Entry and Exit Decisions under Uncertainty in a Duopoly

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Abstract

In this paper, we analyze investment decision on the 'entry-exit' project, which can be active and suspended by paying some cost, in a duopoly setting. The model incorporates Dixit [5] and Huisman and Kort [10]. That is, we propose a new extension of the model that captures competitive nature in the recent trend. Then we show it is optimal that the firm must start producing at the beginning of the project, and the leader is more encouraged to invest in a duopoly than monopoly while the follower is more discouraged.

Keywords: Entry-exit, duopoly, operational options

JEL classification: D81, C73

1 Introduction

Real options approach, in other words, investment under uncertainty is classically studied by Brennan and Schwartz [4] and McDonald and Siegel [13], and basically summarized by Dixit and Pindyck [6]. Dixit and Pindyck [6] provide the most basic model in Ch.5, and analyze investment decision on the project that can be suspended without cost in Ch.6. In Ch.7, they analyze 'entry-exit' decision that the project can be active and suspended by paying some cost.[1]

As the new trend in this field, competitive nature is studied in many researches. Although the standard real options approach assumes a monopoly setting implicitly, there must be competitors in the real world. One can use game theory for decision in competition. In case of preemption, there is the risk that the firm may earn less profit if its competitor invests earlier. Game theory results in that the firm must invest earlier than monopoly, in contradiction to real options approach.

Dixit and Pindyck [6, Ch.9] incorporated competitive nature into real options approach properly at the earliest date. While they did not derive the equilibrium by game

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[1] Dixit [5] and Brekke and Øksendal [2, 3] also study entry and exit decisions in detail.

theory, Grenadier [9] applied their framework to the real-estate investment problem, and succeeded in deriving the equilibrium. His equilibrium remained the problem that simultaneous investment was eliminated, whereas Huisman and Kort [10] extended the theoretical framework by resolving the problem.

While the above literature analyzed only the competitive situation of Ch.5 in Dixit and Pindyck [6],[2] Takashima, Goto, Kimura and Madarame [16] considered that of Ch.6 for the electricity market. They analyzed not only identical two firms but also asymmetric firms. Entry and exit decisions in duopoly are investigated by Lambrecht [12], in particular, market entry, company closure and capital structure. Ruiz-Aliseda [15] and Amir and Lambson [1] investigate it by game theoretic approaches.

However, these literature do not analyze the situation of Ch.7 in Dixit and Pindyck [6]. So, we analyze investment decision on the entry-exit project of Ch.7 in a duopoly setting. This is a natural extension of the above researches in the recent trend.

The rest of the paper is organized as follows. Section 2 describes the firm's problem. Section 3 provides the equilibria, i.e., the firm's optimal investment strategy in our duopoly setting. Next, we present numerical examples in Section 4. Then, we discuss some important results in Section 5. Lastly, Section 6 concludes the paper.

2 The Model

We consider two identical firms which can start a project by investing the initial cost I. The two firms are labeled 1 and 2. By index i we refer to an arbitrary firm and by j to the 'other firm.' We assume that they can sell product in the market at the moment of investing the initial cost.

The project can become active by paying some cost K, then the firm can produce a unit flow of output at the variable cost C. Moreover, the project can be suspended by paying an exit cost E, and the firm can reenter by paying K again at some future time. A product will yield the sales according to a downward-sloping inverse demand function. The demand function will be subject to continuous shocks. The demand function is of the following form:[3]

$$P_t = D(Q_t)X_t, \tag{1}$$

where P_t denotes the output price at time t, Q_t denotes the number of firms which have invested the project, and $D(\cdot)$ is a differential function with $D'(\cdot) < 0$, which ensures the first mover's advantage. This market is characterized by evolving uncertainty in the state of demand. X_t represents a multiplicative demand shock, and follows a geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t, \quad X_0 = x, \tag{2}$$

where μ is the instantaneous expected growth rate of X_t, σ (> 0) is the instantaneous volatility of X_t, and W_t is a standard Brownian motion.

