demand uncertainty. Let B_t denote the price of a riskless bond with dynamics $dB_t = rB_t dt$ where $r > 0$ is the constant riskless rate of interest, and let S_t be the price of a risky asset. The price dynamics of the risky asset is given by

$$dS_t = \mu S_t dt + \sigma S_t dW_t.$$

The risky asset S_t and the industry demand shock share the same stochastics except for the constant drifts. With the existence of the securities B_t and S_t, and the characteristics of the price dynamics for S_t we can construct a portfolio of the bond and the asset S_t that perfectly replicates the industry shocks X_t. This gives us the opportunity to define a risk neutral measure for the shocks X_t. Under the risk neutral measure demand dynamics are given by

$$dX_t = (r - \delta)X_t dt + \sigma X_t d\hat{W}_t, \tag{3}$$

where $\delta \equiv \mu - g > 0$. All the valuations in this section are based on the risk neutral measure (3).

3 Exercise of Expansion Options in a Monopolistic Industry

As a point of reference we start our analysis by repeating the results for a monopoly producer. To make things simple we only look at the case of expansion options and do not consider contraction options. This is the scenario that has been dealt with in the literature by Carlson, Fisher, and Giammarino (2004). The risk implications of growth options, operating leverage, and irreversibility are the results of the firm values in the three different stages of the industry.

In a mature industry the value of the firm is given by the net present value of a risky growing perpetuity associated with the revenue side and a riskless perpetuity arising from the fixed costs. The firm value in the mature industry is given by

$$V_{MI}^{M}(X_t) = \frac{Q_2^{\gamma}}{\delta} X_t - \frac{F_2}{r}. \tag{4}$$

Equation (4) implies that the firm value in a mature industry is entirely given by the value of the assets in place. The value of the assets in place generates risk dynamics that are only driven by operating leverage associated with the fixed costs F_2. We will explore this result later in this section.

In a monopoly industry in which the firm exercised one of the growth options (adolescent industry) the value function consists of two components, the present value of assets in place $V_{AI}^{A(M)}(X_t)$ and the value of the remaining growth option $V_{AI}^{G(M)}(X_t)$, i.e.,

$$V_{AI}^{M}(X_t) = V_{AI}^{A(M)}(X_t) + V_{AI}^{G(M)}(X_t),$$

Standard arguments can be used to derive the explicit form of the value function. It is given by

$$V_{AI}^{M}(X_t) = \frac{Q_1^{\gamma}}{\delta} X_t - \frac{F_1}{r} + \frac{F_2 - F_1 + rIC_2}{(\nu_1 - 1)r} \left(\frac{X_t}{X_2^M} \right)^{\nu_1}. \tag{5}$$

where $\nu_1 > 1$ is the positive root of the characteristic equation

$$\frac{1}{2}\sigma^2 \nu(\nu - 1) + (r - \delta)\nu - r = 0$$

with solutions

$$\nu_{1,2} = \frac{1}{2} - \frac{r - \delta}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{r - \delta}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}}, \tag{6}$$

and X_2^M is the demand trigger for the exercise of the second growth option. This trigger level is given by

$$X_2^M = \frac{\delta \nu_1 (F_2 - F_1 + rIC_2)}{(\nu_1 - 1) r [Q_2^{\gamma} - Q_1^{\gamma}]}. \tag{7}$$

On the basis of the value function (5) the value of the assets in place is

$$V_{AI}^{A(M)}(X_t) \equiv \frac{Q_1^{\gamma}}{\delta} X_t - \frac{F_1}{r},$$

and the option value is

$$V_{AI}^{G(M)}(X_t) \equiv \frac{F_2 - F_1 + rIC_2}{(\nu_1 - 1)r} \left(\frac{X_t}{X_2^M} \right)^{\nu_1}.$$

With the formulation it is obvious that the monopoly's risk in an adolescent industry is driven by operating leverage and the option risk associated with the exercise of the growth option. This decomposition has already been explored in the paper by Carlson, Fisher and Giammarino (2004).

In case of a juvenile industry the monopoly has a compound growth option (two consecutive growth options) which together with the assets in place result in the firm's value given by

$$V_{JI}^{M}(X_t) = V_{JI}^{A(M)}(X_t) + V_{JI}^{G(M)}(X_t), \tag{8}$$

where X_1^M is the option trigger associated with the first capacity expansion given by

$$X_1^M = \frac{\delta \nu_1 (F_1 - F_0 + rIC_1)}{(\nu_1 - 1) r [Q_1^{\gamma} - Q_0^{\gamma}]}. \tag{9}$$

The value of the assets in place is

$$V_{JI}^{A(M)}(X_t) \equiv \frac{Q_0^{\gamma}}{\delta} X_t - \frac{F_0}{r},$$

and the value of the compound growth option is

$$V_{JI}^{G(M)}(X_t) \equiv \frac{F_1 - F_0 + rIC_1}{(\nu_1 - 1)r} \left(\frac{X_t}{X_1^M}\right)^{\nu_1} + \frac{F_2 - F_1 + rIC_2}{(\nu_1 - 1)r} \left(\frac{X_t}{X_2^M}\right)^{\nu_1}.$$

The value of the compound option is the sum of the individual options.

The firm values derived above can be used to characterize the risk dynamics in a monopoly industry. We do this by making use of the firm beta. The firm beta in any particular industry stage is defined as

$$\beta_k^M = \frac{\frac{\partial V_k^M(X)}{\partial X} X}{V_k^M(X)}, \ k = JI, AI, MI.$$

Using the value functions for the different industry stages the betas are given by

$$\beta_k^M(t) = 1 + \frac{V_k^{G(M)}(t)}{V_k^M(t)}(\nu_1 - 1) + \frac{F_k/r}{V_k^M(t)} \ k = JI, AI, MI.$$

The beta values confirm our intuition stated above. Risk in a monopolistic industry is driven by operating leverage (the size depending on the level of fixed costs) and the growth option. The risk implications of the growth options depend on the volatility of the demand shock σ, and the size of the firm (whether or not it already exercised one option). Carlson, Fisher and Giammarino (2004) have pointed out that the beta, and hence the firm risk, varies with the size (related to the option) as well as the book to market ratio.

4 Firm Specific and Industry Risk Implications of Investment Dynamics

We now generalize the results of the previous section by considering a duopolistic industry structure in which two competing firms produce identical goods to satisfy a common, dynamic industry demand. Surprisingly, we show that at each point in time firm risk depends not only on own-firm values of assets-in-place and real options, but also on the investment dynamics of the firm's rival.

To most clearly distinguish firm-specific and industry risk, we assume that one firm is flexible and has the option to expand or contract while the other firm is inflexible, with fixed capacity and no options. This minimizes strategic considerations and focuses on the interdependencies inherent in a common product market. We defer analysis of the more natural situation where both firms have investment opportunities and interact strategically to the following section.

4.1 Firm Values

We begin by examining the optimal exercise decision of the flexible firm. This firm takes the operating decisions of the inflexible firm as given and chooses to expand or contract in a way that maximizes its value. We refer to the critical demand level at which the firm *expands* capacity as X_E and the *contraction* demand level as X_C.

The first component of the flexible firm value is the value of assets in place

$$V^{A(1)}(X_t) = \frac{R^1_{0,0} X_t}{\delta},$$

reflecting the initial capacity and output of both firms, $K^1_0 = K^2_0$ and $Q^1_0 = Q^2_0$, respectively.

