Corporate Optimal Investment Rule under incomplete information: A Real option method

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Abstract

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1. Introduction

It is well know that option pricing models can be used to value projects in corporate finance and to search for the optimal investment rule. In our paper, we assume a manager can invest in one project, which leads to production with output price and input price. The cash flow of the project depends on the quantity of the productions and the difference between the output price and the input price. However the quantity producted depend on the output price.

In section 2, we use the option pricing model to value a project under incomplete information and taxes. Then, we assume that, if the corporate invests, it must incur the sunk investment cost, which is irreversible (and like the exercise price of one option). In section 3, we get the value of the option to invest in the project for this corporate and give the threshold value of the project. In section 4, we give some numerical examples to show the characteristics of the optimal investment rule.

2. The corporate project value with incomplete information and taxes

We consider that a manager can invest in one project. The output price of the production is P_t and the input price is C_t, Q_t is the quantity of output, τ is the tax rate, so the cash flow of the project can be written in the following form:

$$R_t = (P_t - C_t)Q_t - \tau(P_t - C_t)Q_t, \quad Q_t = P_t^b \tag{1}$$

The term b is a constant.

We assume the following dynamics for the output price and the input price of the production:

$$dP_t = \alpha_p P_t dt + \sigma P_t dB_t \tag{2}$$

$$dC_t = \alpha_c C_t dt + \sigma C_t dB_t \tag{3}$$

where α_p and α_c represent the instantaneous expected rates respectively for the output and input prices of the production. The term σ is the instantaneous volatility for this production. The terms B_t is one-dimensional Brownian motion, which represents the external source of uncertainty in the market.

From (2) and (3), we know that:

$$P_t = P_0 e^{(\alpha_p - \frac{1}{2}\sigma^2)t} e^{\sigma B_t}, \quad C_t = C_0 e^{(\alpha_c - \frac{1}{2}\sigma^2)t} e^{\sigma B_t}$$

This analysis does not assume that we are applying the risk-neutral approach. The firm's cash flows can be written as :

$$\begin{aligned} R_t &= (1-\tau)P_t^b(P_t - C_t) \\ &= (1-\tau)P_0^b e^{b(\alpha_p - \frac{1}{2}\sigma^2)t} e^{(b+1)\sigma B_t}(P_0 e^{(\alpha_p - \frac{1}{2}\sigma^2)t} - C_0 e^{(\alpha_c - \frac{1}{2}\sigma^2)t}) \\ &= K e^{(b+1)\sigma B_t} F(t) \end{aligned}$$

with $K = (1-\tau)P_0^b$ and

$$F(t) = e^{b(\alpha_p - \frac{1}{2}\sigma^2)t}(P_0 e^{(\alpha_p - \frac{1}{2}\sigma^2)t} - C_0 e^{(\alpha_c - \frac{1}{2}\sigma^2)t}).$$

Using these expressions, the changes in the cash flow of the project are given by:

$$\begin{aligned} dR_t &= K(b+1)\sigma e^{(b+1)\sigma B_t} F(t) dB_t + \frac{1}{2}(b+1)^2\sigma^2 K e^{(b+1)\sigma B_t} F(t)dt \\ &\quad + K e^{(b+1)\sigma B_t} F'(t)dt \\ &= R_t(b+1)\sigma dB_t + R_t[\frac{1}{2}(b+1)^2\sigma^2 + \frac{F'(t)}{F(t)}]dt \\ &= R_t(b+1)\sigma dB_t + R_t f(t)dt \end{aligned} \tag{4}$$

with $f(t) = \frac{1}{2}(b+1)^2\sigma^2 + \frac{F'(t)}{F(t)}$ and $F(t) \neq 0$.

Now, we use the option methodology to value the above project, denoted by V, with the cash flows R satisfying equation (4).

