Real Options, Competition, and the Valuation of Pharmaceutical Licensing Agreements

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Abstract

This paper combines a real options model with a Cournot-Nash equilibrium game to value a common pharmaceutical industry licensing arrangement. It further extends Schwartz (2003) and Hoe and Diltz (2007) to consider the effect of competition on optimal investment policies in R&D projects. Similar to earlier work, we incorporate the phases required to bring a pharmaceutical product from patent approval to market as well as a deterministic product life cycle variable. We now place all parties into a Cournot-Nash equilibrium game to see how optimal policies are altered by competitive interaction. We focus on the allocation of profit between licensor and licensee, i.e., the "profit split" ratio (PSR) because of its widespread use by practitioners. We find competition alters the PSR, depending on whether a firm is the winner, loser, or in a monopoly situation.

I. Introduction

Pharmaceutical and biotechnology companies must regularly introduce new products to secure profitability and growth opportunities. Competition is intense, and new products must withstand a lengthy, complex, and risky development process. A new product begins with discovery and pre-clinical research, followed by three clinical phases, and FDA regulatory review. Firms adept at performing basic R&D may license a patent to another firm to test, produce, and market the drug. The licensor receives a series of milestone and royalty payments. A licensing agreement thus transfers advanced development and marketing risks from licensor to licensee.

Accurate valuation of the opportunity embodied in the license agreement is crucial for effective negotiation between the parties. It also allows both parties to estimate synergies created by the licensing agreement. In practice, the licensee usually negotiates payments to achieve a target profit split ratio (PSR). The target represents desired compensation for development risks. We focus on the PSR because managers depend heavily on this measure.

Pharmaceutical firms develop and market products in a competitive environment. Frequently, several firms develop different patent-protected drugs targeted for the same disease. Potential competition during the marketing phase plays a crucial role in R&D investment decisions in the development phases since the competing products must undergo the same approval processes. The dynamic equilibrium game strategy complicates the optimal investment/abandonment decisions during product development, thus affecting project value. When there is potential competition, both licensee and licensor should negotiate terms by explicitly considering competitive interactions into valuing the patented R&D project.

Discounted cash flow techniques are not well suited to multi-stage R&D projects, and competition exacerbates the problem. Real option models augmented with simple games have been proposed to deal with this problem. Pindyck (1993) presents a valuation model with uncertain completion costs to capture a "learning effect". Childs and Triantis (1999) examine R&D investment policies and real options valuation for a firm that manages multiple projects with interactions. Schwartz and Moon (2000) extend Pindyck's model to include project revenue uncertainty and catastrophic events. They derive an elliptical partial differential equation and solve (numerically) for investment opportunity value and comparative statics. Trigeorgis (1991) studied the impact of competition on the optimal timing of project initiation using option methodology.

Schwartz (2003) develops a real options simulation framework to value patents and patent protected R&D projects. He specifies stochastic processes for completion costs, sales cash flows, and catastrophic events that render the project worthless. He implements the model using a hypothetical pharmaceutical example, solving the model using the Longstaff & Schwartz (2001) simulation method. Schwartz and Moon (2000) present a simpler model that yields a partial differential equation solved using the successive overrelaxation (SOR) method. Miltersen and Schwartz (2002) augment Schwartz and Moon (2000) model with duopoly settings, using Longstaff & Schwartz (2001) simulation. Berk et al (2004) differs from Schwartz (2003) with respect to (among other things) exogenous variable choices and by assuming perpetual cash flows from product sales.

This paper extends Schwartz (2003) and Hoe and Diltz (2007) by modeling competitive interactions in the valuation of, and optimal investment in, licensing agreements. Our paper differs from Miltersen and Schwartz (2002) in that they focus on total project valuation, while we focus on the valuation of licensee and licensor. They focus on "societal benefits", such as reqired development time, R&D success rates, optimal production level, product prices, aggregate R&D investment costs, aggregate R&D project values, and so on. We focus on analyzing the possible impact on negotiating licensing terms, milestone payments versus royalties, to mutual benefits due to competitive interactions. We explicitly incorporate the phases required to bring the project from patent approval to market. Additionally, we focus on incorporating project lifecycle effects in the marketing phase.

The paper is organized as follows. Section II presents background information, model assumptions, and the basic model. Section III discusses application of the model and sensitivity analysis. We also compare results with analogous results from DCF analysis. We draw conclusions in Section IV.

II. Background, Assumptions, and Model

We assume two firms, X and Y, specialize in developing new drugs in the same therapeutic area. Firm X has finished discovery and pre-clinical research, and X's management has filed for patent protection on a promising new drug. Due to financial, technical, or logistical constraints, X's management decides to license this patent-protected intellectual property to another firm for further development, FDA approval, production, and marketing. Firm Y (the licensee) expresses interest in an agreement with X to shepherd the new drug through the approval process and on to market. Negotiations result in a licensing agreement.

We assume:

(1) Duopolistic Setting: We introduce a competitor to Firm Y to complete the duopoly. The competitor is developing a different molecule targeted at the same disease, and we assume that this molecule is at the same development stage as the targeted licensing compound. Patents on both drugs are assumed to expire simultaneously[1]. Both groups have managerial flexibilities during the development stages, i.e., they can optimally exercise abandonment/investment decisions along the development phases.

[1] These assumptions allow us to focus on symmetric scenario analysis and they can be relaxed.

(2) Licensing Agreement Expiration: The licensing agreement and patent protection expire simultaneously. Following previous research, we assume no residual cash flow after patent expiry.

(3) Licensee's Responsibility and Right: Four developmental phases remain, namely, Clinical Phases I, II, III, and FDA Regulatory Review. Firm Y is responsible for all developmental phases, assumes all development risks, pays periodic milestone payments, and preserves the flexibility to abandon the project. If Y decides to abandon the project, the licensing agreement is terminated. Following FDA Regulatory Review, the project generates sales revenues, which will be either monopolistic or duopolistic revenues depending on the timing of the drug marketed and the other firm's optimal investment/abandonment decision, and Y pays royalties until the licensing agreement expires.

(4) Milestone Payments and Royalties: Milestone payments are made (approximately) upon passage of a clinical phase, thus occurring randomly. Royalties are a fixed percentage of revenues,[2] thereby following the same stochastic process. The size of milestone and royalty payments are exogenously determined and fixed over time.

(5) Stochastic Sales, Development Costs and Development Time: Upon FDA approval, revenues depend on demand for the drug and both competitors face the same stochastic cash flow (revenue) process described in detail in the following sub-section. The winning group enjoys the monopoly profit before the losing group enters the

[2] In practice, the royalty rate based on the sales is preferred rather than the net sales since it prevents the licensor from observing the licensee's internal operating information. In addition, sometimes sales milestone payments are negotiated when sales achieve some targeted levels; this in turn highlights the importance of imposing product life cycle in the evaluation.

markets; both parties share the sales revenue, beginning with the duopoly phase until patent expiry. Development capital costs vary based on the assumed controlled diffusion process (details in the following sub-section). For tractability, we assume that past and present development costs are publicly available. Development time needed is obtained when development capital cost process hits zero, thus stochastically determined.

(6) Catastrophic Events: There is the (Poisson) probability of a catastrophic event (e.g., toxicity, ineffectiveness, etc.) that renders the project worthless. If λ_i is the average rate per unit of time that the project will become worthless in different phases and is independent from each other and uncorrelated with the market (no risk premium associated to them), as shown by Brennan and Schwartz (1985), the failure rate enters with the addition of phase dependent failure rates λ_i to the discount rate.[3] For tractability, we assume that the probability of catastrophic event is publicly available information.

Time Index Specification

Let T be the patent expiry date and also be the licensing agreement expiry date. Let τ_i^j be the completion time for phase with i = 1,2, 3, 4 representing Clinical Phases I through FDA Regulatory Review. Let $j \in \{1,2\}$, i.e., group 1 and 2, such that 1 = licensing group, 2 = competing group. We have $\tau_i^j = \inf\{t \geq 0 \mid K_{it}^j = 0\}$ and τ_{cali}^j is defined as the elapsed time to completion, i.e., $\tau_{cali}^j = \sum_{k=0}^{i} \tau_k^j$ with $\tau_0^j = 0$; we assume current calendar time as zero. The completion time is defined as $\tau_{cal4}^j = Min\{\tau_{cal4}^j, T\}$. Let abn be abandonment time, and $\bar{\tau}$ is the first drug marketed, defined as the first time the

[3] The alternative and equivalent way of modeling the catastrophic events is, as in Merton (1976), to append the Poisson process that can suddenly drive the project value to zero to the stochastic project value process (in our case, it's the sales process equation (1)).

total cost process approaches zero, that is, $\bar{\tau} = \min\{\tau^1_{cal4}, \tau^2_{cal4}\}$ (i.e., $\bar{\tau} = \tau^1_{cal4} \wedge \tau^2_{cal4}$). $\underline{\tau}$ is the second drug marketed defined as $\underline{\tau} = \max\{\tau^1_{cal4}, \tau^2_{cal4}\}$ (i.e., = $\bar{\tau} = \tau^1_{cal4} \vee \tau^2_{cal4}$).

