Simplified Investment Valuation Model for Projects with Technical Uncertainty and Time to Build

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ABSTRACT

Closed form solutions for valuation of projects that takes into account technical uncertainty and takes time to build are not currently available. Approximate solutions are typically obtained by implementing a complex numerical algorithm to solve non-linear partial differential equations or computer intensive simulation techniques. An equivalent Brownian motion stochastic process is proposed in this paper to model the nonlinear stochastic process typically used to describe the variation of technical uncertainty with time for projects that take time to build. The proposed approximation allows estimating the value of projects with technical uncertainty using the well-known Black and Scholes closed form solutions for European-type options as well as the binomial approach. The proposed approximation was compared to the results obtained using numerical Monte Carlo simulations. A parametric study of the error in the approximation shows that the proposed simplified approximation provides a good estimate of the option values. The main advantage of the proposed approximation is its simplicity and straight forward implementation.

INTRODUCTION

Many long term projects (e.g., construction projects, remediation of contaminated sites, real estate investments of contaminated sites, and R&D projects such as software development or pharmaceutical drug development) usually require the consideration of technical and financial risks to quantify the true economic cost associated with the project. Technical risk is associated to the ability to complete the project as originally planned (within the budget and schedule). Contrary to market risk, technical risk cannot be quantified unless the project is started (i.e., endogenous). Projects that take longer to complete increase their associated financial risk because market conditions may change in the time between project initiation and completion. Unlike financial risk where the rate of return is independent of the investor's risk preference and is dictated by market conditions, the required rate of return for a project that only includes technical uncertainty should be the risk-free interest rate. In theory, an investor should be able to diversify technical risk, and hence, no risk premium should be required for a rational investor. However, in reality, diversification of technical risk cannot be easily obtained because of a lack of fluid markets. As a result, investors demand a risk premium that depends upon each investor's risk preference. For instance, most investors consider real

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estate transactions that include remediation of a contaminated land to be riskier than similar projects without the contamination component and typically demand a significantly higher return on investment before agreeing to take on the project. Often, the rate of return associated with environmental risk that the investor demands may be significantly higher than the actual risk associated with the project because the complexity of the project and environmental risk are poorly understood.

Significant effort has been made in recent past to quantify non-financial risk using derivative pricing methods [e.g., Copeland and Keenan, 1998; Dixit and Pyndick, 1994; Trigeorgis, 1999]. Although several models to value technical risk have been derived [e.g., Madj and Pyndick, 1987; Schwartz 2001], these models are difficult to implement and often require the use of complex numerical techniques. Because of the difficulties inherent to the implementation of numerical techniques, evaluation of contingent liability considering technical risk is often neglected. It is clear that simple techniques that allow speedy and reasonable accurate estimations of contingent liability due to technical uncertainty are needed. In what follows, the well-known equation describing the stochastic variation of technical uncertainty for projects that take time to build is described, followed by a description of the derivation of the proposed simplification.

MODEL DEVELOPMENT

Because projects take time to complete, the cost to complete the project usually includes two different kinds of uncertainties: technical and market uncertainty. For instance, in the case of a contaminated site, technical uncertainty would be: How much contamination is present and, therefore, how much it is going to cost to remediate the site? Technical uncertainty can only be resolved by undertaking the remediation project; actual cost unfolds as the project proceeds. The second kind of uncertainty, referred to as input cost uncertainty, is external to the project. It arises because input prices for labor and materials fluctuate over time. The most widely used expression for the evaluation of technical uncertainty and input is given by Pindyck [1993] who proposed the following stochastic process to model these two uncertainties:

$$dK = -Idt + \upsilon(IK)^{1/2}\,dz + \gamma K dw \qquad (1)$$

where K is the remaining project cost; dz and dw are the increments of uncorrelated Wiener processes; I is the investment rate; and υ and γ are the standard deviations of the project cost stochastic process (technical and input costs). The second and third terms in Equation (1) represent the technical and cost uncertainty, respectively. In the absence of technical and input cost uncertainty, for t = 0, K is equal to the total expected cost, and for $t = T_c$ (time at which the project is actually completed), $K = 0$. The total cost of the project (Q) can be related to the remaining total cost (K) at time t as:

$$Q(t) = K(t) + \int_0^t (Idt + \upsilon(IK)^{1/2} dz + \gamma K dw) \tag{2}$$

where t is the elapsed time since the beginning of the project. It follows from Equation (2) that in the absence of technical and/or cost uncertainty component (i.e., $\upsilon = 0$ and $\gamma = 0$), Q would be a constant. It follows from Equation (2) that:

$$dQ = \upsilon(IK)^{1/2} dz + \gamma K dw \tag{3}$$

Assuming that the technical uncertainty is much larger than the input cost uncertainty, only technical uncertainty is considered in the analysis (i.e., $\gamma = 0$).