The second component is the investment option value,

$$V^{G(1)}(X_t) = B^1_1 X_t^{\nu_1} + B^1_2 X_t^{\nu_2},$$

where, ν_1 and ν_2 are as defined in equation (6), and B^1_1 and B^2_2 are constants. Option value maximization implies that the constants are derived from the value matching and smooth pasting conditions:

$$B^1_1 X_E^{\nu_1} + B^1_2 X_E^{\nu_2} = \frac{R^1_{1,0} - R^1_{0,0}}{\delta} X_E - IC_1 \tag{10}$$

$$B^1_1 X_C^{\nu_1} + B^1_2 X_C^{\nu_2} = \frac{R^1_{-1,0} - R^1_{0,0}}{\delta} X_C + SV_1 \tag{11}$$

$$\nu_1 B^1_1 X_E^{\nu_1 - 1} + \nu_2 B^1_2 X_E^{\nu_2 - 1} = \frac{R^1_{1,0} - R^1_{0,0}}{\delta} \tag{12}$$

$$\nu_1 B^1_1 X_C^{\nu_1 - 1} + \nu_2 B^1_2 X_C^{\nu_2 - 1} = \frac{R^1_{-1,0} - R^1_{0,0}}{\delta}. \tag{13}$$

The first two equations are the value matching conditions and specify that the option value at the critical boundaries are exactly equal to the present value of the incremental revenues net of adjustment costs. The last two equations are the smooth pasting conditions which are necessary for value maximization. This system of equations has no convenient analytical solution due to the nonlinearity in X_E and X_C.

Thus, total firm value is simply the sum of the value of the assets in place and the options to adjust capacity.

$$V^1(X_t) = \frac{R^1_{0,0} X_t}{\delta} + B^1_1 X_t^{\nu_1} + B^1_2 X_t^{\nu_2}.$$

The value of the inflexible firm derives only from the assets in place. In turn, the value of the assets in place can be thought of as having two parts. One, is the present value of the revenue stream, assuming there are no capacity adjustments,

$$V^{A(2)}(X_t) = \frac{R^2_{0,0}}{\delta} X_t.$$

The other component is the present value of the revenue gain or loss due to the potential capacity changes elsewhere in the industry. This present value reflects the random time at which the flexible firm adjusts capacity and can be shown to have the form [4]:

$$V^{SE(2)}(X_t) = B^2_1 X_t^{\nu_1} + B^2_2 X_t^{\nu_2}.$$

The values of B^2_1 and B^2_2 satisfy the value matching equations:

$$B^2_1 X_E^{\nu_1} + B^2_2 X_E^{\nu_2} = \frac{R^2_{1,0} - R^2_{0,0}}{\delta} X_E \tag{14}$$

$$B^2_1 X_C^{\nu_1} + B^2_2 X_C^{\nu_2} = \frac{R^2_{-1,0} - R^2_{0,0}}{\delta} X_C, \tag{15}$$

where X_C and X_E are the critical investment boundaries for the flexible firm, as given above.

Thus, inflexible firm value is simply the sum of the two components of the assets in place value

$$V^2(X_t) = \frac{R^2_{0,0} X_t}{\delta} + B^2_1 X_t^{\nu_1} + B^2_2 X_t^{\nu_2}.$$

[4]See Karlin and Taylor (1973) or Dixit and Pindyck (1994).

4.2 Risk

As with the analysis of the monopolist, the risk of each of the firms is given by the elasticity of value with respect to X_t. For the flexible firm, the instantaneous beta is given by

$$\beta_0^1(X_t) = 1 + (\nu_1 - 1)\frac{B_1^1 X_t^{\nu_1}}{V_0^1(X_t)} + (\nu_2 - 1)\frac{B_1^2 X_t^{\nu_2}}{V_0^1(X_t)}.$$

This formula mirrors the equation for the monopolist but has an extra component that accounts for the ability to contract capacity.

The inflexible firm has beta given by

$$\beta_0^2(X_t) = 1 + (\nu_1 - 1)\frac{B_1^2 X_t^{\nu_1}}{V_0^2(X_t)} + (\nu_2 - 1)\frac{B_1^2 X_t^{\nu_2}}{V_0^2(X_t)}.$$

Although the form of the beta is the same for each firm, the economic interpretation is very different. The flexible firm risk depends only on firm specific decisions, as with the monopolist. In contrast, the inflexible firm, by construction, has no firm specific decisions to make. The risk dynamics are entirely due to industry effects. Furthermore, the inflexible firm risk is lower than it would be if it had no competitor. Intuitively, competitors expansion and contraction decisions dampen demand shocks. Near the expansion boundary, increases in the demand level are accompanied by a greater likelihood that the flexible firm will add capacity, increase output, and thereby mitigate the positive impact of a demand increase. Similarly, near the contraction boundary, the likelihood that the flexible firm will reduce capacity increases when demand levels fall, again offsetting demand shocks.

This simple case highlights the fact that when real options are present within an industry they have risk implications for all firms, even those without options. In the next section, we expand on this insight in a setting where all firms can adjust capacity, so that every firm's risk has firm-specific and industry components.

5 Strategic Exercise of Options in a Duopolistic Industry

In a duopoly market exercise of the growth and contraction options is the outcome of a strategic game. If every firm can exercise a single option equilibrium behavior can either consist of sequential or simultaneous exercise of options. In this paper we are primarily interested in an equilibrium with sequential exercise. This implies that one of the firms must act as the leader and the other one as the follower. As we will point out later, the relative level of investment costs and salvage values determine sequential exercise. For now we assume that the low cost (high salvage value) firm is the leader and the high cost (low salvage value) firm is the follower. We make use of standard dynamic programming techniques to derive optimal value functions for the different cases. Since every firm has a single option we distinguish two different cases. In the first case we exclusively allow for expansion options, i.e., each firm can exercise an expansion option. In the second case we assume that firms only exercise contraction options.

Our approach to analyze firm behavior by looking at different scenarios separately is motivated by the fact that we want to explore the role of firm's own option exercise as well as strategic interactions in the risk dynamics of firms.

5.1 Exercise of Expansion Options

In this subsection we assume that both firms hold an expansion option that is exercised sequentially. It can be shown that sequential exercise is an equilibrium strategy if both firms face asymmetric investment costs and that the cost differential is large enough. We assume that these conditions are satisfied.

Based on the time line of Figure 1 and the focus on sequential exercise we apply backward induction to derive the value functions for the two firms in the duopolistic industry. We have to distinguish the following three industry structures. In a mature industry firm values equal the present value of profits minus the present value of fixed costs. In an adolescent industry the leader already exercised his option but the follower needs to decide when optimally to exercise his option. Since both firms are operating in an imperfectly competitive output market the firm value of the leader necessarily depends on the exercise strategy of the follower. In a juvenile industry the leader has to exercise his option. His exercise strategy, however, depends on the strategic choices of the follower. Hence, we derive the leader's value function by taking the followers response into account.

Industry structure and sequential exercise of options give rise to three different value functions of firms.

1. Value function for mature firms.
2. Value function for the follower who exercises his option at τ_F but already experienced an option exercise of the leader.

3. Value function for the leader who exercises his option at τ_L but does take the response of the follower into account.

To derive equilibrium behavior and hence optimal firm values we make use of backward induction and start out with a mature industry. Next, we derive the value of the follower in an adolescent industry and then continue with the derivation of the leader's value function in a juvenile industry.