We present the whole timeline of our model in Figure 1 to help readers visualize the whole process.

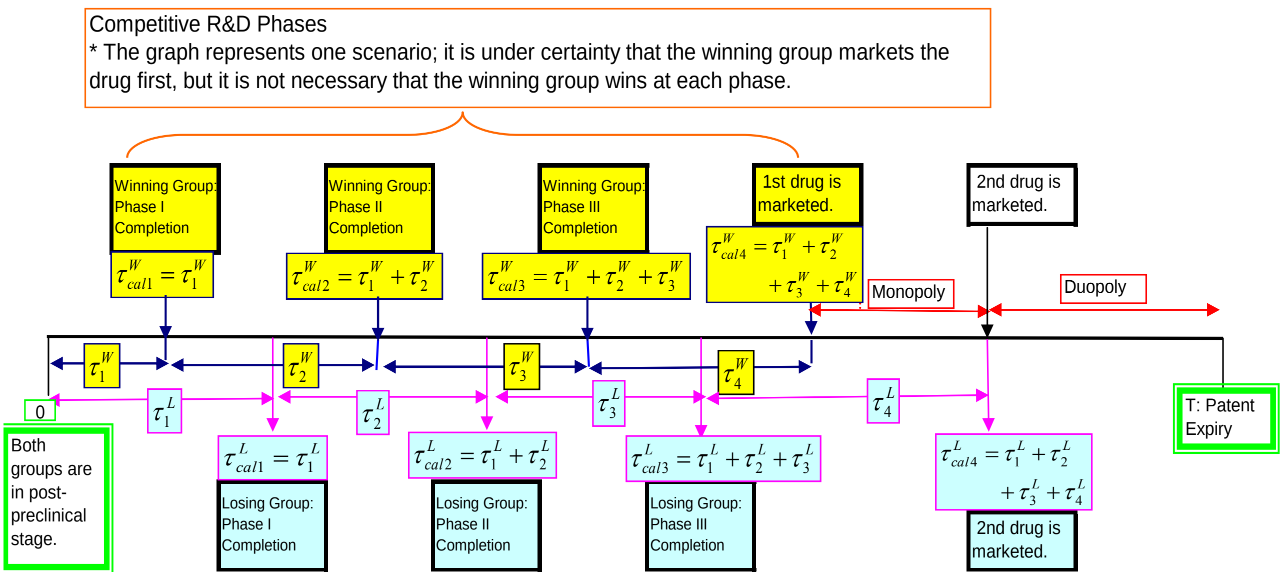

Figure 1. Model Timeline

We work on a probability space (Ω, $\mathcal{F}$, $\mathbb{Q}$) and a filtration, F={ $\mathcal{F}_t\}_{0\leq t\leq T}$,where Brownian motion is defined and the expectation $\mathbb{E}\{\bullet\}$is computed, $\mathbb{Q}$ is the equivalent martingale measure.

Investment Cost Uncertainty and Phase Completion Time

The expected cost to completion in each phase is assumed to follow the controlled diffusion process[4]:

[4] As pointed out in Dixit and Pindyck (1994), this is a special case of the controlled diffusion process: $dK_t = -Idt + g(I, K_t)dB_t$ where $\frac{\partial g}{\partial I} \geq 0$, $\frac{\partial^2 g}{\partial I^2} \leq 0$ and $\frac{\partial g}{\partial K_t} \geq 0$. The equation indicates that expected cost to completion declines with ongoing investment, but also changes stochastically. Stochastic

$$
\begin{aligned}
dK_{it}^{j} &= -I_i^j dt + \sigma_i^j \sqrt{I_i^j K_{it}^j} dB_{it}^j \\
K_{i\tau_{cali}}^{j} &= M_i^j
\end{aligned}
\tag{1}
$$

where i =1,2, 3, 4 represent Clinical Phase I, Phase II, Phase III, and FDA Regulatory Review, respectively. I_i^j is the investment rate (the control), and dB_{it}^j is the increment of a standard Weiner process assumed uncorrelated with the market at each phase, τ_{cali}^j is the elapsed time to completion defined as $\tau_{cali}^j = \sum_{k=0}^{i} \tau_k^j$ with $\tau_0^j = 0$; τ_i^j represents the completion time of each phase, when the cost process hits zero, and M_i^j is the corresponding milestone payment. If no milestone payment is required for advancing through that particular phase, M_i^j equals zero; moreover, in our situation, M_i^j occurs only when j = licensing group. The term $\sigma_i^j \sqrt{I_i^j K_{it}^j} dB_{it}^j$ is described by Pindyck (1993) as technical uncertainty since it is resolved only by additional investment. The term shows that the more R&D investments the firm estimates it still has to conduct and the higher the current R&D investment rate is, the more uncertainty will be revealed per time unit.

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changes in K might be due to technical uncertainty for $g(0,K)=0$ and $g_I > 0$, input cost uncertainty for $g(0,K)>0$, or both. By defining $F(K;V,I_m)$ as the value of the investment opportunity, where I_m is the maximum investment rate, the above controlled diffusion process makes economic sense only if the following conditions hold: (i) $F(K;V,I_m)$ is homogeneous of degree 1 in K, V, and I_m; (ii) $F_K < 0$, that is, an increase in the expected cost of an investment always reduces its value; (iii) the instantaneous variance of dK_t is bounded for all finite K_t and approaches zero as $K_t \to 0$; and (iv) if the firm invests at the maximum rate I_m until the project is completed, time τ, $E_0 \int_0^{\tau} I_m dt = K$, so that K is indeed the expected cost to completion. The general structure $g(I,K) = \beta K (I/K)^{\alpha}$ with $0 \le \alpha \le 1/2$ satisfies the four conditions; note that $0 \le \alpha < 1$ does not satisfy the condition $F_K < 0$. The specification of $\alpha = 0$ or $\alpha = \frac{1}{2}$ results in simple corner solutions for the optimal investment problem.

$\sigma_i^j \sqrt{I_i^j K_{it}^j} dB_{it}^j$ is affected by a stochastic term with mean zero and a variance linear both in the level of investment I and the expected cost. The square root $\sqrt{I_i^j K_i^j}$ [5], in the diffusion term is linear in investment, giving rise to a bang-bang solution for the optimal control. The optimal investment strategy takes one of two possible values: zero or the maximum rate. We assume zero correlation among uncertainties across phases. The expected cost to completion of a phase may change only after investment in that phase has begun. This implies that "learning" about cost to completion occurs only for the current phase. The variance of cost to completion for each phase i has the following analytical representation, $Var(\widetilde{K}_i^j) = (\frac{\sigma_i^{j2}}{2-\sigma_i^{j2}})K_i^j$ [6] and the first passage time of the cost process hitting zero can be obtained explicitly[7].

We summarize the stochastic cost process for each phase as follows:

For $0 \le t \le \tau_{cal1}^j$

$$\begin{aligned} dK_{1t}^j &= -I_1^j dt + \sigma_1^j \sqrt{I_1^j K_{1t}^j} dB_{1t}^j \\ K_{1\tau_{cal1}^j}^j &= 1_{\{milestone\ payment=yes\}} M_1^j \end{aligned} \qquad (2)$$

---

[5] See footnote 5; we can have general specification of $(IK)^\alpha$ with $0 \le \alpha < \frac{1}{2}$.

[6] For a detailed derivation see Karlin and Taylor (1981, p.203), appendix in Pindyck (1993), Dixit and Pindyck (1994, p. 351 footnote 12)

[7] The probability that the total cost to completion of the project in phase i is less than *k*, conditional on an initial expected cost to completion K_i, is given by

$$H(k) = 1 - \sum_{n=0}^{\infty} \frac{e^{-\frac{2K_i}{k\sigma_i^2}} (\frac{2K_i}{k\sigma_i^2})^{n+1-\frac{2}{\sigma_i^2}}}{\Gamma(n+2+\frac{2}{\sigma_i^2})} \quad \textit{where } \Gamma(\bullet) \textit{ is the cumulative gamma distribution} \ ;$$

detailed derivation see Schwartz and Moon (2000).