$$dQ = \upsilon(IK)^{1/2} dz \tag{4}$$

The standard deviation of the technical uncertainty (υ) is related to the time-independent expectation (χ) and standard deviation (σ) of the project cost evaluated at the beginning of the project (i.e., $t = 0$) using a non-trivial relationship first derived by Dixit and Pindyck [1994]:

$$\sigma^2 = \left(\frac{\upsilon^2}{2 - \upsilon^2} \right) \chi^2 \tag{5}$$

This relationship has been used to model the technical volatility of projects that take time to build and for which information is only available prior to project initiation. If the total cost has been obtained through competitive bidding, the average of the bids can be viewed as the market's consensus of the true cost of the project and can be used as the time-independent cost expectation (χ) at $t = 0$ when the remaining project cost is equal to the total project cost. Similarly, the standard deviation of the bids (σ) measures the variation about the project cost and it is an indicator of the bidders' uncertainty about the final project cost. Variation about the mean given by the standard deviation represents judgment differences, assumptions, or minor bidding errors. Both of these values (i.e., χ and σ) can be used to estimate the time-independent technical volatility (υ) using Equation (5). The technical volatility can then be obtained as:

$$\upsilon = \left(\frac{2V^2}{1+V^2} \right)^{1/2} \tag{6}$$

where V is the variance of the project defined as σ/χ. If it is assumed that Q is a security that can be traded in the financial markets and create a portfolio ϕ that is composed of a long position of shares with contingent security $F(Q)$ and a short position of a numbers of shares Q, then the portfolio value ϕ can be obtained as:

$$\phi = F - nQ \tag{6}$$

where n is the number of shares Q that will be selected such that the return of the portfolio is risk-free. The value of F derives from (i.e., is contingent upon) the value of Q, hence it being named as a derivative (or contingent security).

An investor holding a long position of the security Q is expected to receive a payoff of $\mu = \alpha + \delta$, where μ is the market return of the asset Q, α represents the expected return of the asset Q (i.e., the drift term of the stochastic variable Q), and δ represents the dividend (or convenience) yield for holding the asset Q. Furthermore, because technical risks are considered diversifiable, a rational investor should expect a return for technical risk equal to the risk-free interest rate, thus $r = \alpha + \delta$. As shown in Equation (4), the expected drift for the asset Q is zero (i.e., $\alpha = 0$). Hence, the convenience yield for the asset Q should be equal to the risk free interest rate (i.e., $r = \delta$).

For the sake of completeness, the steps followed to derive the value of contingent security for financial securities are presented below. It follows from Equation (6) that a change of the portfolio value ϕ can be written as:

$$d\phi = dF - ndQ \tag{7}$$

From Ito's Lema, the change in the contingent security F can be written as:

$$dF = \frac{\partial F}{\partial Q} dQ + \frac{1}{2} \frac{\partial^2 F}{\partial Q^2} dQ^2 + \frac{\partial F}{\partial t} dt \tag{8}$$

where:

$$dQ^2 = \upsilon^2 (IK) dt \tag{9}$$

The return on the portfolio (R_ϕ) is then given by the change in value of the portfolio given by Equation (7) minus the dividend yield (δ) that the investor holding the long position of Q is expected to receive. This can be expressed as:

$$R_\phi = d\phi - (nQ\delta)dt \tag{10}$$

Hence, an investor holding the short position of n shares of Q should forgo $(nQ\delta)dt$. Because the portfolio is designed to be risk-free, the portfolio return should be the risk-free interest rate (r) and R_ϕ is thus expressed as:

$$R_\phi = (r\phi)dt \tag{11}$$

Then, by setting Equation (10) equal to Equation (11), it follows that:

$$d\phi - (nQ\delta)dt = (r\phi)dt = r(F - nQ)dt \tag{12}$$

Replacing Equations (6) and (7) into Equation (12), yields:

$$\left(\frac{1}{2}\upsilon^2 IKF_{QQ} + rnQ - rF + F_t - \delta nQ\right)dt + (F_Q - n)dQ = 0 \tag{13}$$