The project has three state variables, the demand shock X_t, the number of firms Q_t and discrete variable that indicates whether the project is active (1) or not (0). Let a right-continuous function with left limits Z_t denote the state 0 or 1. The profit function of both firms is given by

$$\pi(X_t, Q_t) = \big(D(Q_t)X_t - C\big)Z_t. \tag{3}$$

[2] However, Huisman [11] analyzed an asymmetric case (Ch.8) and a case of two projects (Ch.9).

[3] This demand structure is similar to that in Grenadier [9] except for the meaning of Q_t.

Suppose $Q_t = q$, the single firm's value function is given by

$$J(x,q) = \sup_{w\in\mathcal{W}} \mathbb{E}\left[\int_0^\infty \mathrm{e}^{-\rho t}\pi(X_t,q)\mathrm{d}t - \sum_{m=1}^{\infty} \mathrm{e}^{-\rho\theta_m} H(Z_{\theta_m^-}, Z_{\theta_m})\mathbf{1}_{\{\theta_m<\infty\}}\right], \tag{4}$$

where ρ ($> \mu$) denotes a discount rate,

$$w = (\theta_1, \theta_2, \ldots, \theta_m, \ldots; \zeta_1, \zeta_2, \ldots, \zeta_m, \ldots) \tag{5}$$

denotes the collection of the stopping time θ_m and the control of the state $\zeta_m = Z_{\theta_m}$, $\mathcal{W}$ denotes the collection of the admissible controls and

$$H(0,0) = H(1,1) = 0, \tag{6}$$
$$H(0,1) = K, \tag{7}$$
$$H(1,0) = E. \tag{8}$$

Equation (4) can be decomposed into the value function of inactive state $J_0(x,q)$ and active state $J_1(x,q)$. From Dixit [5], they are given by

$$J_0(x,q) = G_0(q)x^{\beta_1}, \tag{9}$$
$$J_1(x,q) = G_1(q)x^{\beta_2} + \frac{D(q)x}{\delta} - \frac{C}{\rho}, \tag{10}$$

respectively, where $\delta = \rho - \mu$. The unknown parameters $G_0(q)$ and $G_1(q)$ are found numerically from the value-matching conditions:

$$J_0(\overline{X}(q), q) = J_1(\overline{X}(q), q) - K, \tag{11}$$
$$J_1(\underline{X}(q), q) = J_0(\underline{X}(q), q) - E, \tag{12}$$

and smooth-pasting conditions:

$$J_0'(\overline{X}(q), q) = J_1'(\overline{X}(q), q), \tag{13}$$
$$J_1'(\underline{X}(q), q) = J_0'(\underline{X}(q), q), \tag{14}$$

where $\overline{X}(q)$ and $\underline{X}(q)$ are the optimal threshold to reenter and suspend respectively, which are found from the above conditions.

In a duopoly setting, the firm's decision problem is given by

$$\begin{aligned}
V^i(x) = \sup_{\tau^i\in\mathcal{T}} \mathbb{E}\Bigg[&\sup_{w\in\mathcal{W}} \mathbb{E}\Bigg[\int_{\tau^i\wedge\tau^j}^{\tau^i\vee\tau^j} \mathrm{e}^{-\rho t}\pi(X_t,1)\mathrm{d}t \\
&\qquad - \sum_{m=1}^{\infty} \mathrm{e}^{-\rho\theta_m} H(Z_{\theta_m^-}, Z_{\theta_m})\mathbf{1}_{\{\tau^i\wedge\tau^j\le\theta_m<\tau^i\vee\tau^j\}}\Bigg]\mathbf{1}_{\{\tau^i<\tau^j\}} \\
&+ \sup_{w\in\mathcal{W}} \mathbb{E}\left[\int_{\tau^i\vee\tau^j}^{\infty} \mathrm{e}^{-\rho t}\pi(X_t,2)\mathrm{d}t - \sum_{m=1}^{\infty} \mathrm{e}^{-\rho\theta_m} H(Z_{\theta_m^-}, Z_{\theta_m})\mathbf{1}_{\{\tau^i\vee\tau^j\le\theta_m<\infty\}}\right] - \mathrm{e}^{-\rho\tau^i} I\Bigg], \\
= \sup_{\tau^i\in\mathcal{T}} \mathbb{E}\Bigg[&\mathrm{e}^{-\rho\tau^i\wedge\tau^j} J(X_{\tau^i\wedge\tau^j}, 1)\mathbf{1}_{\{\tau^i<\tau^j\}} \\
&+ \mathrm{e}^{-\rho\tau^i\vee\tau^j}\left(J(X_{\tau^i\vee\tau^j}, 2) - J(X_{\tau^i\vee\tau^j}, 1)\mathbf{1}_{\{\tau^i<\tau^j\}}\right) - \mathrm{e}^{-\rho\tau^i} I\Bigg],
\end{aligned} \tag{15}$$