For $\tau^{j}_{cal1} < t \le \tau^{j}_{cal2}$

$$\begin{aligned} dK^{j}_{2t} &= -I^{j}_{2}dt + \sigma^{j}_{2}\sqrt{I^{j}_{2}K^{j}_{2t}}\,dB^{j}_{2t} \\ K^{j}_{2\tau^{j}_{cal2}} &= 1_{\{milestone\ payment=yes\}}M^{j}_{2} \end{aligned} \tag{3}$$

For $\tau^{j}_{cal2} < t \le \tau^{j}_{cal3}$

$$\begin{aligned} dK^{j}_{3t} &= -I^{j}_{3}dt + \sigma^{j}_{3}\sqrt{I^{j}_{3}K^{j}_{3t}}\,dB^{j}_{3t} \\ K^{j}_{3\tau^{j}_{cal3}} &= 1_{\{milestone\ payment=yes\}}M^{j}_{3} \end{aligned} \tag{4}$$

For $\tau^{j}_{cal3} < t \le \tau^{j}_{cal4}$

$$\begin{aligned} dK^{j}_{4t} &= -I^{j}_{4}dt + \sigma^{j}_{4}\sqrt{I^{j}_{4}K^{j}_{4t}}\,dB^{j}_{4t} \\ K^{j}_{4\tau_{cal4}=} &= 1_{\{milestone\ payment=yes\}}M^{j}_{4} \end{aligned} \tag{5}$$

We perform two simulation runs, each with 110,000 sample paths, by first assuming that the original licensee "wins", and alternatively assuming that the competitor "wins".

Sales Uncertainty

Upon FDA approval, the project generates sales revenue, S. We assume that S follows the Geometric Brownian motion:

$$dS_t = \alpha_t S_t dt + \sigma S_t dZ_t \tag{6}$$

where α_t,[8] is the time dependent instantaneous growth rate with a deterministic life cycle trend, satisfying a Lispchitz assumption with linear growth. σ is the instantaneous annualized sales volatility, assumed constant. dZ_t represents increments of a standard Wiener process. Following Schwartz (2003), cash flow from sales occurs only after investment has been completed. Prior to this time they represent the sales revenues the

[8] For the case $\alpha_t=\alpha$, it indicates the constant instantaneous annualized growth rate, a typical stock price process assumption in the classical Black-Scholes model.

project would have produced if it were completed. We assume zero correlation between the investment cost and the sales revenue processes. We assume both competing products face the same stochastic revenue process.

Net Operating Cash Flows

We assume that net operating cash flows are a proportion, η, of sales revenue. Net operating cash flows follow the same Geometric Brownian motion process as sales:

$$dC_t = \eta dS_t = \eta\ (\alpha_t S_t dt + \sigma S_t dZ_t) \quad (7)$$

where $\eta \in (0,1)$, α_t, σ, and dZ_t are defined above. We assume both parties face the same operating cash flows.

Duopoly Phase Project Value

By simulating the cost process, we can easily identify $\bar{\tau}$ and $\underline{\tau}$. In the duopolistic time period $\underline{\tau}$ *to T*, the revenue for the licensee will be

$$dW_t = 0.5*(dC_t - dR_t) = 0.5*(\eta-\theta)(\alpha_t S_t dt + \sigma S_t dZ_t) \quad (8)$$

where R_t is royalty payments, a percent of sales revenue following the same Geometric Brownian motion process with proportion, θ, of sales:

$$dR_t = \theta dS_t = \theta(\alpha_t S_t dt + \sigma S_t dZ_t) \text{ where } \theta \in (0,1) \quad (9)$$

On the other hand, the revenue for the other competing group will be

$$dC_t = 0.5*\eta*dS_t = 0.5*\eta*\ (\alpha_t S_t dt + \sigma S_t dZ_t) \quad (10)$$

In the first scenario, two firms exist in the duopolistic phase. Project value for the licensee in our licensing group is:

$$
\begin{aligned}
V_{W,D2}^{Licensee}(S_t,t) &= V_{L,D2}^{Licensee}(S_t,t) = V_{D2}^{Licensee}(S_t,t) \\
&= E^Q[\int_t^T e^{-r(u-t)} 0.5\times(\eta-\theta)\times S_u du \mid F_t] \\
&= 0.5\times(\eta-\theta)\times\int_t^T e^{-r(u-t)} E^Q[S_u \mid F_t] du \\
&= 0.5\times(\eta-\theta)\times\int_t^T e^{-r(u-t)} S_t e^{\alpha(u-t)} du \\
&= 0.5\times(\eta-\theta)\times S_t \times e^{(r-\alpha)t}\int_t^T e^{-(r-\alpha)u} du \\
&= \frac{0.5\times(\eta-\theta)}{(r-\alpha)}\times S_t \times e^{(r-\alpha)t}(e^{-(r-\alpha)t}-e^{-(r-\alpha)T}) \\
&= \frac{0.5\times(\eta-\theta)}{(r-\alpha)}\times S_t \times (1-e^{-(r-\alpha)(T-t)}) \qquad \text{where } t\in[\underline{\tau},T]
\end{aligned} \tag{11}
$$

where the subscript W denotes winner, L denotes Loser and D2 denotes that two groups exist in the duopoly phase.

Project value for the competing party is:

$$
\begin{aligned}
V_{L,D2}^{CG}(S_t,t) &= V_{W,D2}^{CG}(S_t,t) = V_{D2}^{CG}(S_t,t) \\
&= E^Q[\int_t^T e^{-r(u-t)} 0.5\times\eta\times S_u du \mid F_t] \\
&= 0.5\times\eta\times\int_t^T e^{-r(u-t)} E^Q[S_u \mid F_t] du \\
&= 0.5\times\eta\times\int_t^T e^{-r(u-t)} S_t e^{\alpha(u-t)} du \\
&= 0.5\times\eta\times S_t \times e^{(r-\alpha)t}\int_t^T e^{-(r-\alpha)u} du \\
&= \frac{0.5\times\eta}{(r-\alpha)}\times S_t \times e^{(r-\alpha)t}(e^{-(r-\alpha)t}-e^{-(r-\alpha)T}) \\
&= \frac{0.5\times\eta}{(r-\alpha)}\times S_t \times (1-e^{-(r-\alpha)(T-t)}) \qquad \text{where } t\in[\underline{\tau},T]
\end{aligned} \tag{12}
$$

where the superscript CG denotes competing group.

In the second scenario, only one firm exists in the duopolistic phase. Project value for the licensee in our licensing group is:

$$
\begin{aligned}
V_{W,D1}^{Licensee}(S_t,t) &= V_{L,D1}^{Licensee}(S_t,t) = V_{D1}^{Licensee}(S_t,t) \\
&= E^Q[\int_t^T e^{-r(u-t)} \times (\eta-\theta) \times S_u du \mid F_t] \\
&= (\eta-\theta) \times \int_t^T e^{-r(u-t)} E^Q[S_u \mid F_t] du \\
&= (\eta-\theta) \times \int_t^T e^{-r(u-t)} S_t e^{\alpha(u-t)} du \\
&= (\eta-\theta) \times S_t \times e^{(r-\alpha)t} \int_t^T e^{-(r-\alpha)u} du \\
&= \frac{(\eta-\theta)}{(r-\alpha)} \times S_t \times e^{(r-\alpha)t} (e^{-(r-\alpha)t} - e^{-(r-\alpha)T}) \\
&= \frac{(\eta-\theta)}{(r-\alpha)} \times S_t \times (1 - e^{-(r-\alpha)(T-t)}) \qquad \text{where } t \in [\underline{\tau}, T]
\end{aligned} \tag{13}
$$

where D1 denotes that only one group exists in the duopoly phase.