5.1.1 Firm Values in a Mature Industry

In case both firms already exercised their growth options the instantaneous profit function of firm i is given by

$$\pi^i(X_t) = X_t R^i_{i=1,j=1} - f^i_1 = X_t(Q^i_1 + Q^j_1)^{\gamma-1} Q^i_1 - f^i_1,$$

and the value functions defined as

$$V^i_{MI}(X_t) \equiv E_t \left\{ \int_t^\infty e^{-r(s-t)} [X_s R^i_{i=1,j=1} - f^i_1] ds \right\}.$$

Applying risk neutral valuation results in

$$V^i_{MI}(X_t) = \frac{R^i_{i=1,j=1}}{\delta} X_t - \frac{f^i_1}{r}. \tag{16}$$

In a mature industry the value of each firm given by (16) corresponds to the classic Gordon growth formula. The value is driven by the value of the assets in place. This value corresponds to the one we derived for the monopolistic industry. To compare the monopoly with the duopolistic industry we construct a portfolio that consists of a unit of the follower and one of the leader. The value of this portfolio is given by

$$V^D_{MI}(X_t) = V^L_{MI}(X_t) + V^F_{MI}(X_t) = \frac{X_t Q^\gamma_2}{\delta} - \frac{F_2}{r}.$$

In case firms are not assumed to use any operating flexibility the value of an industry portfolio in the duopoly market is identical to that of a monopoly market. This implies that the risks in the two different market structures are identical. Thus, we can conclude that in case industry risk is driven by operating leverage and irreversibility[5] only, industry structure has no effect on the risk dynamics.

[5] In this case irreversibility refers to the inability of firms to flexibly adjust production to demand uncertainty and have unused capacity.

Industry beta is given as the weighted sum of the individual betas, i.e.

$$\beta^D_{MI} = w_L \beta^L_{MI} + w_F \beta^F_{MI}$$

where w_L is the weight of the leader and w_F the weight of the follower,

$$w_L = \frac{V^L_{MI}}{V^D_{MI}}, \;\; w_F = \frac{V^F_{MI}}{V^D_{MI}}.$$

Individual firm betas do only deviate from market beta normalized to one if firms face operating leverage. i.e.,

$$\beta^i_{MI} = 1 + \frac{f^i_1/r}{V^i_{MI}}, \; i = L, F.$$

In a mature industry operating leverage and irreversibility drive individual firm and industry risk.

5.1.2 Firm Values in an Adolescent Industry

Next, consider an adolescent industry in which the leader already exercised his option. The follower will exercise his option when the demand shock exceeds or hits the threshold level X^D_2 for the first time. We define $\tau_F \equiv \min\{t | X_t \geq X^D_2\}$. This is the level of demand that triggers an option exercise by the follower in the duopolistic industry. Given the sequential exercise of options we know that this trigger level is larger than the corresponding level for the option exercise of the leader, which we denote as X^D_1.

Assume that firm i is the follower and that $t > \tau_L$. At X_t the value of the follower ($i = F$) equals

$$\begin{aligned} V^i(X_t) \;\; &= \;\; E_t \left\{ \int_{\min[t,\tau_F]}^{\tau_F} e^{-r(s-t)}[X_s R^i_{i=0,j=1} - f^i_0]ds \right. \\ &+ \left. \int_{\max[t,\tau_F]}^{\infty} e^{-r(s-\max[t,\tau_F])}[X_s R^i_{i=1,j=1} - f^i_1]ds - e^{-r(t-\max[t,\tau_F])} IC_i \right\}. \end{aligned}$$

In a first step we derive the value function for the follower under the assumption that the leader already exercised his option. This is the scenario that we refer to as adolescent industry. We have to keep in mind, however, that we also need to derive the follower's value function in a juvenile industry. We will come back to this issue after we derived the leader's value function in the next subsection.

Using standard dynamic programming techniques it is easy to derive the value function for the follower.

Lemma 1 *Assume that the leader already exercised his growth option. Let i be the follower F and let X_2^D be the trigger at which the follower exercises his growth option. Then the follower's value function is given by*

$$V^i(X_t) = \begin{cases} \frac{R^i_{i=0,j=1}}{\delta} X_t - \frac{f^i_0}{r} + \frac{f^i_1 - f^i_0 + rIC_i}{r(\nu_1 - 1)} \left(\frac{X_t}{X_2^D}\right)^{\nu_1} & X_t \leq X_2^D, \\ \frac{R^i_{i=1,j=1}}{\delta} X_t - \frac{f^i_1}{r} & X_t > X_2^D. \end{cases} \tag{17}$$

where ν_1 is the positive root of (6).

The proof of this result is given in the Appendix.

The firm value of the follower in an adolescent industry consists of the value of the assets in place plus the value of the growth option. Only after the follower exercised his growth option and the industry changes to a mature industry will the firm value be given by the value of the assets in place. The value of the assets in place in an adolescent industry is

$$V_{AI}^{A(i)}(X_t) \equiv \frac{R^i_{i=0,j=1}}{\delta} X_t - \frac{f^i_0}{r}, \; i = F,$$

and the value of the growth option is given by

$$V_{AI}^{G(i)} \equiv \frac{f^i_1 - f^i_0 + rIC_i}{r(\nu_1 - 1)} \left(\frac{X_t}{X_2^D}\right)^{\nu_1}, \; i = F$$

where the option trigger is given by

$$X_2^D = \frac{\delta \nu_1 (f^i_1 - f^i_0 + rIC_i)}{(\nu_1 - 1) r [R^i_{i=1,j=1} - R^i_{i=0,j=1}]} > 0. \tag{18}$$

In total the value of the follower in an adolescent industry is the sum of the value of the assets in place and the value of the growth option.

$$V_{AI}^F(X_t) = V_{AI}^{A(F)}(X_t) + V_{AI}^{G(F)}(X_t).$$

5.1.3 Firm Values in a Juvenile Industry

The optimal firm value for the leader in a juvenile industry can be derived using the same reasoning as in the case of the follower, except for an important difference. When we derive the value function for a rational, forward looking leader, we need to take into account that the leader anticipates the follower's option exercise. Whenever the follower exercises his option and hence expands capacity, industry output is increased and product price decreased. Since the leader cannot flexibly adjust his capacity level, the decrease in market prices causes the leader's cash flows to decrease and hence he is forced to adjust his market value. This adjustment is the consequence of irreversibility and sequential exercise. Altogether the leader understands that when the follower exercises his option, he (the leader) will experience a reduction in revenues which he has to anticipate prior to the exercise of the rival's option. This behavior strategically alters the valuation process.

Let us assume that the leader exercises his option at the trigger X_1^D at time τ_L and that $\tau_L < \tau_F$. The value function of the leader ($i = L$) is defined as

$$\begin{aligned} V^i(X_t) &= E_t\left\{\int_t^{\tau_L} e^{-r(s-t)}[X_s R^i_{i=0,j=0} - f^i_0]ds \right. \\ &+ \int_{\tau_L}^{\tau_F} e^{-r(s-t)}[X_s R^i_{i=1,j=0} - f^i_1]ds - e^{-r(t-\tau_L)} IC_i \\ &+ \left. \int_{\tau_F}^{\infty} e^{-r(s-t)}[X_s R^i_{i=1,j=1} - f^i_1]ds \right\}. \end{aligned}$$

The leader's value function changes along the different stages of the industry. These changes are the result of the different levels of revenues earned in the different industry stages. In a juvenile industry the leader earns revenues equal to $X_t R^i_{i=0,j=0}$, in an adolescent industry the leader earns $X_t R^i_{i=1,j=0}$, and in a mature industry he earns $X_t R^i_{i=1,j=1}$.