where τ^i denotes the stopping time for firm i to invest and $\mathcal{T}$ denotes the collection of admissible stopping times. The second equality holds by strong Markov property of X_t. Since $J(x,q)$ can be decomposed into two states, equation (15) can be rewritten into

$$\begin{aligned} V_k^i(x) &= \sup_{\tau^i\in\mathcal{T}} \mathbb{E}\Big[\mathrm{e}^{-\rho\tau^i\wedge\tau^j} J_k(X_{\tau^i\wedge\tau^j},1)\mathbf{1}_{\{\tau^i<\tau^j\}} \\ &\qquad + \mathrm{e}^{-\rho\tau^i\vee\tau^j}\big(J_k(X_{\tau^i\vee\tau^j},2) - J_k(X_{\tau^i\vee\tau^j},1)\mathbf{1}_{\{\tau^i<\tau^j\}}\big) - \mathrm{e}^{-\rho\tau^i} I\Big], \qquad (16) \\ V^i(x) &= \max\{V_0^i(x), V_1^i(x)\}. \qquad (17) \end{aligned}$$

Equation (17) claims that both firms must choose initial state k as well as investment time τ^i.

There are three patterns of investment. If $\tau^i < \tau^j$, firm i can earn higher profit until firm j enters into the market at τ^j. In this case, firm i is called *the leader*, and $V^i(x) := L^i(x)$. If $\tau^i > \tau^j$, firm i waits to enter and can earn no profit until τ^i. In this case, firm i is called *the follower*, and $V^i(x) := F^i(x)$. If $\tau^i = \tau^j$, both firms earn lower profit since they enter into the market simultaneously. this case is called *simultaneous investment*, and $V^i(x) := M^i(x)$.

3 Equilibria

Further ahead, we solve the maximum problem (17) at the moment the leader has invested, i.e., $\tau^i \wedge \tau^j = 0$. Dynamic games are usually solved backwards. Since two firms are identical, we omit the index i, j.

First, we derive the value function of simultaneous investment. In this case, both firms invest simultaneously such that $Q_t = 2$ for all time, so we have

$$M_k(x) = J_k(x,2) - I. \qquad (18)$$

Next, since the leader has already invested the project, the value function of the follower is

$$\begin{aligned} F(x) &= \sup_{\tau\in\mathcal{T}} \mathbb{E}\left[\sup_{w\in\mathcal{W}} \mathbb{E}\left[\int_\tau^\infty \mathrm{e}^{-\rho t}\pi(X_t,2)\mathrm{d}t - \sum_{m=1}^\infty \mathrm{e}^{-\rho\theta_m} H(Z_{\theta_m^-}, Z_{\theta_m})\mathbf{1}_{\{\tau\le\theta_m<\infty\}} \right] - \mathrm{e}^{-\rho\tau} I \right], \\ &= \sup_{\tau\in\mathcal{T}} \mathbb{E}\Big[\mathrm{e}^{-\rho\tau}\big(J(X_\tau,2) - I\big)\Big], \qquad (19) \end{aligned}$$