Project value for the competing party:

$$
\begin{aligned}
V_{L,D1}^{CG}(S_t,t) &= V_{W,D1}^{CG}(S_t,t) = V_{D1}^{CG} \\
&= E^Q[\int_t^T e^{-r(u-t)} \times \eta \times S_u du \mid F_t] \\
&= \eta \times \int_t^T e^{-r(u-t)} E^Q[S_u \mid F_t] du \\
&= \eta \times \int_t^T e^{-r(u-t)} S_t e^{\alpha(u-t)} du \\
&= \eta \times S_t \times e^{(r-\alpha)t} \int_t^T e^{-(r-\alpha)u} du \\
&= \frac{\eta}{(r-\alpha)} \times S_t \times e^{(r-\alpha)t} (e^{-(r-\alpha)t} - e^{-(r-\alpha)T}) \\
&= \frac{\eta}{(r-\alpha)} \times S_t \times (1 - e^{-(r-\alpha)(T-t)}) \qquad \text{where } t \in [\underline{\tau}, T]
\end{aligned} \tag{14}
$$

Monopoly Phase Project Value

In the monopolistic phase $\bar{\tau}$ *to* $\underline{\tau}$, the revenue for the licensee will be

$$dW_t = (dC_t - dR_t) = (\eta\text{-}\theta)(\alpha_t S_t dt + \sigma S_t dZ_t) \tag{15}$$

where R_t is royalty payments, a percent of sales revenue following the same Geometric Brownian motion process with proportion, θ, of sales:

$$dR_t = \theta dS_t = \theta(\alpha_t S_t dt + \sigma S_t dZ_t) \text{ where } \theta \in (0,1) \tag{16}$$

On the other hand, the revenue for the other competing group will be

$$dC_t = \eta^* dS_t = \eta^* (\alpha_t S_t dt + \sigma S_t dZ_t) \tag{17}$$

In the first scenario our licensee is the "winner". Project value for the licensee, winner, if the competing group's project is still alive is as follows. We must consider that (1) the losing firm is still investing in R&D and is still exposed to catastrophic events, and (2) the losing firm (competing group) will follow its optimal R&D investment/abandonment strategy. Given that we model the catastrophic events through Poisson probability, the conditional probability (under an equivalent martingale measure, ℚ) that the competing group, the losing firm, is not hit by catastrophic events throughout the period from date t to date u in the monopoly phase, given that its project was alive at date t is:

$$1_{\{\tau^l_{cal3} \le t < \tau^l_{cal4}\}} e^{-\lambda_4(u-t)} + 1_{\{\tau^l_{cal2} \le t < \tau^l_{cal3}\}} e^{-\lambda_4(u-\tau^l_{cal4})-\lambda_3(\tau^{l-}_{cal4}-t)} + 1_{\{\tau^l_{cal1} \le t <^l_{\tau cal2}\}} e^{-\lambda_4(u-\tau^l_{cal4})-\lambda_3(\tau^{l-}_{cal4}-\tau^l_{cal3})-\lambda_2(\tau^{l-}_{cal3}-t)}$$
$$+1_{\{0 \le t < \tau^l_{cal1}\}} e^{-\lambda_4(u-\tau^l_{cal4})-\lambda_3(\tau^{l-}_{cal4}-\tau^l_{cal3})-\lambda_2(\tau^{l-}_{cal3}-\tau^l_{cal2})-\lambda_1(\tau^{l-}_{cal2}-t)} = \sum \lambda$$

As a result, the conditional probability (under an equivalent martingale measure, ℚ) that the competing group (losing firm), is hit by catastrophic events during a period from date t to date u in the monopoly phase, given that its project was alive at date t is $1-\sum\lambda$.

At any given date t in the monopoly phase, i.e. $t \in [\overline{\tau}, \underline{\tau})$, if the competing group's project is still alive, the project value for the licensee, the winning group is:

$$V_{W,M2}^{Licensee}(S_t, K_{L,t}, t) = E^Q[\int_t^{\underline{\tau}} e^{-r(u-t)} \times (\eta - \theta) \times S_u du + 1_{\{abn_{L,t}^{2,*} = \underline{\tau}\}} \times \sum \lambda \times e^{-r(\underline{\tau}-t)} \times V_{D2}^{Licensee}(S_{\underline{\tau}}, \underline{\tau})$$

$$+ ((1 - \sum \lambda) 1_{\{abn_{l,t}^{2,*} = \underline{\tau}\}} + 1_{\{abn_{l,t}^{2,*} < \underline{\tau}\}}) \times e^{-r(\underline{\tau}-t)} V_{D1}^{Licensee}(S_{\underline{\tau}}, \underline{\tau}) =| F_t]$$

(18)

where $t \in [\overline{\tau}, \underline{\tau})$, and $abn_{l,t}^{2,*}$ is losing group's optimal R&D investment/abandonment strategy. The first term in equation (18) represents the winning firm's (licensee's) monopoly profit from date t until the end of the monopoly phase. The second term in equation (18) is the winning group's (licensee's) share of the duopoly profit in the duopoly phase in the event that the losing group (the competitor) is not hit by catastrophic events and does not abandon its project. The third term in equation (18) is the winning group's (licensee's) monopoly profit in the duopoly phase where the losing group (competing group) is either hit by catastrophic events before the duopoly phase or the losing group (competing group) finds it optimal to abandon.

Project value for the licensee, winner, if the competing group's project is no longer alive is:

$$V_{W,M1}^{Licensee}(S_t, t) = \frac{(\eta - \theta)}{(r - \alpha)} \times S_t \times (1 - e^{-(r-\alpha)(T-t)}) \qquad \text{where } t \in [\overline{\tau}, \underline{\tau}) \qquad (19)$$

Project value for the competing party given our licensee is the winner and the winning group's project is still alive at the entrance date of the monopoly phase:

$$V_{L,M2}^{CG}(S_t, K_t^{CG}, t) = \underset{abn_l \in \{t | \overline{\tau} < t \leq \underline{\tau}\}}{Max} E^Q[-\int_t^{abn_l} 1_{\{\tau_{cal3}^l \leq \overline{\tau}\}} e^{-\lambda_4(u-t)} e^{-r(u-t)} I_{4,CG} du$$

$$-1_{\{\tau_{cal2}^l \leq \overline{\tau} < \tau_{cal3}^l\}}[(1_{\{abn_l \geq \tau_{cal3}^l\}}(\int_t^{abn_l} I_{4,l} e^{-\lambda_4(u-\tau_{cal3}^l)-\lambda_3(\tau_{cal3}^{l-}-t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal3}^{l-}-t)} K_{3,t}^{CG} | F_t]))$$

$$+(1_{\{abn_l \leq \tau_{cal3}^l\}} \int_t^{abn_l} I_{3,CG} e^{-\lambda_3(u-t)} e^{-r(u-t)} du)]$$

$$-1_{\{\tau_{cal1}^l \leq \overline{\tau} < \tau_{cal2}^l\}}[(1_{\{abn_l \geq \tau_{cal3}^l\}}(\int_t^{abn_l} I_{4,CG} e^{-\lambda_4(u-\tau_{cal3}^l)-\lambda_3(\tau_{cal3}^{l-}-\tau_{cal2}^l)-\lambda_2(\tau_{cal2}^{l-}-t)} e^{-r(u-t)} du$$

$$+E^Q[e^{-r(\tau_{cal3}^{l-}-t)} K_{3,t}^{CG} | F_t] + E^Q[e^{-r(\tau_{cal2}^{l-}-t)} K_{2,t}^{CG} | F_t]))$$

$$+(1_{\{\tau_{cal2}^l < abn_l \leq \tau_{cal3}^l\}}(\int_t^{abn_l} I_{3,CG} e^{-\lambda_3(u-\tau_{cal2}^l)-\lambda_2(\tau_{cal2}^{l-}-t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal2}^{l-}-t)} K_{2,t}^{CG} | F_t]))$$

$$+(1_{\{\tau_{cal1}^l < abn_l \leq \tau_{cal2}^l\}}(\int_t^{abnl} I_{2,CG} e^{-\lambda_2(u-t)} e^{-r(u-t)} du)]$$

$$-1_{\{\overline{\tau} < \tau_{cal1}^l\}}[(1_{\{abn_l \geq \tau_{cal3}^l\}}(\int_t^{abnl} I_{4,CG} e^{-\lambda_4(u-\tau_{cal3}^l)-\lambda_3(\tau_{cal3}^{l-}-\tau_{cal2}^l)-\lambda_2(\tau_{cal2}^{l-}-\tau_{cal1}^l)-\lambda_1(\tau_{cal1}^{l-}-t)} e^{-r(u-t)} du$$

$$+E^Q[e^{-r(\tau_{cal3}^{l-}-t)} K_{3,t}^{CG} | F_t] + E^Q[e^{-r(\tau_{cal2}^{l-}-t)} K_{2,t}^{CG} | F_t] + E^Q[e^{-r(\tau_{cal1}^{l-}-t)} K_{1,t}^{CG} | F_t]))$$

$$+(1_{\{\tau_{cal2}^l < abn_l \leq \tau_{cal3}^l\}}(\int_t^{abnl} I_{3,CG} e^{-\lambda_3(u-\tau_{cal2}^l)-\lambda_2(\tau_{cal2}^{l-}-t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal2}^{l-}-t)} K_{2,t}^{CG} | F_t]$$