To eliminate the stochastic (i.e., uncertainty) component of Equation (13), and thus eliminating the portfolio risk, the numbers of shares n must be equal to F_Q. Thus, Equation (13) reduces to:

$$\frac{1}{2}\upsilon^2 IKF_{QQ} + (r - \delta)QF_Q - rF + F_t = 0 \tag{14}$$

Defining the time variable as the available time before the option expiration date as $\tau = T_c - t$, where T_c is the project completion date, Equation (14) can be written as:

$$\frac{1}{2}\upsilon^2 IKF_{QQ} + (r - \delta)QF_Q - rF - F_\tau = 0 \tag{15}$$

Equation (15) together with a set of boundary conditions can be used to estimate the contingent security F. Because the coefficient of the first term depends upon the parameter K (which changes with time), Equation (15) is a non-linear partial differential equation (PDE). Hence, numerical techniques are required to solve this equation.

LINEARIZATION TECHNIQUE

Linearization of Stochastic Process

Closed form solutions for Equation (15) are not available, only numerical finite difference solutions have been provided for particular cases [Dixit and Pyndick, 1994]. To facilitate the application of Equation (15) to practical problems (e.g., R&D projects, construction cost), a linearization of Equation (4) was performed. By dividing Equation (4) by the project total cost (Q), the following expression is obtained.

$$\frac{dQ}{Q} = \upsilon\left(\frac{I}{Q}\right)^{1/2}\left(\frac{K}{Q}\right)^{1/2} dz \qquad (16)$$

For a constant rate of investment, Q/I represents the expected time of project completion (T_c). The second term in Equation (16) represents the ratio between remaining project cost and the total actual cost. At the beginning of the project (t=0) the ratio is equal to 1 (i.e., K=Q) whereas at the end of the project (t=T_c), the ratio is zero. Assuming a linear variation between these two values, the average cost ratio is 0.5. Defining a parameter β as the average cost ratio $[K/Q]_{avg}$, the value of β before project completion is defined as follows:

$$\beta = \left[\frac{K}{Q}\right]_{avg} = \frac{1}{2}\left(2 - \frac{t}{T_c}\right) \qquad (17)$$

It follows from Equation (17) that for t=T_c, the parameter β is equal to ½. Replacing Equation (17) into Equation (16), yields:

$$\frac{dQ}{Q} = \sigma_Q \, dz \qquad (18)$$

where:

$$\sigma_Q = \upsilon(\alpha_Q \beta)^{1/2} \qquad (19)$$

$$\alpha_Q = \left(\frac{I}{Q}\right) = \frac{1}{T_c} \qquad (20)$$

where the parameters α_Q is the instantaneous (constant) equivalent return on the asset *K*; σ_Q is the instantaneous (constant) standard deviation of asset return. Equation (18) is the equivalent linear representation of the more accurate non-linear stochastic process given by Equation (4). Also, the stochastic process given by Equation (18) is similar to the well known stochastic process used for modeling stocks.

Closed Form Solution

Using the proposed linearization, the non-linear Equation (15) can be rewritten as:

$$\frac{1}{2}\upsilon^2\left(\frac{I}{Q}\right)\left(\frac{K}{Q}\right)Q^2F_{QQ}+(r-\delta)QF_Q-rF-F_\tau=0 \tag{21}$$

and replacing Equation (19) into Equation (21), the following well-known linear partial differential equation is obtained.

$$\frac{1}{2}\sigma_Q\ Q^2F_{QQ}+(r-\delta)QF_Q-rF-F_\tau=0 \tag{22}$$

If Equation (22) is subjected to the following boundary condition, is then a call option.

$$C=F(Q,X,T_C)=\max\{Q-X,0\} \tag{23}$$

The partial differential equation (22) subject to the boundary condition given by Equation (23) can be used to answer the following question: What is the expected cost overrun (*F*) over the invested amount *X* in a project that has an unknown total cost to complete (*Q*) at the end of period (T_c)?