where τ denotes the stopping time for the follower to invest. Decomposing equation (19) into the two states, we have

$$F_k(x) = \sup_{\tau_k\in\mathcal{T}} \mathbb{E}\Big[\mathrm{e}^{-\rho\tau_k}\big(J_k(X_\tau,2) - I\big)\Big], \qquad (20)$$

$$F(x) = \max\{F_0(x), F_1(x)\}, \qquad (21)$$

where τ_k denotes the stopping time of the follower with state k. Given the constant threshold of the follower X_k^{F}, τ_k is the form of

$$\tau_k = \inf\{t > 0 : X_t \ge X_k^{\mathrm{F}}\}. \qquad (22)$$

Equation (20) satisfies the following ODE:

$$\frac{1}{2}\sigma^2 x^2 F_k''(x) + \mu x F_k'(x) - \rho F_k(x) = 0 \qquad (23)$$

with boundary conditions:

$$F_k(0) = 0, \tag{24}$$
$$F_k(X_k^{\mathrm{F}}) = M_k(X_k^{\mathrm{F}}), \tag{25}$$
$$F_k'(X_k^{\mathrm{F}}) = M_k'(X_k^{\mathrm{F}}). \tag{26}$$

Here, the second condition is called the value-matching condition, and the third is called the smooth-pasting condition. By solving equation (23) with equation (24)–(26), we have

$$F(x) = \begin{cases} A_1 x^{\beta_1} & \text{if } x < X_1^{\mathrm{F}}, \\ M_1(x) & \text{if } x \geq X_1^{\mathrm{F}}, \end{cases} \tag{27}$$

where A_1 and X_1^{F} are the solutions of nonlinear simultaneous equation (25) and (26) with $k = 1$. And the optimal X_0^{F} does not exist.

Suppose that the follower plays the optimal policy τ_1, the value function of the leader is

$$\begin{aligned} L(x) &= \mathbb{E}\left[\sup_{w\in\mathcal{W}} \mathbb{E}\left[\int_0^{\tau} \mathrm{e}^{-\rho t}\pi(X_t, 1)\mathrm{d}t - \sum_{m=1}^{\infty} \mathrm{e}^{-\rho\theta_m} H(Z_{\theta_m^-}, Z_{\theta_m})\mathbf{1}_{\{\theta_m<\tau\}}\right]\right. \\ &\qquad \left. + \sup_{w\in\mathcal{W}} \mathbb{E}\left[\int_{\tau}^{\infty} \mathrm{e}^{-\rho t}\pi(X_t, 2)\mathrm{d}t - \sum_{m=1}^{\infty} \mathrm{e}^{-\rho\theta_m} H(Z_{\theta_m^-}, Z_{\theta_m})\mathbf{1}_{\{\tau\leq\theta_m<\infty\}}\right] - I\right], \\ &= \mathbb{E}\Big[J(x, 1) - I + \mathrm{e}^{-\rho\tau_1}\big(J(X_{\tau_1}, 2) - J(X_{\tau_1}, 1)\big)\Big], \end{aligned} \tag{28}$$

which can be rewritten into

$$L_k(x) = \mathbb{E}\Big[J_k(x, 1) - I + \mathrm{e}^{-\rho\tau_1}\big(J_k(X_{\tau_1}, 2) - J_k(X_{\tau_1}, 1)\big)\Big], \tag{29}$$
$$L(x) = \max\{L_0(x), L_1(x)\}. \tag{30}$$

Let the last term of equation (29) be $\tilde{L}_k(x)$, which satisfies the following ODE:

$$\frac{1}{2}\sigma^2 x^2 \tilde{L}_k''(x) + \mu x \tilde{L}_k'(x) - \rho \tilde{L}_k(x) = 0 \tag{31}$$

with boundary conditions

$$\tilde{L}_k(0) = 0, \tag{32}$$
$$L_k(X_1^{\mathrm{F}}) = M_k(X_1^{\mathrm{F}}). \tag{33}$$