Suppose that we construct a portfolio at time t, that contains one unit of the project, and a short position of n unit the cash flow of the productions, where we choose n to make the portfolio risk-less. The holder of the project will get the revenue or profit flow Rdt over the small interval of time $(t, t+dt)$. A holder of each unit of the short position must pay to the holder of the corresponding long position an amount equal to the dividend or convenience yield, that the latter would have earned, namely $\delta R dt$. Thus holding the portfolio yields a net dividend $(R - n\delta R)dt$, the capital gain of the portfolio equals to:

$$\begin{aligned} dV(R) - ndR_t =& [f(t)R(V'(R) - n) + \frac{1}{2}\sigma^2(b+1)^2R^2V''(R)]dt \\ &+ R(b+1)\sigma(V'(R) - n)dB_t \end{aligned}$$

Now, we choose $n = V'(R)$, so that the terms in dB_t disappear and the portfolio becomes risk-less. The total return to the portfolio is given by:

$$[R - \delta RV'(R) + \frac{1}{2}\sigma^2(b+1)^2R^2V''(R)]dt$$

To avoid riskless arbitrage, the value of this portfolio must be the riskless rate. However, since there are information costs embedded in the project, and on its profit flow, the return rate must be equal to $(r+\lambda_V)$ for the project and $(r+\lambda_R)$ for the profit flow of the project, where r is the risk-less rate, λ_V and λ_R refer respectively to the information costs on the project and the cash flow of productions. We also assume that $\lambda_V \geq \lambda_R$. These parameters represent sunk cost, which are necessary before entering into a project. Therefore, the cost of gathering information and data about the project and the productions are present in the discounting procedure. In this context, we have

$$\begin{aligned} &[R_t - \delta RV'(R) + \frac{1}{2}\sigma^2(b+1)^2R^2V''(R)]dt \\ &= (r+\lambda_V)V(R)dt - RV'(R)(r+\lambda_R)dt \end{aligned}$$

So the value of the project satisfies the following equation:

$$\frac{1}{2}\sigma^2(b+1)^2R^2V''(R)+(r+\lambda_R-\delta)RV'(R)-(r+\lambda_V)V(R)+R=0 \tag{5}$$

The general solution of the equation (5) is:

$$V(R)=B_1R^{\bar{\beta}_1}+B_2R^{\bar{\beta}_2}+\frac{R}{\delta+\lambda_V-\lambda_R} \tag{6}$$

here $\bar{\beta}_1$ and $\bar{\beta}_2$ are the roots of the fundamental quadratic equation:

$$\frac{1}{2}\sigma^2(b+1)^2\bar{\beta}(\bar{\beta}-1)+(r+\lambda_R-\delta)\bar{\beta}-(r+\lambda_V)=0 \tag{7}$$

and

$$\bar{\beta}_1=\frac{1}{2}-\frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2}+\sqrt{[\frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2}-\frac{1}{2}]^2+\frac{2(r+\lambda_V)}{(b+1)^2\sigma^2}}>1 \tag{8}$$

$$\bar{\beta}_2=\frac{1}{2}-\frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2}-\sqrt{[\frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2}-\frac{1}{2}]^2+\frac{2(r+\lambda_V)}{(b+1)^2\sigma^2}}<0 \tag{9}$$

However, we should have $V(0)=0$. As similar analysis as that in Dixit and Pindyck (1994) (Chapter 6, section 1.C) is suitable for our case with incomplete information. We show that the value of the project with the profit flow R should be given by:

$$V(R)=\frac{R}{\delta+\lambda_V-\lambda_R} \tag{10}$$

3. The investment decision and the option's value of the project with incomplete information

Since we know the project's value, it is possible to determine the firm's option to invest. This option depends on the value and the profit flow of the project. We give the value of the option to invest in the project and also the critical level of the cash flow of the project R^* as well as V^*. At this level, the manager exercises the option by paying an amount I in exchange for the project.