Lemma 2 *Suppose firm i is the leader L and let X_1^D be the investment trigger of the leader and X_2^D that of the follower. The leader's value function is given by*

$$V^i(X_t) = \begin{cases} \frac{R^i_{i=0,j=0}}{\delta} X_t - \frac{f^i_0}{r} + \frac{f^i_1 - f^i_0 + rIC_i}{r(\nu_1 - 1)} \left(\frac{X_t}{X_1^D}\right)^{\nu_1} & \\ + \frac{X_2^D}{\delta}\left[R^i_{i=1,j=1} - R^i_{i=1,j=0}\right]\left(\frac{X_t}{X_2^D}\right)^{\nu_1} & X_t < X_1^D, \\ \frac{R^i_{i=1,j=0}}{\delta} X_t - \frac{f^i_1}{r} & \\ + \frac{X_2^D}{\delta}\left[R^i_{i=1,j=1} - R^i_{i=1,j=0}\right]\left(\frac{X_t}{X_2^D}\right)^{\nu_1} & X_t \in [X_1^D, X_2^D], \\ \frac{R^i_{i=1,j=1}}{\delta} X_t - \frac{f^i_1}{r} & X_t > X_2^D, \end{cases} \quad (19)$$

with an expansion trigger X_1^D equal to

$$X_1^D = \frac{\delta \nu_1 (f_1^i - f_0^i + rIC_i)}{(\nu_1 - 1) r [R^i_{i=1,j=0} - R^i_{i=0,j=0}]} > 0. \tag{20}$$

The proof of this result is given in the Appendix.

In a juvenile industry before any growth option is exercised the firm value of the leader consists of three components: (i) the present value of the assets in place, (ii) the option value for the leader's growth option which is strictly positive, and (iii) a strategic option value that is the result of strategic product market behavior of the rival firms. The strategic option value is negative from the point of view of the leader. When the follower exercises his option and expands capacity the product price in the industry drops. This necessarily results in a revenue loss for the leader. Since the leader anticipates this loss the consequences of the price decline in the industry are reflected in the firm value. In total we get

$$V_{JI}^L(X_t) = V_{JI}^{A(L)}(X_t) + V_{JI}^{G(L)}(X_t) + V_{JI}^{SE(L)}(X_t).$$

The first component of the leader's firm value is the present value of the assets in place

$$V_{JI}^{A(L)}(X_t) \equiv \frac{R^i_{i=0,j=0}}{\delta} X_t - \frac{f_0^i}{r}.$$

The second term reflects the value of the growth option

$$V_{JI}^{G(L)}(X_t) \equiv \frac{f_1^i - f_0^i + rIC_i}{r(\nu_1 - 1)} \left(\frac{X_t}{X_1^L} \right)^{\nu_1} > 0,$$

and the third term measures the strategic effect, i.e., it is the leader's valuation of the follower's option exercise

$$V_{JI}^{SE(L)}(X_t) \equiv \frac{X_2^D}{\delta} \left[R^i_{i=1,j=1} - R^i_{i=1,j=0} \right] \left(\frac{X_t}{X_2^D} \right)^{\nu_1} < 0.$$

This last term is fundamentally different to the monopoly case. It is the result of strategic interactions among the rival firms. Hence, in an industry in which the leader already exercised his growth option but the follower did not, risk in the duopoly market must be fundamentally different to the monopoly market. This part of the firm value can only directly be observed if firms move sequentially. The asymmetric cost structure in our model is the basis for sequential exercise and therefore responsible for this strategic effect.

The insight into the valuation of the leader in a juvenile industry can also be used to derive the firm value of the follower in a juvenile industry. Recall that in the preceding subsection we have derived the value of the follower under the assumption that the leader already exercised his option. In case the leader did not exercise his option and both firms act in a juvenile industry, the strategic option effect also applies to the follower prior to the leader's exercise of his growth option. We therefore can summarize the followers value function in all three industry stages as follows.

Lemma 3 *Let i be the follower F and let X_1^D and X_2^D be the triggers at which the leader and the follower exercise their growth options, respectively. The follower's value function over all three industry stages is given by*

$$V^i(X_t) = \begin{cases} \frac{R^i_{i=0,j=0}}{\delta} X_t - \frac{f_0^i}{r} + \frac{f_1^i - f_0^i + rIC_i}{r(\nu_1 - 1)} \left(\frac{X_t}{X_2^D}\right)^{\nu_1} & \\ + \frac{X_1^D}{\delta} \left[R^i_{i=0,j=1} - R^i_{i=0,j=0}\right] \left(\frac{X_t}{X_1^D}\right) & X_t \leq X_1^D, \\ \frac{R^i_{i=0,j=1}}{\delta} X_t - \frac{f_0^i}{r} + \frac{f_1^i - f_0^i + rIC_i}{r(\nu_1 - 1)} \left(\frac{X_t}{X_2^D}\right)^{\nu_1} & X_t \in [X_1^D, X_2^D], \\ \frac{R^i_{i=1,j=1}}{\delta} X_t - \frac{f_1^i}{r} & X_t > X_2^D. \end{cases} \tag{21}$$

The proof of this result is given in the Appendix.
The firm value of the follower consists of the value of the assets in place, the value of the growth option and the strategic effect related to the option exercise of the leader, i.e.,

$$V^F_{JI}(X_t) = V^{A(F)}_{JI}(X_t) + V^{G(F)}_{JI}(X_t) + V^{SE(F)}_{JI}(X_t),$$

where

$$V^{A(i)}_{JI}(X_t) \equiv \frac{R^i_{i=0,j=0}}{\delta} X_t - \frac{f_0^i}{r}, \quad i = F,$$

is the value of the assets in place,

$$V^{G(i)}_{JI} \equiv \frac{f_1^i - f_0^i + rIC_i}{r(\nu_1 - 1)} \left(\frac{X_t}{X_2^D}\right)^{\nu_1}, \quad i = F$$

is the value of the growth option that is exercised at the trigger level X_2^D and

$$V^{SE(i)}_{JI} \equiv \frac{X_1^D}{\delta} \left[R^i_{i=0,j=1} - R^i_{i=0,j=0}\right] \left(\frac{X_t}{X_1^D}\right) < 0, \quad i = F$$

is the strategic option value which is negative because of

$$R^i_{i=0,j=1} - R^i_{i=0,j=0} < 0.$$

We are now in a position to summarize the valuation of both the leader and the follower for the different industry stages.

Theorem 1 *The firm values of the leader $i = L$ and the follower $i = F$, respectively, are given by*

$$V_k^i(X_t) = V_k^{A(i)}(X_t) + V_k^{G(i)}(X_t) + V_k^{SE(i)}(X_t), \;\; k = JI, AI, MI.$$

This result is an immediate consequence of our preceding discussion. The value functions for both firms can now be used to derive the risk implications in an industry in which each firm has a single growth option. Our risk analysis can either be based on the individual firm level or on the industry level. Individual firm betas are defined as

$$\beta_k^i = \frac{\frac{\partial V_k^i(X)}{\partial X} X}{V_k^i(X)}, \;\; k = JI, AI, MI \text{ and } i = L, F.$$

Industry beta is defined as the beta of a portfolio that consists of both firms in the industry.

Theorem 2 *Consider a growing industry in which each firm has a single expansion option. Systematic firm risks for both the follower ($i = F$) and the leader ($i = L$) over the different industry stages are given by*

$$\beta_k^i(t) = 1 + \frac{V_k^{G(i)}(t) + V_k^{SE(i)}(t)}{V_k^i(t)}(\nu_1 - 1) + \frac{f_k^i/r}{V_k^i(t)}, \;\; k = JI, AI, MI.$$

where $V_k^{SE(i)}(t) < 0$ holds. Hence, strategic competition is risk reducing.