$$+E^Q[e^{-r(\tau_{cal1}^{l-}-t)} K_{1,t}^{CG} | F_t])) \tag{20}$$

$$+(1_{\{\tau_{cal1}^l < abn_l \leq \tau_{cal2}^l\}}(\int_t^{abnl} I_{2,CG} e^{-\lambda_2(u-\tau_{cal1}^l)-\lambda_1(\tau_{cal1}^{l-}-t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal1}^{l-}-t)} K_{1,t}^{CG} | F_t]))$$

$$+(1_{\{0 < abn_l \leq \tau_{cal1}^l\}}(\int_t^{abnl} I_{1,CG} e^{-\lambda_1(u-t)} e^{-r(u-t)} du)]$$

$$+1_{\{abn_l = \underline{\tau}\}} \times \sum \lambda \times e^{-r(\underline{\tau}-t)} \times V_{D2}^{CG}(S_{\underline{\tau}}, \underline{\tau}) | F_t\} \qquad \text{where } t \in [\overline{\tau}, \underline{\tau}]$$

where the next to last term represents the losing group's (i.e. the competing group's) R&D investment costs in the monopoly phase after date t and until (1) it is hit by catastrophic events, (2) it decides to abandon its R&D investment project, or (3) it completes the R&D investment project. The last term is the losing group's (i.e. the competing group's) share of the duopoly profit in the duopoly phase where the losing group is neither hit by catastrophic events, nor does it abandon.

The value obtained in the above equation is a maximization problem because the losing group (competing group) needs to decide at each instant whether to continue investing in R&D or abandon for maximizing the project value. Given that the losing group has not abandoned and the winning group's (licensee's) project is still alive at that date, the losing group's (competing group's) optimal R&D investment/abandonment strategy, abn_l^{2*}, is an optimal stopping time related to the filtration $\mathfrak{F}$. The maximization problem can be solved by applying dynamic programming with boundary condition:

$$V_{L,M2}^{CG}(S_{\underline{\tau}}, 0, \underline{\tau}) = V_{D2}^{CG}(S_{\underline{\tau}}, \underline{\tau}) \tag{21}$$

Project value for the competing party given our licensee is a winner and the winning group's project is no longer alive at the entrance date of the monopoly phase:

$$V_{L,M1}^{CG}(S_t, K_t^{CG}, t) = \underset{abn_l \in \{t | \overline{\tau} \le t < \underline{\tau}\}}{Max} E^Q[-\int_t^{abn_l} 1_{\{\tau_{cal3}^l \le \overline{\tau} < \tau_{cal4}^l\}} e^{-\lambda_4 (u-t)} e^{-r(u-t)} I_{4,CG} du$$

$$-1_{\{\tau_{cal2}^l \le \overline{\tau} < \tau_{cal3}^l\}} [(1_{\{abn_l \ge \tau_{cal3}^l\}} (\int_t^{abnl} I_{4,CG} e^{-\lambda_4 (u - \tau_{cal3}^l) - \lambda_3 (\tau_{cal3}^{l-} - t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal3}^{l-} - t)} K_{3,t}^{CG} | F_t]))$$

$$+ (1_{\{abn_l \le \tau_{cal3}^l\}} \int_t^{abnl} I_{3,CG} e^{-\lambda_3 (u-t)} e^{-r(u-t)} du)]$$

$$-1_{\{\tau_{cal1}^l \le \overline{\tau} < \tau_{cal2}^l\}} [(1_{\{abn_l \ge \tau_{cal3}^l\}} (\int_t^{abnl} I_{4,CG} e^{-\lambda_4 (u - \tau_{cal3}^l) - \lambda_3 (\tau_{cal3}^{l-} - \tau_{cal2}^l) - \lambda_2 (\tau_{cal2}^{l-} - t)} e^{-r(u-t)} du$$

$$+ E^Q[e^{-r(\tau_{cal3}^{l-} - t)} K_{3,t}^{CG} | F_t] + E^Q[e^{-r(\tau_{cal2}^{l-} - t)} K_{2,t}^{CG} | F_t]))$$

$$+ (1_{\{\tau_{cal2}^l < abn_l \le \tau_{cal3}^l\}} (\int_t^{abnl} I_{3,CG} e^{-\lambda_3 (u - \tau_{cal2}^l) - \lambda_2 (\tau_{cal2}^{l-} - t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal2}^{l-} - t)} K_{2,t}^{CG} | F_t]))$$

$$+ (1_{\{\tau_{cal1}^l < abn_l \le \tau_{cal2}^l\}} (\int_t^{abnl} I_{2,CG} e^{-\lambda_2 (u-t)} e^{-r(u-t)} du)]$$

$$-1_{\{\overline{\tau} < \tau_{cal1}^l\}} [(1_{\{abn_l \ge \tau_{cal3}^l\}} (\int_t^{abnl} I_{4,CG} e^{-\lambda_4 (u - \tau_{cal3}^l) - \lambda_3 (\tau_{cal3}^{l-} - \tau_{cal2}^l) - \lambda_2 (\tau_{cal2}^{l-} - \tau_{cal1}^l) - \lambda_1 (\tau_{cal1}^{l-} - t)} e^{-r(u-t)} du$$

$$+ E^Q[e^{-r(\tau_{cal3}^{l-} - t)} K_{3,t}^{CG} | F_t] + E^Q[e^{-r(\tau_{cal2}^{l-} - t)} K_{2,t}^{CG} | F_t] + E^Q[e^{-r(\tau_{cal1}^{l-} - t)} K_{1,t}^{CG} | F_t]))$$

$$+ (1_{\{\tau_{cal2}^l < abn_l \le \tau_{cal3}^l\}} (\int_t^{abnl} I_{3,CG} e^{-\lambda_3 (u - \tau_{cal2}^l) - \lambda_2 (\tau_{cal2}^{l-} - t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal2}^{l-} - t)} K_{2,t}^{CG} | F_t]$$

$$+ E^Q[e^{-r(\tau_{cal1}^{l-} - t)} K_{1,t}^{CG} | F_t]))$$

$$+ (1_{\{\tau_{cal1}^l < abn_l \le \tau_{cal2}^l\}} (\int_t^{abnl} I_{2,CG} e^{-\lambda_2 (u - \tau_{cal1}^l) - \lambda_1 (\tau_{cal1}^{l-} - t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal1}^{l-} - t)} K_{1,t}^{CG} | F_t]))$$

$$+ (1_{\{0 < abn_l \le \tau_{cal1}^l\}} (\int_t^{abnl} I_{2,CG} e^{-\lambda_1 (u-t)} e^{-r(u-t)} du)]$$

$$+1_{\{abn_l = \underline{\tau}\}} \times \sum \lambda \times e^{-r(\underline{\tau} - t)} \times V_{D1}^{CG}(S_{\underline{\tau}}, \underline{\tau}) | F_t\} \qquad \text{where } t \in [\overline{\tau}, \underline{\tau}] \tag{22}$$

where the next to last and last terms have the same interpretation as those in $V_{L.M2}^{CG}(S_t, K_{lt}, t)$. Here the losing firm's R&D investment/abandonment strategy, abn_l^{1*}, again can be solved through dynamic programming with boundary condition:

$$V_{L,M1}^{CG}(S_{\underline{\tau}},0,\underline{\tau}) = V_{D1}^{CG}(S_{\underline{\tau}},\underline{\tau}) \tag{23}$$

Competitive R&D Phase Project Value

In the competitive R&D phase before any drug is marketed (i.e. prior to the time that the first drug is commercialized), the two groups race to commercialize their drug first. When both groups are still investing in the competitive R&D phase, there is a competitive interaction element to the optimal R&D investment/abandonment strategy. That is, the value to one of the groups of all future cash flows is negative if the other group continues investing, but the value becomes positive if the other group abandons. As Milterson and Schwartz (2002), a standard Cournot-Nash equilibrium is applied to find the optimal R&D investment/abandonment strategies for both groups. In this phase, at any given date t the two groups' date t optimal R&D investment/abandonment decisions are obtained as a reaction (i.e., response function) to their competitor's given date t R&D investment/abandonment decision.

First, we assume that the other group is hit by catastrophe or it abandons. It becomes a standard optimal stopping problem. The same methods applied to the losing group in the monopoly phase can be used to solve the problem.