Once again, we follow the steps of contingent claims valuation suitable for our case with the incomplete information. Now, the portfolio consists of the option to invest in the project with value $F(R)$, and a short position of n units the cash flow of the productions of the project. We also choose n to make the portfolio riskless. Also the holder of each unit of the short position must pay to the holder of the corresponding long position an amount equal to the dividend, or convenience yield, namely δRdt. The capital gain of the portfolio equals to

$$\begin{aligned} dF(R)-ndR_t =&[f(t)R(F'(R)-n)+\frac{1}{2}\sigma^2(b+1)^2R^2F''(R)]dt \\ &+R(b+1)\sigma(F'(R)-n)dB_t \end{aligned}$$

We also choose $n = F'(R)$ so that the terms in dB_t disappear and the portfolio becomes riskless. The total return to the portfolio is then:

$$[\frac{1}{2}\sigma^2(b+1)^2R^2F''(R) - \delta RF'(R)]dt$$

To avoid riskless arbitrage, the value of this portfolio must lead to the riskless rate. However, since there are information costs embedded in the option of the investment and the profit flow of the project, the return rate must be equal to $(r + \lambda_F)$ for the option to invest in the project and $(r + \lambda_R)$ for the profit flow of the project. The term λ_F refers to the information costs on the option of the project. Therefore, the cost of gathering information and data about the option and the productions are present in the discounting procedure. In this context, we have:

$$\begin{aligned}&[\frac{1}{2}\sigma^2(b+1)^2R^2F''(R) - \delta RF'(R)]dt \\ &= (r+\lambda_F)F(R)dt - RF'(R)(r+\lambda_R)dt\end{aligned}$$

So the option value of the investment opportunity in the project, $F(R)$, satisfies:

$$\frac{1}{2}\sigma^2(b+1)^2R^2F''(R) + (r+\lambda_R-\delta)RF'(R) - (r+\lambda_F)F(R) = 0 \tag{11}$$

The equation (11) is a homogeneous linear equation of second order, so its solution is a linear combination of any two linearly independent solutions.

$$F(R) = A_1R^{\beta_1} + A_2R^{\beta_2}$$

where A_1 and A_2 are constants to be determined, β_1 and β_2 are two roots of the fundamental quadratic equation:

$$\frac{1}{2}\sigma^2(b+1)^2\beta(\beta-1) + (r+\lambda_R-\delta)\beta - (r+\lambda_F) = 0 \tag{12}$$

and

$$\beta_1 = \frac{1}{2} - \frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} + \sqrt{[\frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} - \frac{1}{2}]^2 + \frac{2(r+\lambda_F)}{(b+1)^2\sigma^2}} > 1 \tag{13}$$

$$\beta_2 = \frac{1}{2} - \frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} - \sqrt{[\frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} - \frac{1}{2}]^2 + \frac{2(r+\lambda_F)}{(b+1)^2\sigma^2}} < 0 \tag{14}$$

We also need to determine the investment threshold cash flow R^* of the productions .

From $F(0) = 0$, we know $A_2 = 0$, so that:

$$F(R) = A_1R^{\beta_1} \tag{15}$$

We know at the threshold cash flow R^*, it is optimal to exercise the option, thereby acquire the value of project $V(R^*)$ by incurring the exercise price (sunk investment cost) I.

So, we have the first condition, stating that the value of the option at threshold R^*, must be equal to the net value by exercising it (which is called the value matching condition):

$$F(R^*) = V(R^*) - I \tag{16}$$

Secondly, the graphs of $F(R)$ and $V(R) - I$ should meet tangentially at R^*, this is called the smooth-pasting condition:

$$F'(R^*) = V'(R^*) \tag{17}$$

From the expression functions form of $F(R)$ in (15) and $V(R)$ in (10), we can write the value-matching and smooth-pasting conditions as

$$A_1(R^*)^{\beta_1} = \frac{R^*}{\delta + \lambda_V - \lambda_R} - I$$

$$\beta_1 A_1(R^*)^{\beta_1 - 1} = \frac{1}{\delta + \lambda_V - \lambda_R}$$

This yields:

$$R^* = \frac{\beta_1}{\beta_1 - 1}(\delta + \lambda_V - \lambda_R)I \tag{18}$$

and

$$A_1 = \frac{(\beta_1 - 1)^{\beta_1 - 1} I^{-(\beta_1 - 1)}}{((\delta + \lambda_V - \lambda_R)\beta_1)^{\beta_1}} \tag{19}$$