The solution to this problem is equivalent to the equation derived by Black and Scholes [1973] to evaluate European-type options. The solution to the proposed linear equation for call option is then:

$$F(Q)=Qe^{\delta\,T_c}N(d_1)-Xe^{r\,T_c}N(d_2) \tag{24}$$

where:

$$d_1=\frac{\ln(Q/X)+\left((r-\delta)+\frac{1}{2}\sigma_Q^2\right)T_c}{\sigma_Q\sqrt{T_c}} \tag{25}$$

and:

$$d_2 = d_1 - \sigma_Q \sqrt{T_c} \tag{26}$$

and the operator $N(\ )$ is the cumulative standard normal distribution function. A comparison between the value of a contingent security and a contingent liability of real project is shown in the table below. The closed form solution given by Equations (23) and (24) can be used to evaluate the contingent liability for the cost overrun on a real project. As discussed above, because the expected drift is zero, the project convenience yield is equal to the risk-free interest rate (i.e., $\delta = r$).

Table 1 – Comparison of Real Options and Financial Options

Variable	Project Contingent Liability	Contingent Security (Call/Put Option)
Q	Expected total project cost	Stock price
σ_Q	Project cost uncertainty	Volatility of the stock
X	Project target cost	Exercise price
R	Risk free interest rate	Risk free interest rate
δ	Project convenience yield	Stock dividend yield
T_c	Expected project completion time	Time to maturity
F	Contingence liability above cost X	Call (C)/Put option (P)

If Equation (22) is subjected to the following boundary condition, is then a put option.

$$P = F(Q, X, T_C) = \max\{X - Q, 0\} \tag{27}$$

The partial differential equation (22) subject to the boundary condition given by Equation (27) can be used to answer the following question: What is the expected project saving (P) below the invested amount X in a project that has an unknown cost to complete (Q) at the end of period (T_c)?

Lattice Model

The advantage of the simplified model derived above is that it can also be implemented using the binomial option pricing model [Cox, et al, 1979]. The binomial option pricing model (Figure 1), a special case of lattice models, is generally more intuitive, simpler, and more flexible in handling different stochastic processes such as option payoffs, several underlying variables, early exercise, or other intermediary decisions. Also, a binomial model can be more easily communicated to non-technical parties than its partial differential equation counterpart.

The binomial option pricing model assumes that the price of the underlying asset (Q) in each period can only move up (by a multiplicative factor u) or down (by a multiplicative factor d) – that is, the asset price follows a binomial distribution (Figure 2). This approach to pricing options conveys much of the depth and intuition of more complex and seemingly more realistic models. Binomial pricing is especially useful in pricing American-type options due to its backward, dynamic programming-type process. For the proposed linear solution, the multiplicative factors u and d are given by:

$$u = \exp\left(\sigma_Q \sqrt{\frac{T_c}{N}} \right) \tag{28}$$

$$d = 1/u \tag{29}$$

where N is the number of time steps used to describe the binomial distribution (Figure 2), and σ_Q is standard deviation of the proposed modified solution and it is given by Equation (19).