At any given date t in the competitive R&D phase, i.e. t $\in$ $[0, \bar{\tau})$, if the competing group's project is no longer alive, the total value to the winner, licensee, of all cash flows after that date is:

$$
\begin{aligned}
&V_{R\&D1}^{Licensee}(S_t, K_t^{Licensee}, t) = \underset{abn_{licensee} \in \{t|0 \le t < \overline{\tau}\}}{Max} E^Q[-\int_t^{abn_{licensee}} 1_{\{\tau_{cal3}^{licenseel} \le t < \tau_{cal4}^{licensee}\}} e^{-\lambda_4(u-t)} e^{-r(u-t)} I_{4,licensee} du \\
&-1_{\{\tau_{cal2}^{licensee} \le t < \tau_{cal3}^{licensee}\}}[(1_{\{abn_{licensee} \ge \tau_{cal3}^{l}\}}(\int_t^{abn_{licensee}} I_{4,licensee} e^{-\lambda_4(u-\tau_{cal3}^{licensee}) - \lambda_3(\tau_{cal3}^{licensee-} - t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal3}^{l-} - t)} K_{3,t}^{Licensee} | F_t])) \\
&+(1_{\{abn_{licensee} \le \tau_{cal3}^{licensee}\}} \int_t^{abn_{licensee}} I_{3,licensee} e^{-\lambda_3(u-t)} e^{-r(u-t)} du)] \\
&-1_{\{\tau_{cal1}^{licensee} \le t < \tau_{cal2}^{l}\}}[(1_{\{abn_l \ge \tau_{cal3}^{licensee}\}}(\int_t^{abn_{licensee}} I_{4,licensee} e^{-\lambda_4(u-\tau_{cal3}^{licensee}) - \lambda_3(\tau_{cal3}^{licensee-} - \tau_{cal2}^{licensee}) - \lambda_2(\tau_{cal2}^{licensee-} - t)} e^{-r(u-t)} du \\
&+E^Q[e^{-r(\tau_{cal3}^{licensee-} - t)} K_{3,t}^{Licensee} | F_t] + E^Q[e^{-r(\tau_{cal2}^{licensee-} - t)} K_{2,t}^{Licensee} | F_t])) \\
&+(1_{\{\tau_{cal2}^{licensee} < abn_{licensee} \le \tau_{cal3}^{licensee}\}}(\int_t^{abn_{licensee}} I_{3,licensee} e^{-\lambda_3(u-\tau_{cal2}^{licensee}) - \lambda_2(\tau_{cal2}^{licensee-} - t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal2}^{licensee-} - t)} K_{2,t}^{Licensee} | F_t])) \\
&+(1_{\{\tau_{cal1}^{l} < abn_{licensee} \le \tau_{cal2}^{l}\}}(\int_t^{abn_{licensee}} I_{2,licensee} e^{-\lambda_2(u-t)} e^{-r(u-t)} du))] \\
&-1_{\{0 \le t < \tau_{cal1}^{licensee}\}}[(1_{\{abn_l \ge \tau_{cal3}^{licensee}\}}(\int_t^{abn_{licensee}} I_{4,licensee} e^{-\lambda_4(u-\tau_{cal3}^{licensee}) - \lambda_3(\tau_{cal3}^{licensee-} - \tau_{cal2}^{licensee}) - \lambda_2(\tau_{cal2}^{licensee-} - \tau_{cal1}^{licensee}) - \lambda_1(\tau_{cal1}^{licensee-} - t)} e^{-r(u-t)} du \\
&+E^Q[e^{-r(\tau_{cal3}^{licensee-} - t)} K_{3,t}^{Licensee} | F_t] + E^Q[e^{-r(\tau_{cal2}^{licensee-} - t)} K_{2,t}^{Licensee} | F_t] + E^Q[e^{-r(\tau_{cal1}^{licensee-} - t)} K_{1,t}^{Licensee} | F_t])) \\
&+(1_{\{\tau_{cal2}^{l} < abn_l \le \tau_{cal3}^{l}\}}(\int_t^{abn_{licensee}} I_{3,licensee} e^{-\lambda_3(u-\tau_{cal2}^{l}) - \lambda_2(\tau_{cal2}^{l-} - t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal2}^{l-} - t)} K_{2,t}^{Licensee} | F_t] \\
&+E^Q[e^{-r(\tau_{cal1}^{l-} - t)} K_{1,t}^{Licensee} | F_t])) \\
&+(1_{\{\tau_{cal1}^{l} < abn_l \le \tau_{cal2}^{l}\}}(\int_t^{abn_{licensee}} I_{2,licensee} e^{-\lambda_2(u-\tau_{cal1}^{licensee}) - \lambda_1(\tau_{cal1}^{licensee-} - t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal1}^{licensee-} - t)} K_{1,t}^{Licensee} | F_t])) \\
&+(1_{\{0 < abn_{licensee} \le \tau_{cal1}^{licensee}\}}(\int_t^{abn_{licensee}} I_{2,licensee} e^{-\lambda_1(u-t)} e^{-r(u-t)} du))] \\
&+1_{\{abn_{licensee} = \overline{\tau}\}} \sum \hat{\lambda} \times e^{-r(\overline{\tau}-t)} \times V_{M1}^{Licensee}(S_{\overline{\tau}}, \overline{\tau}) | F_t\} \qquad \text{where } t \in [0, \overline{\tau}]
\end{aligned}
\tag{24}
$$

where

$$
\begin{aligned}
&1_{\{\tau_{cal3}^{w} \le t < \tau_{cal4}^{w}\}} e^{-\lambda_4(u-t)} + 1_{\{\tau_{cal2}^{w} \le t < \tau_{cal3}^{w}\}} e^{-\lambda_4(u-\tau_{cal4}^{w}) - \lambda_3(\tau_{cal4}^{w-} - t)} + 1_{\{\tau_{cal1}^{w} \le t < \tau_{cal2}^{w}\}} e^{-\lambda_4(u-\tau_{cal4}^{w}) - \lambda_3(\tau_{cal4}^{w-} - \tau_{cal3}^{w}) - \lambda_2(\tau_{cal3}^{w-} - t)} \\
&+1_{\{0 \le t < \tau_{cal1}^{w}\}} e^{-\lambda_4(u-\tau_{cal4}^{w}) - \lambda_3(\tau_{cal4}^{w} - \tau_{cal3}^{w}) - \lambda_2(\tau_{cal3}^{w-} - \tau_{cal2}^{w}) - \lambda_1(\tau_{cal2}^{w-} - t)} = \sum \hat{\lambda}
\end{aligned}
$$

represents the conditional probability (under equivalent martingale measure $\mathbb{Q}$) that the licensee (winner) is not hit by catastrophe from date t to u in the development phase, given that its project was alive at t, and the conditional probability (under equivalent martingale measure $\mathbb{Q}$) that the competing group (loser) is hit by catastrophe during a period from date t to date u in the monopoly phase, given that its project was alive at date t is $1-\sum^{\wedge}\lambda$.

This is an optimal stopping problem, and we can derive the optimal abandonment/investment strategy, $abn_{licensee}^{1*}$, through dynamic programming with boundary condition:

$$V_{R\&D1}^{Licensee}(S_{\bar{\tau}},0,\bar{\tau}) = V_{M1}^{Licensee}(S_{\bar{\tau}},\bar{\tau}) \tag{25}$$

Similarly, the total value to the competing group after that date can be derived as:

$$V_{R\&D1}^{CG}(S_t, K_t^{CG}, t) = \underset{abn_l \in \{t|0\le t<\underline{\tau}\}}{Max} E^Q[-\int_t^{abn_l} 1_{\{\tau_{cal3}^l \le t < \tau_{cal4}^l\}} e^{-\lambda_4(u-t)} e^{-r(u-t)} I_{4,CG} du$$

$$-1_{\{\tau_{cal2}^l \le t < \tau_{cal3}^l\}}[(1_{\{abn_l \ge \tau_{cal3}^l\}}(\int_t^{abn_l} I_{4,l} e^{-\lambda_4(u-\tau_{cal3}^l)-\lambda_3(\tau_{cal3}^{l-}-t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal3}^{l-}-t)} K_{3,t}^{CG} \mid F_t]))$$

$$+(1_{\{abn_l \le \tau_{cal3}^l\}} \int_t^{abn_l} I_{3,CG} e^{-\lambda_3(u-t)} e^{-r(u-t)} du))]$$

$$-1_{\{\tau_{cal1}^l \le t < \tau_{cal2}^l\}}[(1_{\{abn_l \ge \tau_{cal3}^l\}}(\int_t^{abn_l} I_{4,CG} e^{-\lambda_4(u-\tau_{cal3}^l)-\lambda_3(\tau_{cal3}^{l-}-\tau_{cal2}^l)-\lambda_2(\tau_{cal2}^{l-}-t)} e^{-r(u-t)} du$$

$$+E^Q[e^{-r(\tau_{cal3}^{l-}-t)} K_{3,t}^{CG} \mid F_t] + E^Q[e^{-r(\tau_{cal2}^{l-}-t)} K_{2,t}^{CG} \mid F_t]))$$