From the value of the project $V(R)$ in (10), we also know the equivalent threshold value of the project to invest given by:

$$V^* = \frac{\beta_1}{\beta_1 - 1} I \tag{20}$$

4. The simulation results and the characteristics of the optimal investment rule

The results in section 3 give the optimal investment rule in the presence of information costs. The manager should invest only when the cash flow, R, of the project is greater than R^* in (18). When R is less than R^*, then $V(R) < F(R) + I$, where $F(R)$ is the opportunity cost. Hence, the value of the project is less than its full cost i.e. the direct cost I plus the opportunity cost $F(R)$. We give some simulation results and show the critical value R^* and the characteristics of the optimal investment rule.

In Figure 4.1, $F(R)$ is an increasing function of R for the case $\sigma = 0.1$, $\sigma = 0.3$ and $\sigma = 0.5$ when $r = 0.04$, $\delta = 0.02$, $\lambda_R = 0.01$, $\lambda_F = 0.02$, $\lambda_V = 0.03$, $b = -2$ and $I = 20$. From the Figure, we notice that the option's value of the project $F(R)$ increases when σ increases. From Figure 4.2, we know, that the threshold value of the cash flow R^* is also an increasing function of the instantaneous volatility σ. In the Figure, we take $r = 0.04$, $\delta = 0.6$, $\lambda_R = 0.01$, $\lambda_F = 0.02$, $\lambda_V = 0.03$, $b = -2$, and $I = 2$. In Figure 4.3, we show the influence of the instantaneous volatility σ and the dividend (or convenience yield rate δ) to the threshold value V^* of the project to invest. For the case $\delta = 0.01$, $\delta = 0.02$ and $\delta = 0.03$, V^* is the increasing function of σ where $r = 0.04$, $\lambda_R = 0.01$, $\lambda_F = 0.02$,

$\lambda_V = 0.03$, $b = -2$, $I = 2$. An increase in σ will still increase the critical value of the project V^* and hence, tends to depress investment. Thus, the greater uncertainty in the market increases the value of the firm's investment opportunities, $F(R)$, it increases also the investment critical value of the project V^*, and the cash flow in the project R^* for the corporate, but it decreases the amount of the actual investment.

Figure 4.3 we shows that when δ increases, the critical value of the project V^* decreases. The further illustration can be seen in Figure 4.4. In this figure, we notice that the increase in δ from 0.05 to 0.08, then to 0.1 results in the decrease in $F(R)$ and $V(R)$, which are the increasing functions of R, (here $r = 0.04$, $\sigma = 0.3$, $\lambda_R = 0.01$, $\lambda_F = 0.02$, $\lambda_V = 0.03$, $b = -2$, $I = 15$). When δ in the Figure increases, $V(R) - I$ and hence $F(R)$ falls and the tangency point, of the two curves at R^*, moves to the right. In Figure 4.4, when $\delta = 0.05$, the critical cash flow $R^* = 2.4347$, $\delta = 0.08$, $R^* = 2.6930$, and $\delta = 0.1$, $R^* = 2.9111$. So, the increase in δ increases the critical cash flow R^* of the project at which the corporate should invest. In fact, there are two opposing effects of δ on the project. If δ is larger, the expected rate of increase of R is smaller, options on future production are worthless. So, $V(R)$ is smaller. At the same time, the opportunity cost of waiting to invest rises (the expected rate of growth of $F(R)$ is smaller), so there is more incentive to exercise the investment option, rather than keep it alive. The first effect dominates, so that a higher δ results in a higher R^*. This is illustrated in Figure 4.4.