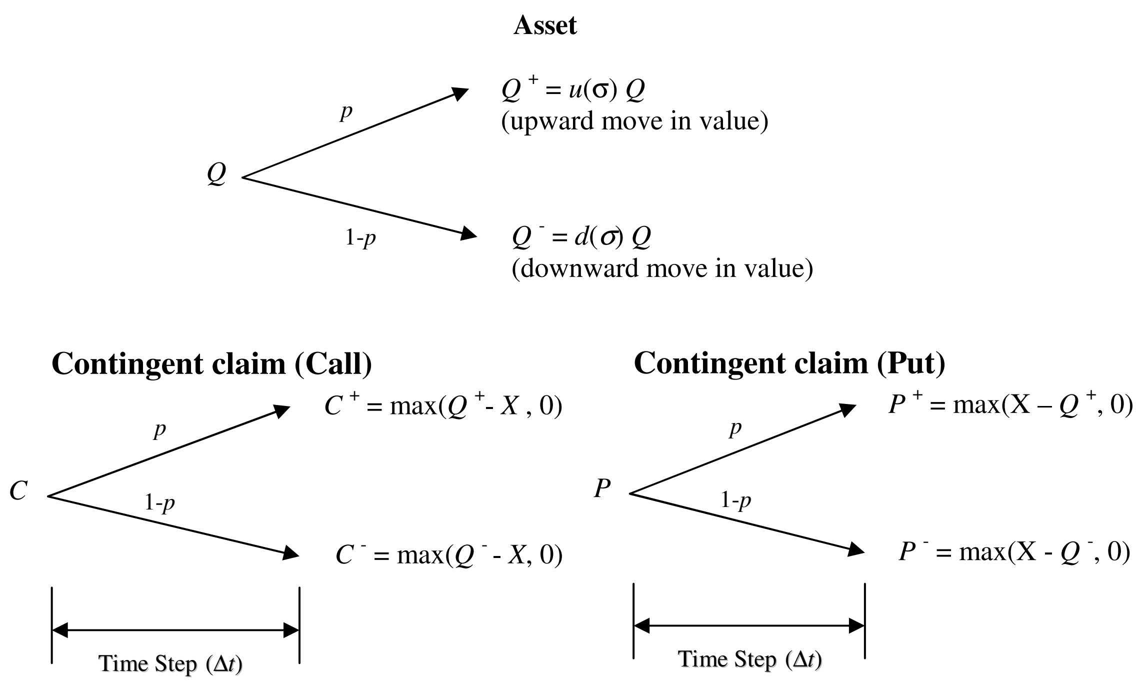

Figure 1 – Binomial Lattice Representation

As described above, the call option can be viewed as the expected total cost overrun of the project (i.e., the project cost over the amount *X*), whereas the put option can be viewed as the expected project savings that may be realized at the end of the project.

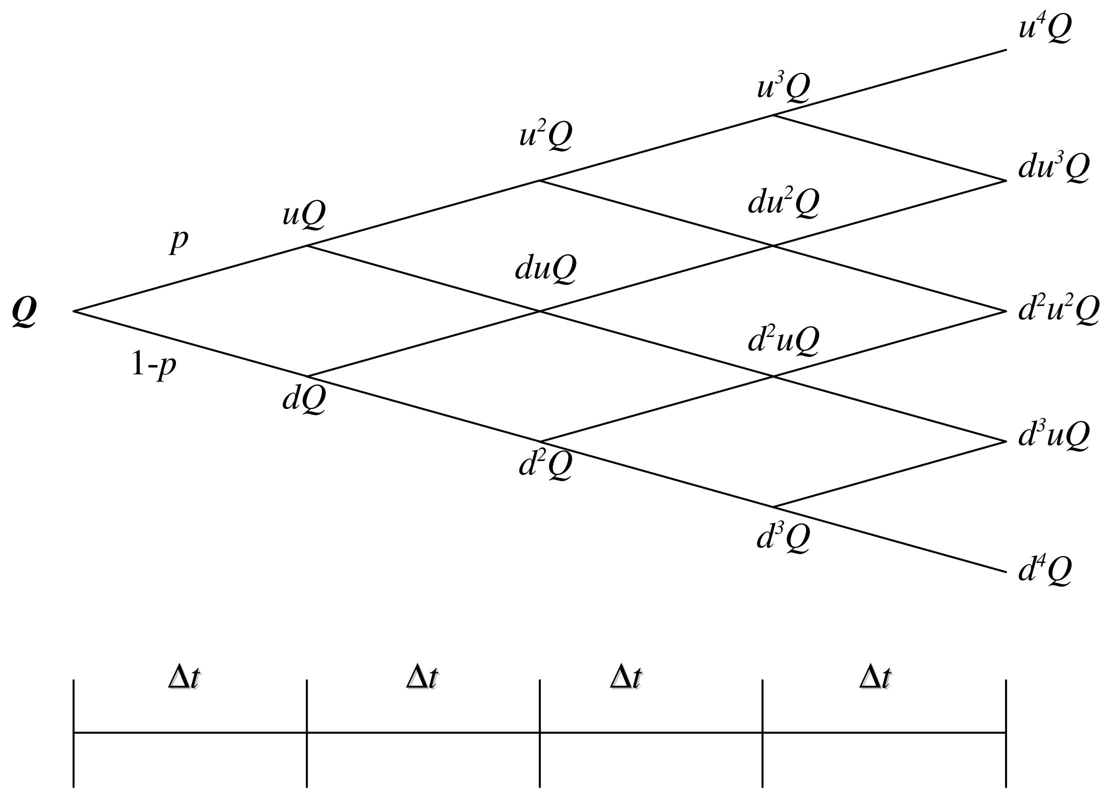

Figure 2 – Multi-step Binomial Lattice Representation

VERIFICATION

A solution to Equation (4) was implemented using Monte Carlo simulation (MCS) to evaluate the contingent liability of a cost overrun above an initial expected project cost. The MCS results were then compared to those obtained using the closed form solution and the binomial solution described above. The input parameters are listed in Table 2.