$$+(1_{\{\tau_{cal2}^l < abn_l \le \tau_{cal3}^l\}}(\int_t^{abnl} I_{3,CG} e^{-\lambda_3(u-\tau_{cal2}^l)-\lambda_2(\tau_{cal2}^{l-}-t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal2}^{l-}-t)} K_{2,t}^{CG} \mid F_t]))$$

$$+(1_{\{\tau_{cal1}^l < abn_l \le \tau_{cal2}^l\}}(\int_t^{abnl} I_{2,CG} e^{-\lambda_2(u-t)} e^{-r(u-t)} du))]$$

$$-1_{\{0\le t<\tau_{cal1}^l\}}[(1_{\{abn_l \ge \tau_{cal3}^l\}}(\int_t^{abnl} I_{4,CG} e^{-\lambda_4(u-\tau_{cal3}^l)-\lambda_3(\tau_{cal3}^{l-}-\tau_{cal2}^l)-\lambda_2(\tau_{cal2}^{l-}-\tau_{cal1}^l)-\lambda_1(\tau_{cal1}^{l-}-t)} e^{-r(u-t)} du$$

$$+E^Q[e^{-r(\tau_{cal3}^{l-}-t)} K_{3,t}^{CG} \mid F_t] + E^Q[e^{-r(\tau_{cal2}^{l-}-t)} K_{2,t}^{CG} \mid F_t] + E^Q[e^{-r(\tau_{cal1}^{l-}-t)} K_{1,t}^{CG} \mid F_t]))$$

$$+(1_{\{\tau_{cal2}^l < abn_l \le \tau_{cal3}^l\}}(\int_t^{abn_l} I_{3,CG} e^{-\lambda_3(u-\tau_{cal2}^l)-\lambda_2(\tau_{cal2}^{l-}-t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal2}^{l-}-t)} K_{2,t}^{CG} \mid F_t]$$

$$+E^Q[e^{-r(\tau_{cal1}^{l-}-t)} K_{1,t}^{CG} \mid F_t])) \tag{26}$$

$$+(1_{\{\tau_{cal1}^l < abn_l \le \tau_{cal2}^l\}}(\int_t^{abn_l} I_{2,CG} e^{-\lambda_2(u-\tau_{cal1}^l)-\lambda_1(\tau_{cal1}^{l-}-t)} e^{-r(u-t)} du + E^Q[e^{-r(\tau_{cal1}^{l-}-t)} K_{1,t}^{CG} \mid F_t]))$$

$$+(1_{\{0<abn_l \le \tau_{cal1}^l\}}(\int_t^{abn_l} I_{2,CG} e^{-\lambda_1(u-t)} e^{-r(u-t)} du))]$$

$$+1_{\{abn_l = \underline{\tau}\}}(1-\sum\lambda) \times e^{-r(\underline{\tau}-t)} \times V_{D1}^{CG}(S_{\underline{\tau}}, \underline{\tau}) \mid F_t\} \qquad \text{where } t \in [0, \underline{\tau}]$$

Again we can derive the optimal abandonment/investment strategy, abn_l^{1*} , through dynamic programming with boundary condition:

$$V_{R\&D1}^{CG}(S_{\underline{\tau}}, 0, \underline{\tau}) = V_{D1}^{CG}(S_{\underline{\tau}}, \underline{\tau}) \tag{27}$$

Having derived the value of each group's project given the other abandons (equation 24 and equation 26), we the turn to derive the date t value of licensee group's and competing group's projects given the other group continues investing in order to find the best response function. We must account for the fact that both groups are exposed to catastrophic events, as well as the fact that both groups follow investment strategies that are Cournot-Nash equilibria at any later date u ≥ t in the competitive R&D phase. Because of the competitive interactions, only the objective function as a solution to a dynamic programming problem can be derived. If both projects are alive in the competitive R&D phase, the boundary conditions for the project values are given by the value at the entrance date into the monopoly phase. That is

$$V_{R\&D2}^{Licensee}(S_{\bar{\tau}},0,K_{CG,\bar{\tau}},\bar{\tau}) = V_{M2}^{Licensee}(S_{\bar{\tau}},K_{CG,\bar{\tau}},\bar{\tau})$$

$$\text{and} \quad V_{R\&D2}^{CG}(S_{\bar{\tau}},0,K_{CG,\bar{\tau}},\bar{\tau}) = V_{M2}^{CG}(S_{\bar{\tau}},K_{CG,\bar{\tau}},\bar{\tau}) \tag{28}$$

This valuation problem in the competitive R&D phase is then solved by backward induction. We solve for project value at t (in the competitive R&D phase) conditional on having already solved for the value at any later date u.

The value at t in the competitive phase to the licensing group (if its project is still alive) of all cash flows after t assuming both groups continue investing at t, denoted as $\hat{V}_{R\&D2}^{Licensee}(S_t, K_t^{Licensee}, K_t^{l}, t)$, can be expressed as an expectation under equivalent ℚ martingale measure (with Filtration F_t) from the next moment, say date $t+dt$, with three main components: (1) licensee investment costs from continuing investing at time $t+dt$, (2) $V_{R\&D2}^{Licensee}(S_{t+dt}, K_{d+dt}^{Licensee}, K_{d+dt}^{CG}, t+dt)$ with the probability that both groups are not hit by catastrophic events, and (3) $V_{R\&D1}^{Licensee}(S_{t+dt}, K_{t+dt}^{Licensee}, t+dt)$ with the probability that the competing group is hit by catastrophic events.

The value at t in the competitive R&D phase to the competing group (if its project is still alive) of cash flows after t, assuming both groups continue investing, denoted as $\hat{V}_{R\&D2}^{CG}(S_t, K_t^{Licensee}, K_t^{l}, t)$, can be expressed similarly to the licensing group described above.

Following Milterson and Schwartz (2002), we consider the game shown in the following table to find the Cournot-Nash type equilibrium R&D investment/abandonment decisions at date t for the two groups in the competitive R&D phase:

		Competing Group			
		Continue Investing		Abandon	
Licensing Group	Continue Investing	$\hat{V}_{R\&D2}^{Licensee}(S_t, K_t^{Licensee}, K_t^{CG}, t)$	$\hat{V}_{R\&D2}^{CG}(S_t, K_t^{Licensee}, K_t^{CG}, t)$	$V_{R\&D1}^{Licensee}(S_t, K_t^{Licensee}, t)$	0
	Abandon	0	$V_{R\&D1}^{CG}(S_t, K_t^{CG}, t)$	0	0

For considering this Cournot-Nash Equilibrium game in the competitive R&D phase, we can summarize the total value to the licensing group and the competing group as follows:

$$V_{R\&D2}^{Licensee}(S_t, K_t^{Licensee}, K_t^{CG}, t) = \begin{cases} \hat{V}_{R\&D2}^{Licensee}(S_t, K_t^{Licensee}, K_t^{CG}, t) & \text{if } \hat{V}_{R\&D2}^{Licensee}(S_t, K_t^{Licensee}, K_t^{CG}, t) \geq 0 \text{ and } \hat{V}_{R\&D2}^{CG}(S_t, K_t^{Licensee}, K_t^{CG}, t) \geq 0 \\ V_{R\&D1}^{Licensee}(S_t, K_t^{Licensee}, t) & \text{if } \hat{V}_{R\&D2}^{Licensee}(S_t, K_t^{Licensee}, K_t^{CG}, t) \geq 0 \text{ and } \hat{V}_{R\&D2}^{CG}(S_t, K_t^{Licensee}, K_t^{CG}, t) < 0 \\ & \text{or if } V_{R\&D1}^{Licensee}(S_t, K_t^{Licensee}, t) \geq 0 \text{ and } V_{R\&D1}^{CG}(S_t, K_t^{CG}, t) < 0 \\ & \text{or if } V_{R\&D1}^{Licensee}(S_t, K_t^{Licensee}, t) > V_{R\&D1}^{CG}(S_t, K_t^{CG}, t) \geq 0 \text{ and } \hat{V}_{R\&D2}^{CG}(S_t, K_t^{Licensee}, K_t^{CG}, t) < 0 \\ 0 & \text{if } V_{R\&D1}^{Licensee}(S_t, K_t^{Licensee}, t) < 0 \\ & \text{or } \hat{V}_{R\&D2}^{Licensee}(S_t, K_t^{Licensee}, K_t^{CG}, t) < 0 \text{ and } \hat{V}_{R\&D2}^{CG}(S_t, K_t^{Licensee}, K_t^{CG}, t) \geq 0 \\ & \text{or } V_{R\&D1}^{Licensee}(S_t, K_t^{Licensee}, t) \geq V_{R\&D1}^{CG}(S_t, K_t^{CG}, t) \geq 0 \text{ and } \hat{V}_{R\&D2}^{Licensee}(S_t, K_t^{Licensee}, K_t^{CG}, t) < 0 \end{cases} \quad (28)$$