Here, since equation (29) is not maximum problem, the smooth-pasting condition is not necessary, and X_1^{F} is known. By solving equation (31) with equation (32) and (33), we have

$$L(x) = \begin{cases} \dfrac{D(1)x}{\delta} - \dfrac{C}{\rho} + B_1 x^{\beta_1} - I & \text{if } x < X_1^{\mathrm{F}}, \\ M_1(x) & \text{if } x \geq X_1^{\mathrm{F}}, \end{cases} \tag{34}$$

where

$$B_1 = G_1(2)(X_1^{\mathrm{F}})^{\beta_2-\beta_1} - \frac{D(1) - D(2)}{\delta}(X_1^{\mathrm{F}})^{1-\beta_1}, \tag{35}$$

and the optimal $L_0(x)$ does not exist.

The fact that the optimal X_0^{F} and $L_0(x)$ do not exist gives the following proposition:

Proposition 1 *The follower and the leader must invest the project with the state* 1*.*

Proposition 1 describes that there is no reason to incur the investment cost I only to keep the project idle for some time. We can find the same property as Dixit and Pindyck [6, Ch.6].

Finally, we derive the equilibrium.

Proposition 2 *There exists a unique value for* x*, which we denote by* X_1^{L}*, such that*

$$L_1(X_1^{\mathrm{L}}) = F_1(X_1^{\mathrm{L}}), \text{ and } 0 < X_1^{\mathrm{L}} < X_1^{\mathrm{F}}. \tag{36}$$

We define the stopping time

$$\lambda_1 = \inf\{t > 0 : X_t \geq X_1^{\mathrm{L}}\}. \tag{37}$$

Unfortunately, X_1^{L} must be found numerically. Due to Proposition 2, we can use the strategy space and equilibrium concept defined by Huisman and Kort [10]. This concept can be traced back to Fudenberg and Tirole [7].

Proposition 3 *There are three types of equilibria depending on the value of* x*.*

1. *If* $x \in (0, X_1^{\mathrm{L}}]$*, there are two possible outcomes. In the first, firm 1 is the leader and invests the project at time* λ_1*, and firm 2 is the follower and invests at time* τ_1 *with probability* $1/2$*. The second is the symmetric counterpart, and the probability that both firms invest simultaneously is zero.*

2. *If* $x \in (X_1^{\mathrm{L}}, X_1^{\mathrm{F}}]$*, there are three possible outcomes. In the first, firm 1 is the leader and invests at time* 0*, and firm 2 is the follower and invests at time* τ_1 *with probability* $\frac{F_1(x)-M_1(x)}{L_1(x)+F_1(x)-2M_1(x)}$*. The second is the symmetric counterpart. In the third, both firms invests simultaneously at time* 0 *with probability* $\frac{L_1(x)-F_1(x)}{L_1(x)+F_1(x)-2M_1(x)}$*.*

3. *If* $x \in (X_1^{\mathrm{F}}, \infty)$*, then both firms invests at time* 0 *with probability* 1*.*

The proof is the exactly same as Huisman and Kort [10] and omitted. Proposition 3 claims that there is no simultaneous investment where both firms can earn less profit, if the game starts with low demand shock.

4 Numerical Examples

In this section, we use the basic parameter values: $\mu = 0.02$, $\sigma = 0.20$, $\rho = 0.04$, $D(1) = 2$, $D(2) = 1$, $C = 5$, $K = 10$, $E = 5$ and $I = 50$ (see Table 1). Then, we have the optimal thresholds: $\underline{X}(1) = 1.53$, $\overline{X}(1) = 3.93$, $\underline{X}(2) = 3.06$, $\overline{X}(2) = 7.87$, $X_1^{\mathrm{L}} = 3.81$ and $X_1^{\mathrm{F}} = 11.12$ (see Table 2).