Figure 4.5 shows the effect of the interest rate r on the option value $F(R)$ when $\delta = 0.02$, $\lambda_R = 0.01$, $\lambda_F = 0.02$, $\lambda_V = 0.03$, $b = -2$, $I = 2$, $\sigma = 0.38$. In the figure, if the risk free rate r is increased, $F(R)$ increases (the cash flow of the project $R = 0.1$, $R = 0.2$ and $R = 0.3$). The reason is that an increase in r reduces the present value of the cost of the investment, but does not reduce its payoff. Hence higher interest rates increases the opportunity cost of investing now and reduces the investment.

Figure 4.6 is done for $r = 0.04$, $\delta = 0.02$, $\sigma = 0.3$, $\lambda_R = 0.01$, $\lambda_V = 0.03$, $b = -2$, $I = 2$. In this Figure, we show the influence of the information λ_F to the option value $F(R)$. For the case $R = 0.25$, $R = 0.3$ and $R = 0.35$, when the information cost rate λ_F increases, the option value of the investment (i.e. the opportunity cost) also increases. This also will reduce the investment. We know that when R increases, the option value $F(R)$ increases. This coincides with the case in Figures 4.1, 4.4 and 4.5.

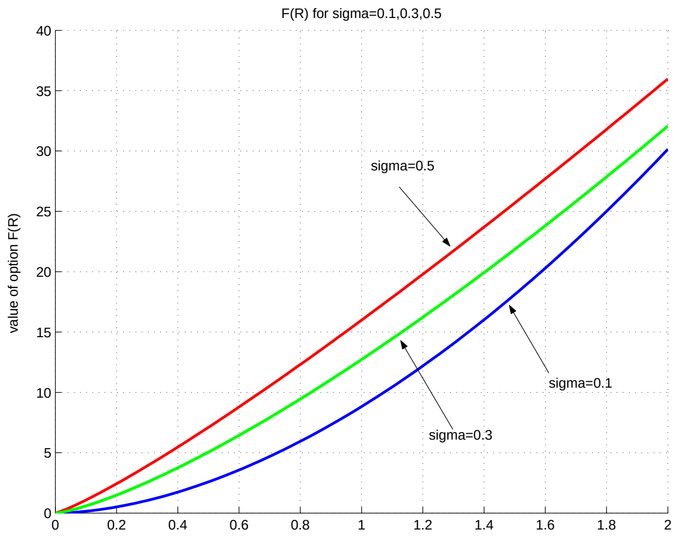

Figure 4.1

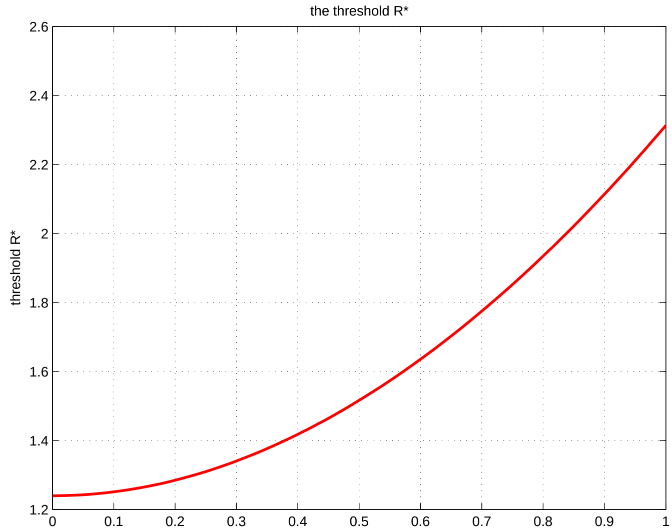

Figure 4.2

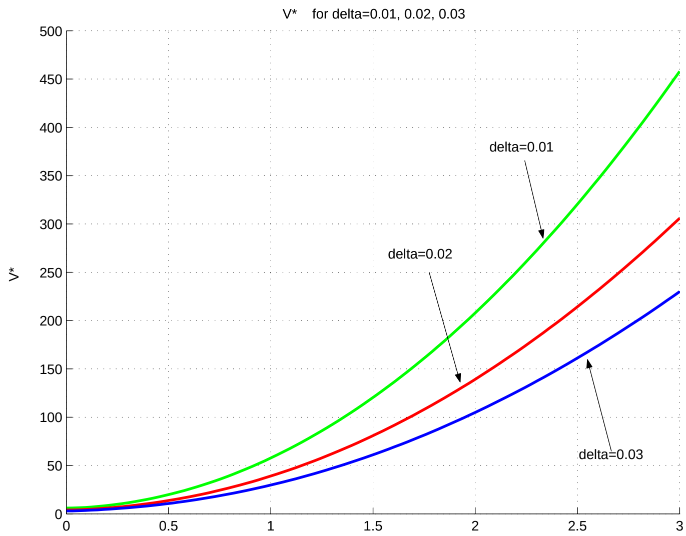

Figure 4.3

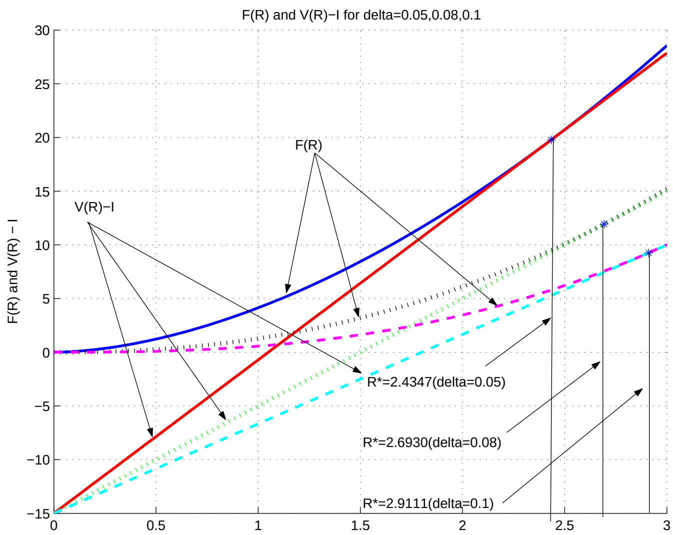

Figure 4.4

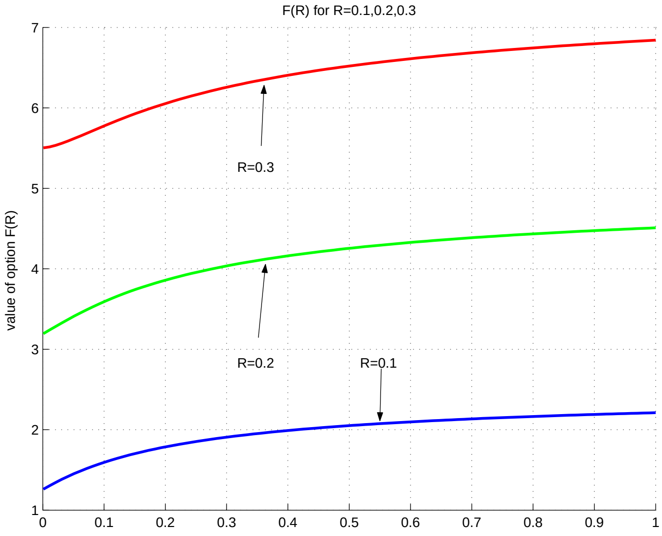

Figure 4.5

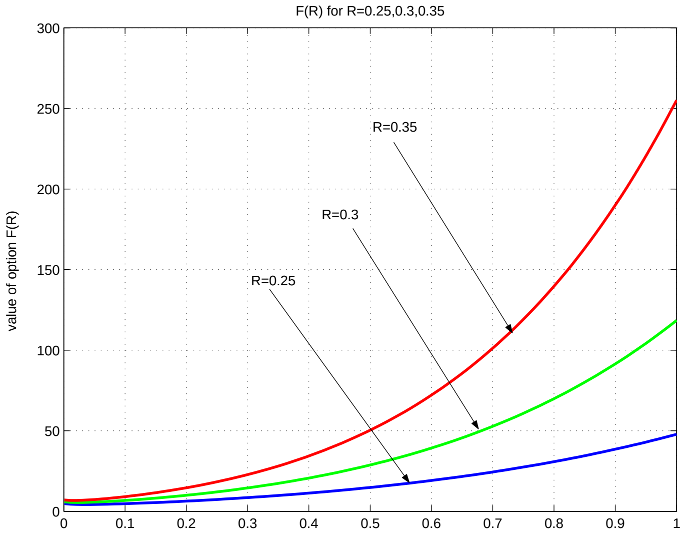

Figure 4.6

References