Table 2 – Input Parameters

Variable	Project Contingent Liability	Value
Q	Expected total project cost	**$6.0** million
V	Project Variance	5%, 10%, 15%,20%, 25%, **30%**
X	Project target cost	**$6**, $6.1, $6.2, $6.3, $6.4, $6.5, $6.6 million
r	Risk free interest rate	**6%**
T_c	Project completion time	1, 2, **3**, 4, 5 years
N	Number of time steps	18, **24**, 36
ns	Number of simulations	**5000**, 10000, 20000, 30000

Quantities shown in bold correspond to the base case

For illustrative purposes, the results of typical random simulations of the variation of the remaining project cost (K) as a function of elapse time (t) for the case analyzed is presented in Figure 3.

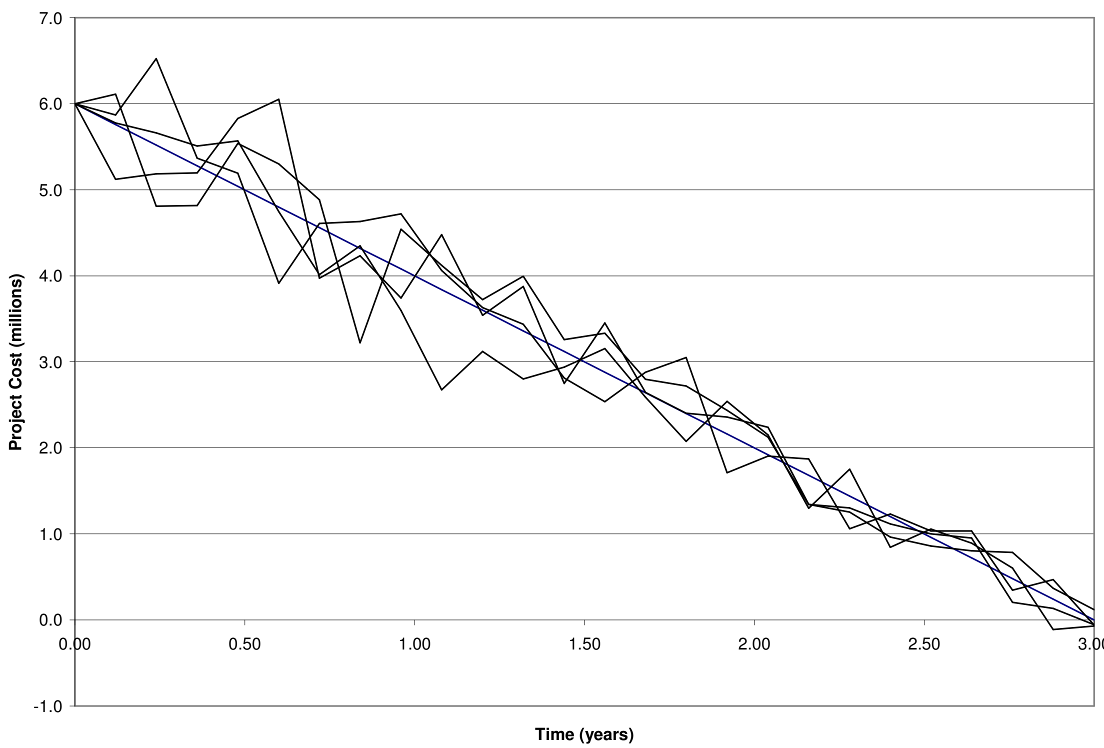

Figure 3 – Random Variation of Remaining Project Cost vs. Time

The results of the contingent liability (i.e., call option) for the base case are presented in Table 3. Because the results of MCS are not unique, each simulation yields slightly different results. Table 3 presents the results of several simulations for 5,000 trials and shows that the average error of the modified binomial and closed form solution is 4% and 3%, respectively. Hence, as the number of trials increases, the error should trend towards the calculated numbers averages.

Table 3 –Simulation Results (5,000 trials)

Binomial C_B	Modified C_M	Simulation C_S	Error $(C_S-C_B)/C_{MCS}$	$(C_S-C_M)/C_{MCS}$
$ 566,607	$ 572,538	$ 574,575	1.4%	0.4%
$ 566,607	$ 572,538	$ 605,484	6.4%	5.4%
$ 566,607	$ 572,538	$ 595,411	4.8%	3.8%
$ 566,607	$ 572,538	$ 596,060	4.9%	3.9%
$ 566,607	$ 572,538	$ 580,819	2.4%	1.4%
	Average	$ 590,470	4.0%	3.0%

Table 4 shows the comparison between: (i) the MCS and the proposed closed solution; and (ii) the MCS and the binomial solution for various cases for the parameters listed above. As shown in the table, error associated with the binomial and modified techniques are small (i.e., approximately less than 5%).