Figure 1 displays the value functions of the leader, the follower and simultaneous investment. Their shapes are almost same as Huisman and Kort [10].

Figure 2 displays the comparative statics of the thresholds with respect to σ. The leader's threshold X_1^{L} could be less than $\underline{X}(2)$ but surely greater than $\underline{X}(1)$, so that lemma 5 holds. Although the leader's threshold X_1^{L} could be less than $\overline{X}(1)$, the follower's threshold X_1^{F} is much greater than $\overline{X}(2)$. This implies that the leader is more encouraged to invest in a duopoly than monopoly and the follower is more discouraged.

Table 1: The parameter values for the base case

parameter	value
μ	0.02
σ	0.20
ρ	0.04
$D(1)$	2
$D(2)$	1
C	5
K	10
E	5
I	50

Table 2: The threshold values for the base case

threshold	value
$\underline{X}(1)$	1.53
$\overline{X}(1)$	3.93
$\underline{X}(2)$	3.06
$\overline{X}(2)$	7.87
X_1^{L}	3.81
X_1^{F}	11.12

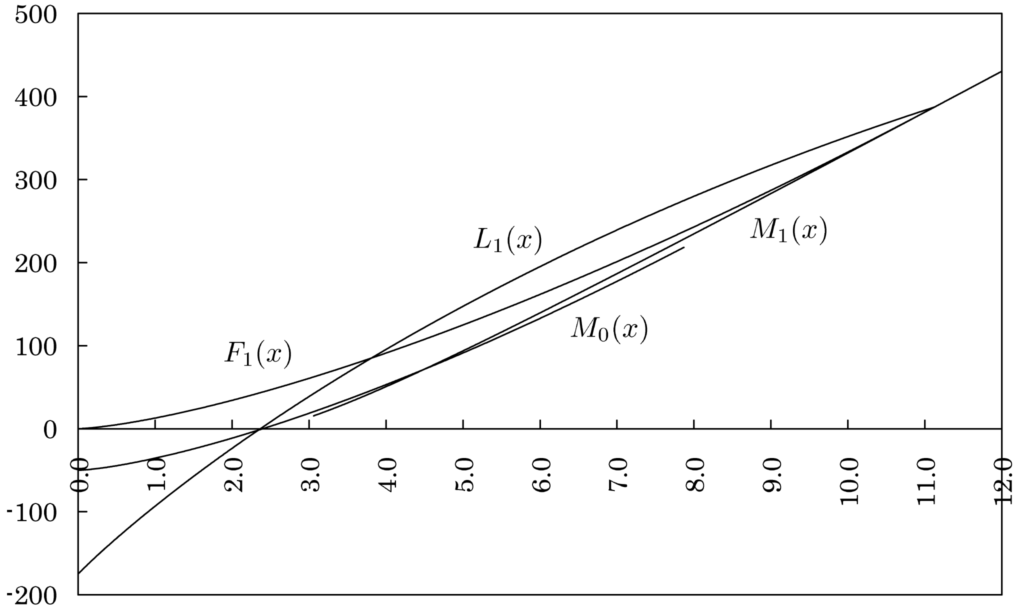

Figure 1: The value functions

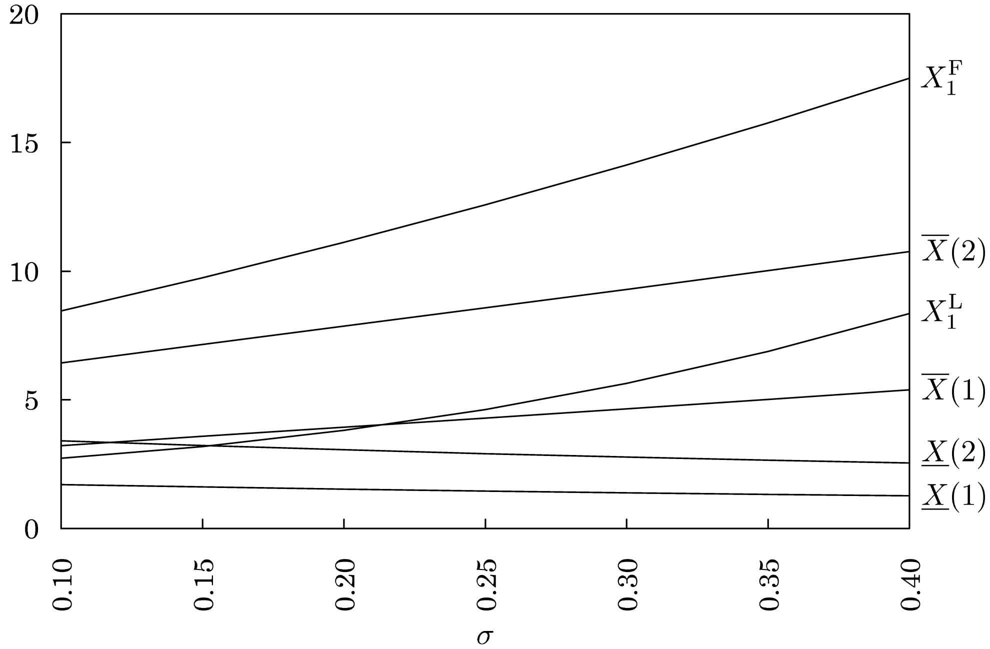

Figure 2: The comparative statics of the thresholds with respect to σ

5 Discussion

Numerical results in the previous section give some important results below. We need them for the completion of the equilibrium.

First, there is the question that the leader suspends the project when the follower invests. If it is ture, then equation (19) does not hold because $Q_{\tau_1} = 1$. However, numerical results show X_1^F is greater than $\overline{X}(1)$. The leader is sure to activate the project when $x \geq \overline{X}(1)$, so that $Q_{\tau_1} = 2$. Therefore, we obtain the following proposition:

Proposition 4 *The fact that*

$$\overline{X}(1) < X_1^F \tag{38}$$

ensures the validity of equation (19).

Next, there is the question that X_1^L is low enough to damage the leader's value much. The leader is sure to idle the project when $x \leq \underline{X}(1)$. If the leader activates the project in this region, the threat of preemption makes the leader's value much lower. However, numerical results show X_1^L is greater than $\underline{X}(1)$. Therefore, we obtain the following proposition:

Proposition 5 *The optimality of leader's investment is ensured by the fact that*

$$\underline{X}(1) < X_1^L. \tag{39}$$

Due to Proposition 4 and 5, we use Proposition 3 in relief. Finally, the following proposition explains the firm's optimal actions in the equilibrium.

Proposition 6 *We assume that the initial value x is sufficiently low. Then, the equilibrium is as follows:*

1. *At the first moment that X_t exceeds X_1^{L}, firm i becomes the leader with state* 1 *with probability* 1/2*,*

2. *when X_t falls below $\underline{X}(1)$, firm i suspends the project,*

3. *when X_t exceeds $\overline{X}(1)$, firm i reenters the project,*

4. *at the first moment that X_t exceeds X_1^{F} $(> \overline{X}(1))$, firm j invests with state* 1*,*

5. *when X_t falls below $\underline{X}(2)$, both firms suspend the project,*

6. *when X_t exceeds $\overline{X}(2)$, both firms reenter the project.*

6 Conclusion

In this paper, we have analyzed investment decision on the entry-exit project in a duopoly setting. Then we have shown it is optimal that the firm must start producing at the beginning of the project, and there are no simultaneous investment if the initial demand shock is sufficiently low. Furthermore, the comparative statics of thresholds imply the leader is more encouraged to invest and the follower is more discouraged.

For future research, we will analyze the case in which Q_t means the supply of products in the market. It is natural that the supply determines the price, however, Q_t could change after the firm invests the project. Consequently, there is possibility that equation (15) no longer holds. Furthermore, we will try an abandonment decision similar to Murto [14] or Goto and Ohno [8].

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