• Aliber R. (1970), *A Theory of foreign direct investment. In Charles P. Kindleberger, ed.* , The International Corporation, Cambridge, Mass : MIT Press.

• Aliber R. (1983), *Money, Multinationals and sovereigns. In Charles P. Kindleberger D.* , Audretsch, ed.,The Multinational corporation in the 1980s, Cambridge,Mass : MIT Press.

• Adler M. and Dumas B. (1983), *International portfolio choice and corporation finance : A synthesis* , Journal of Finance, June, 925-984.

• Adler M. and Dumas B. (1984), *Exposure to currency risk: definition and measurement*, Financial Management, Summer, 41-50.

• Bellalah, M., (1990), *Quatres Essais Sur L'évaluation des Options : Dividendes, Volatilités des Taux d'intérêt et Information Incomplète.*, Doctorat de l'université de Paris-Dauphine, (June).

• Bellalah M., Jacquillat B., (1995), *Option Valuation with Information Costs: Theory and Tests*, Financial Review, August : 617-635.

• Bellalah M., (1999), *The valuation of futures and commodity options with information costs*, Journal of Futures Markets, September.

• Bellalah, M. (2001), *A Re-examination of Corporate Risks Under Incomplete Information*, International Journal of Finance and Economics, 6, 59-67.

• Bellalah M., (2001), *Market imperfections, information costs and the valuation of derivatives : some general results*, International Journal of Finance, Vol 13, 1895-1928.