The results in Theorem 2 are surprising and have the following implications. As stated, competition is risk reducing. This is counter intuitive but is the consequence of $V_k^{SE(i)}(t) < 0$ and a hedging argument that goes along with that. The intuition for this result can be gained as follows. In case neither firm changes its capacity level all the profit uncertainty arises from the demand shock. Demand shocks directly translate into changes in the firm's cash flows. If, however, the leader who already exercised his growth option and hence operates with fixed capacity, faces an increase in the capacity of the follower upon the follower's option exercise, demand shocks are hedged by an output increase. This hedge is larger the closer the follower comes to exercising his growth option. As a consequence, the leader's risk is reduced and lies below market risk. This implies a counter-intuitive result and does not confirm the empirical findings of Hou and Robinson (2006). Systematic firm risk in a growing oligopolistic industry is driven by the growth option (i.e. the size of the firm), by operating leverage (i.e. firm's book to market), and an industry effect (the strategic effect). While the growth option and operating leverage are risk increasing, the strategic or industry effect is risk reducing. Hence, we find that firm's own and industry characteristics have opposite risk implications. Figure 2 in the Appendix gives a graphical presentation of the hedging argument. Before the follower exercises his growth option, industry output is given by the level Q_1. Since both firms have to produce at full capacity levels price fluctuates along the supply curve Q_1. Let's suppose that demand increases by an efficient amount so that the follower finds it optimal to exercise his growth option. Option exercise results in an increase in industry output to the level Q_2. The increase in industry supply causes prices to increase less than to the level indicated by the old supply curve P^*. The new price level is P_2 instead of P^*. This dampening corresponds to the hedging effect.

Figure 2 about here

Employing this hedging effect risk dynamics over different industry structures exhibit the following pattern. For ease of exposition let us look at the leader. Prior to the exercise of the growth option risk of the leader is running up until the exercise trigger is reached. Immediately after the exercise and prior to the exercise of the follower's expansion option, risk of the leader drops below market risk which is normalized by 1 and deceases until the follower exercises his option. After both firms have exercised their options risk is only driven by operating leverage. Risks dynamics are sketched in Figure 3.

Figure 3 about here

5.1.4 Sequential Equilibrium Option Exercise

The analysis in the preceding subsections is based on the assumption that firms exercise their growth options sequentially. In a duopoly market sequential exercise, however, must be the outcome of equilibrium behavior. Therefore we are now interested in deriving a sufficient condition on the fundamentals of the model that establishes sequential exercise as equilibrium strategy.

Consider the high cost firm, firm 2, and let us assume that it acts as the leader. In this case its firm value is characterized by the value function (19). In case the firm acts as the follower the value function corresponds to (21). Firm 2 has no incentive to be the leader if the firm value in case it acts as the follower is strictly greater than the firm value when it acts as the leader. This translates into the following inequality

$$V^F(X_t) > V^L(X_t) \;\; \forall X_t \in [X_0, X_D^1]. \tag{22}$$

It can now be established that if the investment costs of the high cost firm are sufficiently larger than those of the rival firm, it is always optimal for firm 2 to act as the follower.

Lemma 4 *There exists a critical level of investment costs $\tilde{IC}$ such that for all $IC_2 > \tilde{IC}$ firm 2 has no incentive to act as the leader, i.e., inequality (22) holds, and the industry is characterized by sequential option exercise.*

The proof of this result is given in the Appendix. This result justifies our assumption of sequential exercise of options. Only in case of sequential exercise are we able to study the different risk dynamics for the follower and for the leader and hence get more detailed insights than existing studies with symmetric firms. It should, however, be pointed out that there are additional equilibria. In particular there might exist an equilibrium with symmetric exercise. Such an equilibrium has interesting implications. It demonstrates that in an asymmetric industry, different firms can have the incentive to simultaneously act and exercise a growth option. Such a behavior can be used to explain merger waves in an asymmetric industry with multiple targets.

5.2 Exercise of Contraction Options

The risk analysis of a growing industry in which firms exercise growth options has revealed two important results. We found that industry effects arising from the strategic interactions of rival firms are risk reducing and that firm's own and industry characteristics have opposite risk implications. In this subsection we explore the robustness of these results for the case of contraction options. We assume that firms are operating with a given initial capacity Q_0^i and that each has a single option to reduce capacity to a level given by Q_{-1}^i. Contraction to a smaller firm size and capacity will be optimal for the firms if demand turns out to be low so that existing capacity levels cannot be sustained. Allowing for contraction options can be seen as a substitute for operating flexibility. Operating flexibility implies that firms can adjust their output levels downwards and omit some idle capacities when industry demand is low.

In analogy to the case of growth options we start out with a juvenile industry in which both firms produce at capacity levels equal to Q_0^i. The juvenile industry is followed by an adolescent industry in which the leader produces with capacity level Q_{-1}^i and the follower with the initial level Q_0^j. The last stage is the mature industry in which both firms have reduced their capacity to Q_{-1}^i.

When we allow for contraction options, we have to make an assumption about the value of the capacity that is sold. We capture this by introducing salvage value terms. These salvage values can be interpreted as the proceeds to the downsizing firm resulting from selling off some capacity units. In our model the salvage values are exogenously given. As in the expansion case we assume asymmetric salvage values and sequential exercise of contraction options. The leader is assumed to benefit from a higher salvage value and the follower is assumed to have the lower one $SV_L > SV_F$.

5.2.1 Firm Values in a Mature Industry

In a mature industry both firms have exercised their contraction options and each operates with given assets in place. Firm values correspond to the present values of the assets in place and are given by

$$V_{MI}^i(X_t) = \frac{R_{i=-1,j=-1}^i}{\delta} X_t - \frac{f_{-1}^i}{r}. \tag{23}$$

Based on the asset values we can derive the beta's of both firms. They are equal to

$$\beta_{MI}^i(t) = 1 + \frac{f_{-1}^i/r}{V_{MI}^i(t)},\ i = L, F.$$

Again, in a mature industry operating leverage and irreversibility drive individual firm and industry risk.

5.2.2 Firm Values in an Adolescent Industry

In an adolescent industry the leader already exercised his option to shrink while the follower exercises his option depending on the level of industry demand. In such an environment the firm value of the follower can be derived along the same lines as in the preceding subsection.

Lemma 5 *Assume that the leader already exercised his contraction option. Let i be the follower F and let X_2^C be the trigger at which the follower exercises his contraction option. Then the follower's value function is given by*

$$V^i(X_t) = \begin{cases} \frac{R^i_{i=0,j=-1}}{\delta} X_t - \frac{f^i_0}{r} + \frac{f^i_0 - f^i_{-1} + rSV_i}{r(1-\nu_2)} \left(\frac{X_t}{X_2^C}\right)^{\nu_2} & X_t \geq X_2^C, \\ \frac{R^i_{i=-1,j=-1}}{\delta} X_t - \frac{f^i_{-1}}{r} & X_t < X_2^C. \end{cases} \tag{24}$$

where ν_2 is the negative root of (6) and the trigger level X_2^C is given by

$$X_2^C = \frac{\delta \nu_2 (f^i_0 - f^i_{-1} + rSV_i)}{(1-\nu_2) r [R^i_{i=-1,j=-1} - R^i_{i=0,j=-1}]} > 0.$$

The proof of this result is given in the Appendix.
In case of downsizing the follower's option becomes a put option instead of a call option when expansion is studied. The put option is visible by the negative root of the characteristic equation (6), which turns the value function into a convex decreasing function of the value of the underlying asset X_t.