As shown by the parametric study of the variability of the construction cost (first group of Table 4), larger construction cost variability results in a larger risk premium (as expected). The second group of Table 4 shows the risk premium associated with a cost overrun higher than the listed values (e.g., for an initial estimated project cost of $6 million, the risk premium associated with a final cost being higher than $6.3 million is $588,493). As shown by the parametric study of the variability of the time to completion of the project (third group of Table 4), the risk premium is not particularly sensitive to this parameter. The last two parameters (number of time steps and number of simulation trials) are related to the accuracy of the MCS algorithm. As shown in the fourth and fifth group in Table 4, for the selected range of these parameters, the risk premium is not sensitive to these parameters.

Table 4 – Results of Simulation Comparison for a Call Option

X	V	T_c	N	n_s	Binomial	Modified	Simulation	Error	
[million]	[%]	[Years]	[-]	[-]	C_B	C_M	C_S	Binomial	Modified
$6.0	30.0%	3	24	5,000	$566,607	$572,538	$574,550	1.4%	0.4%
$6.0	25.0%	3	24	5,000	$478,716	$483,727	$492,845	2.9%	1.8%
$6.0	20.0%	3	24	5,000	$387,421	$391,477	$400,145	3.2%	2.2%
$6.0	15.0%	3	24	5,000	$293,243	$296,312	$309,416	5.2%	4.2%
$6.0	10.0%	3	24	5,000	$196,800	$198,860	$206,786	4.8%	3.8%
$6.0	5.0%	3	24	5,000	$98,798	$99,832	$104,260	5.2%	4.2%
$6.0	30.0%	3	24	5,000	$566,607	$572,538	$574,550	1.4%	0.4%
$6.1	30.0%	3	24	5,000	$536,227	$536,494	$538,623	0.4%	0.4%
$6.2	30.0%	3	24	5,000	$505,848	$502,312	$510,837	1.0%	1.7%
$6.3	30.0%	3	24	5,000	$475,468	$469,940	$483,892	1.7%	2.9%
$6.4	30.0%	3	24	5,000	$445,089	$439,320	$468,929	5.1%	6.3%
$6.5	30.0%	3	24	5,000	$414,710	$410,393	$405,770	-2.2%	-1.1%
$6.6	30.0%	3	24	5,000	$384,330	$383,098	$389,731	1.4%	1.7%
$6.0	30.0%	1	24	5,000	$638,847	$645,535	$651,802	2.0%	1.0%
$6.0	30.0%	2	24	5,000	$601,644	$607,942	$634,290	5.1%	4.2%
$6.0	30.0%	3	24	5,000	$566,607	$572,538	$588,493	3.7%	2.7%
$6.0	30.0%	4	24	5,000	$533,610	$539,196	$559,882	4.7%	3.7%
$6.0	30.0%	5	24	5,000	$502,535	$507,796	$525,532	4.4%	3.4%
$6.0	30.0%	3	36	5,000	$566,607	$572,538	$577,659	1.9%	0.9%
$6.0	30.0%	3	24	5,000	$566,607	$572,538	$574,550	1.4%	0.4%
$6.0	30.0%	3	18	5,000	$566,607	$572,538	$595,709	4.9%	3.9%
$6.0	30.0%	3	24	5,000	$566,607	$572,538	$580,819	2.4%	1.4%
$6.0	30.0%	3	24	10,000	$566,607	$572,538	$573,775	1.2%	0.2%
$6.0	30.0%	3	24	20,000	$566,607	$572,538	$587,973	3.6%	2.6%
$6.0	30.0%	3	24	30,000	$566,607	$572,538	$589,962	4.0%	3.0%

CONCLUSIONS

A simplified model that considers technical uncertainty and time to build has been developed and its results compared to those obtained using Monte Carlo simulation for a specific example. The error appears to be small for the cases analyzed. The advantage of the method is its simplicity and its relation to the well-known Black and Scholes equations. Extension of the proposed procedure to other common applications could therefore be easily achieved.

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