• Bellalah M., (2001), *Irreversibility sunk costs and investment under incomplete information*, R& D Management, Vol.31, No.2, 127-136.

• Bellalah M., and WU. Z. (2002), *A model for market closure and international portfolio management within incomplete information* , International Journal of Theoretical and Applied Finance, Vol 5, No. 5, 479-495.

• Black F. (1974), *International capital market equilibrium with investment barriers*, Journal of Financial Economics, 1, 337-352.

• Choi J. B. (1989), *Corporate international investment*, Journal of International Business Studies, Spring, 145-155.

• Coval J. and Moskowitz T. F. (1999), *Home bias at home : local equity preference in domestic portfolios*, Working Paper, University of Michigan.

• A.K.Dixit and R.S.pindyck (1994), *Investment under uncertainty*, Princeton University Press, Princeton, New Jersey.

• Dornbusch. R. (1980), *Exchange rate risk and the macroeconomics of exchange rate determination* , NBER Working Paper, N 493, June.

• Kang J. and Stulz R. (1997), *Why is there a home bias ? an analysis of foreign portfolio equity in Japan*, Journal of Financial Economics, 46, 3-28.

• Merton, R., (1987), *An equilibrium Market Model with Incomplete Information.*, Journal of Finance 42, 483-510.

• Solnik B. (1974), *An equilibrium model of the international capital market*, Journal of Economic Theory, August, 500-524.

• Stulz R. (1981), *On the effects of barriers to international investment* , Journal of Finance, 36,923-934.