The follower's value function as stated in Lemma 5 only applies to the case that the leader already exercised his option. In a juvenile industry the follower's value has to be adjusted for the impact the leader has when exercising his option.

5.2.3 Firm Values in a Juvenile Industry

In a juvenile industry both firms start out with their initial levels of capacity and sequentially reduce capacity. This sequential exercise, again gives rise to a strategic effect which we want to quantify now. While the contraction options of both firms correspond to put options, and hence are risk reducing, it is not obvious, what the size and the risk implications of the strategic effect is.

Lemma 6 *Suppose firm i is the leader L and let X_1^C be the investment trigger of the leader and X_2^C that of the follower. The leader's value function* *is given by*

$$V^i(X_t) = \begin{cases} \frac{R^i_{i=0,j=0}}{\delta}X_t - \frac{f^i_0}{r} + \frac{f^i_0 - f^i_{-1} + rSV_i}{r(1-\nu_2)}\left(\frac{X_t}{X^C_1}\right)^{\nu_2} & \\ +\frac{X^C_2}{\delta}\left[R^i_{i=-1,j=-1} - R^i_{i=-1,j=0}\right]\left(\frac{X_t}{X^C_2}\right)^{\nu_2} & X_t > X^C_1, \\ \frac{R^i_{i=-1,j=0}}{\delta}X_t - \frac{f^i_{-1}}{r} & \\ +\frac{X^C_2}{\delta}\left[R^i_{i=-1,j=-1} - R^i_{i=-1,j=0}\right]\left(\frac{X_t}{X^C_2}\right)^{\nu_2} & X_t \in [X^C_2, X^C_1], \\ \frac{R^i_{i=-1,j=-1}}{\delta}X_t - \frac{f^i_{-1}}{r} & X_t < X^C_2, \end{cases} \tag{25}$$

with an expansion trigger X^C_1 *equal to*

$$X^C_1 = \frac{\delta\nu_2(f^i_0 - f^i_{-1} + rSV_i)}{(1-\nu_2)r[R^i_{i=-1,j=0} - R^i_{i=0,j=0}]} > 0. \tag{26}$$

The proof of this result is given in the Appendix.
Based on the last result we can decompose the value of the leader into three components. The value of the assets in place, the value of the contraction option given by

$$V^{C(i)}_{JI} \equiv \frac{f^i_0 - f^i_{-1} + rSV_i}{r(1-\nu_2)}\left(\frac{X_t}{X^C_1}\right)^{\nu_2}, \quad i = L$$

and the value associated with the strategic effect equal to

$$V^{SE(i)}_{JI} \equiv \frac{X^C_2}{\delta}\left[R^i_{i=-1,j=-1} - R^i_{i=-1,j=0}\right]\left(\frac{X_t}{X^C_2}\right)^{\nu_2}, \quad i = L.$$

Because the inequality

$$R^i_{i=-1,j=-1} - R^i_{i=-1,j=0} > 0$$

holds, the strategic effect in case of contraction options now becomes positive, i.e., value increasing. Summing up, the value of the leader ($i = L$) in the three different industry stages is composed of the value of the assets in place, the put option value and the strategic value, i.e.,

$$V^i_k(X_t) = V^{A(i)}_k(X_t) + V^{C(i)}_k(X_t) + V^{SE(i)}_k(X_t), \; k = JI, AI, MI.$$

The strategic (industry) effect derived for the leader, also applies to the follower when a juvenile industry is considered. Prior to the exercise of the put option of the leader, the follower's value also depends on the strategic effect.

Lemma 7 *Let i be the follower F and let X_1^C and X_2^C be the triggers at which the leader and the follower exercise their contraction options, respectively. The follower's value function over all three industry stages is given by*

$$V^i(X_t) = \begin{cases} \frac{R^i_{i=0,j=0}}{\delta} X_t - \frac{f_0^i}{r} + \frac{f_0^i - f_{-1}^i + rSV_i}{r(1-\nu_2)} \left(\frac{X_t}{X_2^C}\right)^{\nu_2} & \\ \frac{X_1^C}{\delta}\left[R^i_{i=0,j=-1} - R^i_{i=0,j=0}\right]\left(\frac{X_t}{X_1^C}\right)^{\nu_2} & X_t \geq X_1^C, \\ \frac{R^i_{i=0,j=-1}}{\delta} X_t - \frac{f_0^i}{r} + \frac{f_0^i - f_{-1}^i + rSV_i}{r(1-\nu_2)} \left(\frac{X_t}{X_2^C}\right)^{\nu_2} & X_t \in [X_2^C, X_1^C], \\ \frac{R^i_{i=-1,j=-1}}{\delta} X_t - \frac{f_{-1}^i}{r} & X_t < X_2^C. \end{cases} \tag{27}$$

where

$$R^i_{i=0,j=-1} - R^i_{i=0,j=0} > 0.$$

holds.

The proof of this result is given in the Appendix.
The firm value of the follower is given by the value of the assets in place, the value of the contraction option and the strategic effect related to the option exercise of the leader. The valuation results for both the leader and the follower can now be summarized.

Theorem 3 *The firm values of the leader $i = L$ and the follower $i = F$, respectively, are given by*

$$V_k^i(X_t) = V_k^{A(i)}(X_t) + V_k^{C(i)}(X_t) + V_k^{SE(i)}(X_t), \;\; k = JI, AI, MI.$$

Relative to the results found for the case of expansion options we have to point out, that the strategic effect in case of downsizing is positive and hence value increasing. But as we will find out, it is also risk reducing.

Theorem 4 *Systematic firm risks for both the follower ($i = F$) and the leader ($i = L$) over the different industry stages are given by*

$$\beta_k^i(t) = 1 + \frac{V_k^{C(i)}(t) + V_k^{SE(i)}(t)}{V_k^i(t)}(\nu_2 - 1) + \frac{f_k^i/r}{V_k^i(t)}, \; k = JI, AI, MI.$$

where $\nu_2 < 0$ and $V_k^{SE(i)}(t) > 0$ hold.

The last result has important consequences. First, the strategic effect is again risk reducing so that we can argue that competition in our model is risk reducing, independent of whether we are in a growing or a shrinking industry. Second, option risk in case of contraction is now risk reducing, given the nature of the put option. This together implies that firm own and industry effects do have the same risk implications.

6 Conclusion

In this paper we consider a duopolistic industry with firms producing a homogenous product at given capacity levels. Demand in the industry is stochastic and governed by an industry shock that follows a geometric Brownian motion. Firms produce with given capacity levels that are fixed (i.e. there is no operating flexibility) but can increase (decrease) their output with the exercise of a growth (contraction) option. Although there are no variable production costs, firms operate with fixed costs that change with the level of capacity. Growth option exercise causes the firms to incur investment costs that include adjustment costs and the price of the investment, while in case of the contraction option firms incur a salvage value. We assume that one of the firms is a high cost (high salvage value) and one a low cost (low salvage value) firm. In terms of option exercise we only consider the case of sequential exercise. The low cost (high salvage value) firm acts as the leader and exercises first and the high cost (low salvage value) firm acts as the follower and exercises second. We discuss that this exercise behavior corresponds to a sub-game perfect Nash equilibrium.

Given this industry structure we derive firm values and risk dynamics for individual firms and an industry portfolio. We find interesting novel results. First, we identify a strategic effect that causes risk of the the firms to reduce. Hence we argue that more competition results in a lower firm specific risk. This reduced risk is the consequence of a hedging effect. In case both firms operate with fixed capacity levels any profit uncertainty arises from the industry demand shock. Demand shocks directly translate into changes in the firm's cash flows. If, however, the leader who already exercised his growth option and faces fixed capacity forever, faces an change in the capacity of the follower upon the follower's option exercise, demand shocks are hedged by an output increase or decrease, depending whether we are in a growing or shrinking industry. This hedge is larger the closer the follower comes to exercising his option. As a consequence, the leaders risk is reduced and is below the market risk normalized by 1. This is a counter-intuitive result and does not confirm the empirical findings of Hou and Robinson (2006). The result, however, allows for a better understanding of the driving forces behind a firm's risks. As pointed out, many existing studies identify operating leverage and irreversibility as the two channels that drive the risk dynamics in a market. In this paper we add an additional factor, that we call industry factor. It turns out, that the industry factor behaves differently, depending on the whether the industry grows or shrinks. In case of expansion options, firm own and industry characteristics have opposite risk implications. In case of a contraction option, firm own and industry characteristics have the same risk implications.

There are many open questions for further research. It is important to allow for operating flexibility and derive the risk dynamics in this case. When firms have operating flexibility they need not produce with a given capacity level but can choose to have idle capacities if demand is low. It is clear that operating flexibility substantially changes the risk dynamics. Additionally, it seems interesting to look at risk dynamics in an equilibrium with simultaneous exercise. This simultaneous exercise can exist even if firms are asymmetric. Finally, it is a challenge to allow for a more dynamic capacity expansion in which each firm faces several growth options to expand.

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Appendix

Proof of Lemma 1: Using dynamic programming it can be shown that the value function for the follower ($i = F$) needs to satisfy the Bellman equation,

$$\frac{1}{2}\sigma^2 X^2 V^i_{XX} + (r-\delta)XV^i_X - rV^i + XR^i_{i=0,j=1} - f^i_0 = 0. \tag{28}$$

Ruling out bubbles, the general solution to this differential equation is given by

$$V^i(X) = A^i_0 + A^i_1 X + A^i_2 X^{\nu_1}$$

where ν_1 is the positive root of the characteristic equation (6). The value function needs to satisfy the boundary conditions

$$\begin{aligned} V^i(0) &= -\frac{f^i_0}{r}, \\ V^i(X^D_2) &= \frac{X^D_2 R^i_{i=1,j=1}}{\delta} - \frac{f^i_1}{r} - IC_i, \\ V^i_X(X^D_2) &= \frac{R^i_{i=1,j=1}}{\delta}. \end{aligned}$$

Using these boundary conditions the integration constants become

$$\begin{aligned} A^i_0 &= -\frac{f^i_0}{r}, \\ A^i_1 &= \frac{R^i_{i=0,j=1}}{\delta} \\ A^i_2 &= \frac{f^i_1 - f^i_0 + rIC_i}{r(\nu_1 - 1)} \left(X^i_F\right)^{-\nu_1}, \end{aligned}$$

which results in the value function for the case $X_t \leq X^D_2$. For $X_t > X^D_2$ immediate exercise is optimal which result in the second part of the value function. QED

Proof of Lemma 2: The value function for the leader ($i = L$) needs to satisfy the Bellman equation

$$\frac{1}{2}\sigma^2 X^2 V^i_{XX} + (r-\delta)XV^i_X - rV^i + XR^i_{i=0,j=0} - f^i_0 = 0$$

together with the boundary conditions

$$\begin{aligned}
V^i(0) &= -\frac{f_0^i}{r}, \\
V^i(X_1^D) &= \frac{X_1^D R^i_{i=1,j=0}}{\delta} - \frac{f_1^i}{r} - IC_i + A_3^i (X_1^D)^{\nu_1}, \\
V_X^i(X_1^D) &= \frac{R^i_{i=1,j=0}}{\delta} + \nu_1 A_3^i (X_1^D)^{\nu_1 - 1}, \\
V^i(X_2^D) &= \frac{X_2^D R^i_{i=1,j=1}}{\delta} - \frac{f_1^i}{r}.
\end{aligned}$$

A solution to the Bellman equation is given by

$$V^i(X) = A_0^i + A_1^i X + A_2^i X^{\nu_1}$$

where $A_k^i, k = 0, 1, 2$ are constants that are determined together with the boundary conditions. The constant A_3^i from above expresses the change of the value function for the leader after the capacity expansion of the follower has taken place. It is determined by the boundary condition

$$V^i(X_2^D) = \frac{X_2^D R^i_{i=1,j=1}}{\delta} - \frac{f_1^i}{r}.$$

Solving the Bellman equation together with the boundary conditions results in

$$\begin{aligned}
A_0^i &= -\frac{f_0^i}{r} \\
A_1^i &= \frac{R^i_{i=0,j=0}}{\delta} \\
A_2^i &= \frac{f_1^i - f_0^i + rIC_i}{r(\nu_1 - 1)} (X_1^D)^{-\nu_1} + A_3^i \\
A_3^i &= \frac{X_2^D [R^i_{i=1,j=1} - R^i_{i=1,j=0}]}{\delta} (X_2^D)^{-\nu_1}.
\end{aligned}$$

Substitution results in the value function given by (19). QED

Proof of Lemma 3: We make use of the proof of Lemma 1 for the follower($i = F$) and note that the boundary conditions now become

$$\begin{aligned} V^i(0) &= -\frac{f_0^i}{r}, \\ V^i(X_2^D) &= \frac{X_2^D R_{i=1,j=1}^i}{\delta} - \frac{f_1^i}{r} - IC_i + A_3^i (X_2^D)^{\nu_1}, \\ V_X^i(X_2^D) &= \frac{R_{i=1,j=1}^i}{\delta} + \nu_1 A_3^i (X_2^D)^{\nu_1 - 1}, \\ V^i(X_1^D) &= \frac{X_1^D R_{i=0,j=1}^i}{\delta} - \frac{f_0^i}{r}. \end{aligned}$$

The change of the boundary conditions relative to the proof of Lemma 1 is the consequence of the follower's response to the leader's exercise of the option at the trigger level X_1^D. The constant A_3^i accounts for this change. At the trigger level X_1^D when the leader exercises his option the followers value function needs to satisfy

$$V^i(X_1^D) = \frac{X_1^D R_{i=0,j=1}^i}{\delta} - \frac{f_0^i}{r}$$

which implies a follower's value function equal to (21). QED

Proof of Lemma 4: We assume that firm 1 is the low cost and firm 2 the high cost firm. We need to show that when firm 2 acts as the leader and chooses a value function along the lines of (19) this results in a firm value that is lower than the value if its acts as the follower. In this sense the high cost firm does not have the incentive to act as the leader. If this is the case, then sequential exercise is an equilibrium if the low cost firm has no incentives to ac t as the follower.

Firm 2 has no incentive to be the leader if the firm value in case it acts as the follower is strictly greater than the firm value when it acts as the leader. This translates into the following inequality

$$V^F(X_t)_{Follower} > V^F(X_t)_{Leader} \quad \forall X_t \in [X_0, X_D^1]. \tag{29}$$

Substituting (17) and (19) into equation (29) results in an equality that is satisfied the lager the difference between IC_2 and IC_1 is. An identical argument as here can be found in the paper by Pawlina and Kort (2002). Applying this argument shows the result. QED

Proof of Lemma 5: Again we make use of dynamic programming and assume that the leader (now the high salvage value firm) already exercised his option. The follower's value function ($i = F$) needs to satisfy the Bellman equation,

$$\frac{1}{2}\sigma^2 X^2 V^i_{XX} + (r-\delta)XV^i_X - rV^i + XR^i_{i=0,j=-1} - f^i_0 = 0. \tag{30}$$

A general solution to this differential equation is given by

$$V^i(X) = A^i_0 + A^i_1 X + A^i_2 X^{\nu_2} + A^i_3 X^{\nu_1}$$

where ν_1 is the positive and ν_2 the negative root of the characteristic equation (6). Since we are now dealing with a contraction option the no bubbles condition requires that $A^i_3 \equiv 0$ so that the corresponding contraction option becomes a put option. The value function needs to satisfy the boundary conditions

$$\begin{aligned} V^i(X^C_2) &= \frac{X^C_2 R^i_{i=-1,j=-1}}{\delta} - \frac{f^i_{-1}}{r} + SV_i, \\ V^i_X(X^C_2) &= \frac{R^i_{i=-1,j=-1}}{\delta}. \end{aligned}$$

Using these boundary conditions the integration constants become

$$\begin{aligned} A^i_0 &= -\frac{f^i_0}{r}, \\ A^i_1 &= \frac{R^i_{i=0,j=-1}}{\delta} \\ A^i_2 &= \frac{f^i_0 - f^i_{-1} + rSV_i}{r(1-\nu_2)} \left(X^C_2\right)^{-\nu_2}, \end{aligned}$$

which results in the value function specified in (24). QED

Proof of Lemma 6: The proof of this Lemma follows exactly that of Lemma 2 with the only change that because of the contraction the call option has to be changed to a put option with the corresponding terminal conditions.

Proof of Lemma 7: The proof of this Lemma follows exactly that of Lemma 3 with the change that we are now dealing with a contraction option that corresponds to a put rather than a call option.

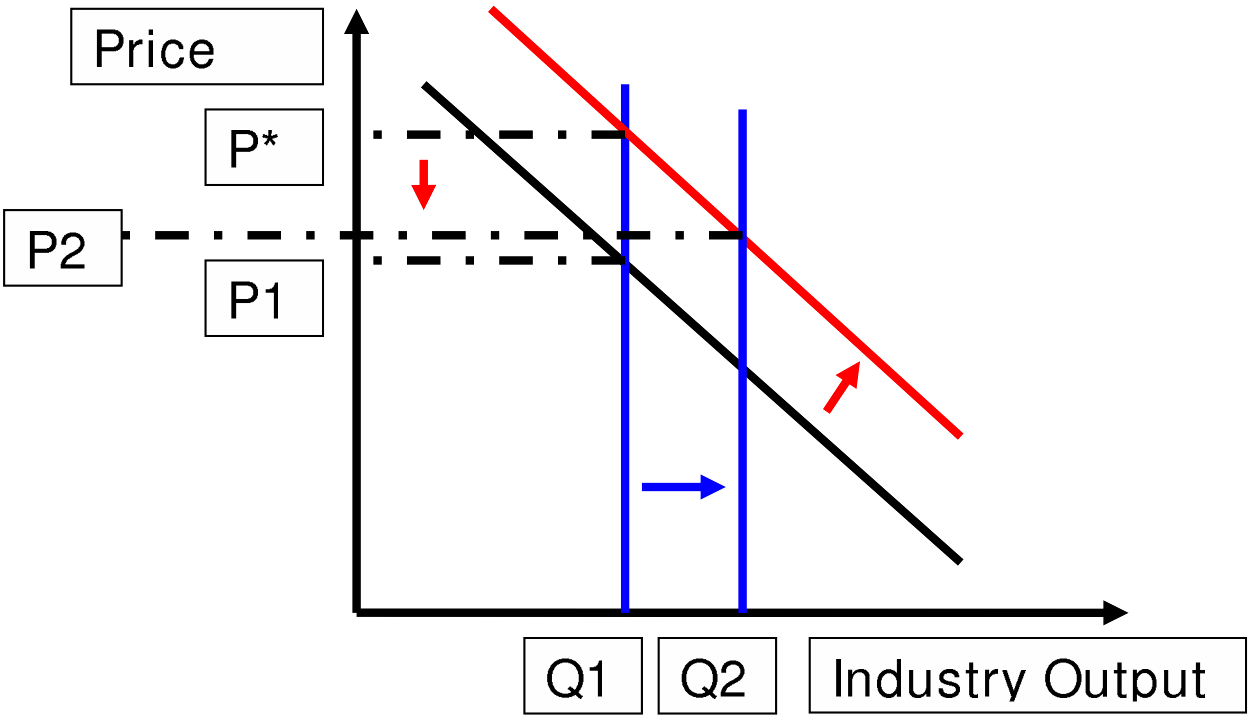

Figure 2: Price dynamics following the option exercise of the follower.

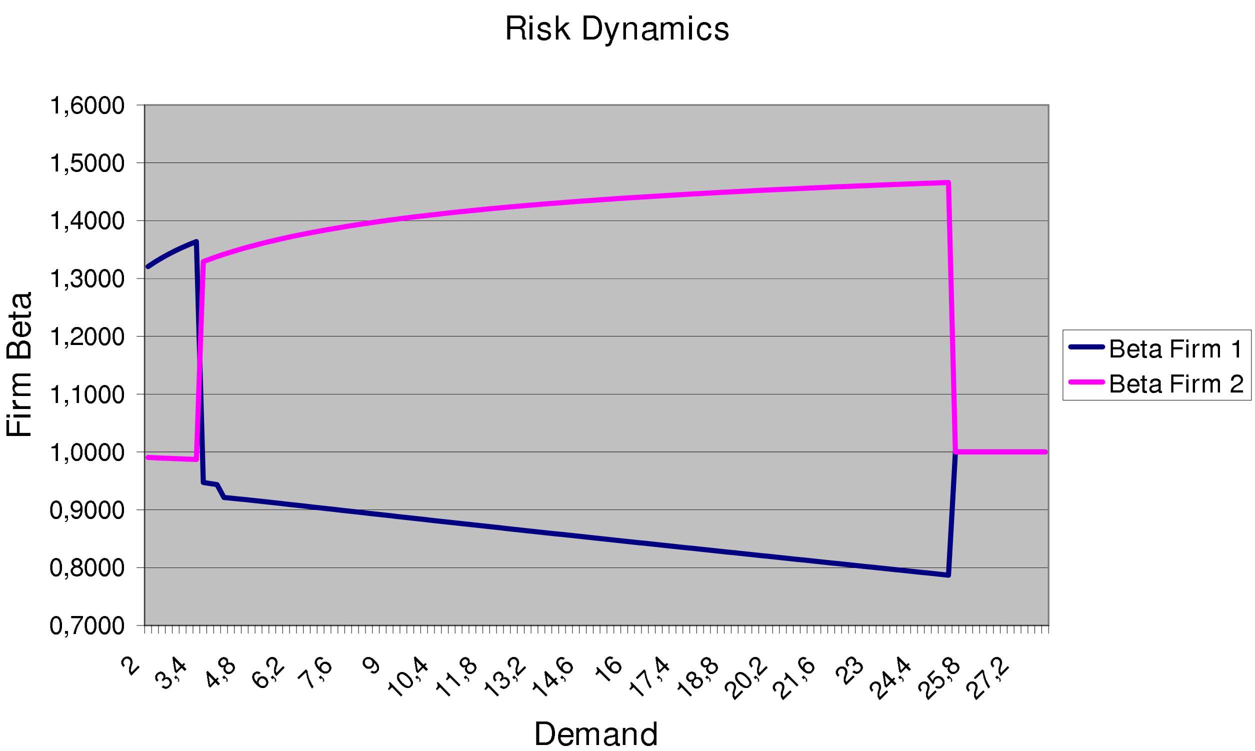

Figure 3: Risk dynamics across the three different industry stages for the leader and the follower.