$$V^{CG}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)=\begin{cases} \hat{V}^{CG}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t) & \text{if } \hat{V}^{CG}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)\geq 0 \text{ and } \hat{V}^{Licensee}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)\geq 0 \\ V^{CG}_{R\&D1}(S_t,K_t^{Licensee},t) & \text{if } \hat{V}^{CG}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)\geq 0 \text{ and } \hat{V}^{Licensee}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)< 0 \\ & \text{or if } V^{CG}_{R\&D1}(S_t,K_t^{CG},t)\geq 0 \text{ and } V^{Licensee}_{R\&D1}(S_t,K_t^{Licensee},t)<0 \\ & \text{or if } V^{CG}_{R\&D1}(S_t,K_t^{CG},t)> V^{Licensee}_{R\&D1}(S_t,K_t^{Licensee},t)\geq 0 \text{ and } \hat{V}^{Licensee}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)<0 \\ 0 & \text{if } V^{CG}_{R\&D1}(S_t,K_t^{CG},t)<0 \\ & \text{or } \hat{V}^{CG}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)<0 \text{ and } \hat{V}^{Licensee}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)\geq 0 \\ & \text{or } V^{CG}_{R\&D1}(S_t,K_t^{CG},t)\geq V^{Licensee}_{R\&D1}(S_t,K_t^{Licensee},t)\geq 0 \text{ and } \hat{V}^{CG}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)<0 \end{cases} \tag{29}$$

It can be seen that $\hat{V}^{CG}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)<V^{CG}_{R\&D1}(S_t,K_t^{CG},t)$ (so as for the licensing group), i.e., the value to any group if it continues investing in R&D is lower if the other group also continues investing than if the other group abandons, ceteris paribus.

If $\hat{V}^{CG}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)\geq 0$ and $\hat{V}^{Licensee}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)\geq 0$, there is a unique Nash equilibrium in simple strategies. This equilibrium has both firms continuing investment.

If $\hat{V}^{CG}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)\geq 0$ and $\hat{V}^{Licensee}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)< 0$ (or the other way around), the unique Nash equilibrium in simple strategies is that the group with positive project value continues investing, whereas the other abandons.

If both $V^{CG}_{R\&D1}(S_t,K_t^{CG},t)$ and $V^{Licensee}_{R\&D1}(S_t,K_t^{Licensee},t) < 0$, the unique Nash equilibrium in simple strategies has both firms abandoning.

If $V^{CG}_{R\&D1}(S_t,K_t^{CG},t)\geq 0$ and $V^{Licensee}_{R\&D1}(S_t,K_t^{Licensee},t) < 0$ (or the other way around), the unique Nash equilibrium in simple strategies is that the group with positive project value continues investing, whereas the other abandons.

If $V^{CG}_{R\&D1}(S_t,K_t^{CG},t)\geq 0$, $\hat{V}^{CG}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)<0$, $V^{Licensee}_{R\&D1}(S_t,K_t^{Licensee},t)\geq 0$ and $\hat{V}^{Licensee}_{R\&D2}(S_t,K_t^{Licensee},K_t^{CG},t)< 0$), there are multiple Nash equilibria in simple strategies. It is a Nash equilibrium in simple strategies that one of the groups continues investing and the other abandons. We need to be able to establish a rule for the two groups, so that we know which of them should continue investing and which should abandon. In this situation, we follow the technique suggested by Miltersen and Schwartz (2002) to set up the rule that the group with the highest value of continuing investing in R&D, given that the other firm abandons its R&D investment project, continues investing in R&D, and the other firm abandons its R&D investment project. As suggested by Miltersen and Schwartz (2002), this Nash equilibrium among all Nash equilibria gives the highest ex ante values of the two firms' projects and thus should be the one Nash equilibrium that both firms would prefer to play. As a result, we have found the Cournot-Nash type equilibrium investment decisions for date t. The date t values for each of the two projects corresponding to the outcome of this Cournot-Nash type investment game can then be assigned to $V_{R\&D2}^{CG}(S_t, K_t^{Licensee}, K_t^{CG}, t)$ and $V_{R\&D2}^{Licensee}(S_t, K_t^{Licensee}, K_t^{CG}, t)$ (In our summarized equation above).

III. Model Application and Sensitivity Analyses

Numerical Solution Procedure

We solve the model by applying the Longstaff and Schwartz (2001) simulation method. We simulate 110,000 discretizised (quarterly) sample paths for governing state variables representing sales revenue (See Figure 3 and 4), all of the Clinical Phase cost processes, and cost upon completion of the FDA regulatory review (each with two different stochastic cost processes, one for each group) (See Figure 1). The completion time for each phase for a particular firm can be obtained through simulation of cost processes (See Figure 2). We can also find the time for the first marketed the drug and the time for the second marketed drug (i.e., the monopoly and duopoly phases).

First, we calculate the future profits after the duopoly phase both for the duopolistic situation and monopolistic situation. The losing firm's value in the monopolistic phase involves optimal investment/abandonment decisions and can be obtained by backward induction, regressing the continuation value along each of the sample paths which are in the monopoly phase at the same date onto a set of basis functions of the state variables, K and S at the same date. Regression coefficients are used to estimate the conditional expectation (continuation value). If the regression forecast exceeds the costs of investing in R&D for another quarter, the losing firm should continue investing. This procedure gives us: (1) the losing firm's optimal R&D investment/abandonment decisions along each sample path for each quarter in the monopoly phase both in the case where both projects are alive and in the case where there is only one project alive, and (2) the losing firm's value of all future cash flows back to the date that the winning firm commercializes the drug. The winning firm's project value and optimal investment/abandonment decision back to the first commercialization date can be easily obtained through similar procedures making use of losing firm's optimal investment/abandonment decisions. In the competitive R&D phase, we use the same approximation techniques and go through the Cournot-Nash game to find the Nash equilibrium R&D investment/abandonment decisions.

After we obtain all the R&D investment/abandonment decisions for the two groups, we implement a forward procedure that evaluates the profit along each sample path taking into account the R&D investment/abandonment decisions. We then average the results over all sample paths. We obtain the licensing group's value in terms of loser and winner in this R&D competing game. The licensee's value is obtained directly whereas the licensor's value is a byproduct when obtaining the licensee's values.

Illustrative Examples

We present post-phase I examples with time-varying drift to capture life cycle effects. Our analysis focuses on the profit split ratio (PSR) between licensor and licensee because this metric is used by pharmaceutical industry managers. Licensor value is the present value of licensing fees, milestone payments, and royalties. Licensee value is the present value of the residual difference between total project value and licensor value. We define the PSR as the ratio of licensee value to licensor value.

The licensee is a residual claimant somewhat like a common stock shareholder. The licensor expects a fixed set of payments in development stages, followed by royalties. In practice, PSRs of three (i.e., 75% to licensee, 25% to licensor) to four (i.e., 80% to licensee, 20% to licensor) are common. The parties typically negotiate the PSR based on the costs incurred from the project. Costs include those incurred by the licensor from project inception to the present versus development and commercial costs anticipated by the licensee. Although PSRs are usually greater than one, increasing competition for licensing agreements in recent years has shifted the balance of power in favor of licensors, occasionally resulting in a PSR less than one. Occasionally, the licensee may adopt a portfolio approach to establish the lowest acceptable PSR. The profit and risk of the licensing agreement are evaluated relative to existing licensing agreements and internal product pipelines. We classify PSRs below one as unacceptable to our licensee.

Following Schwartz (2003) and Schwartz and Moon (2000), we compare project values from our real option model to an approximate NPV by setting volatility parameters to zero and applying an appropriate risk-adjusted discount rate (see Schwartz (2003)). Licensee management monitors the development process and optimally abandons if continuation value is less than the investment required to keep the project alive.

Our base case situations are as follows. Firm X (the licensor) and Y (the licensee) negotiate a 20-year agreement for X's patent-protected compound that has passed the Phase I clinical trial. Y is responsible for subsequent phases, and it pays milestone payments as the drug advances through Clinical Phases II and III, and FDA Regulatory Review. The completed project generates sales revenues, and Y pays a 10.5 percent royalty. Expected Clinical Phase II development time is 2.5 years, three years for Clinical Phase III, and one year for FDA Regulatory Review. The annual failure probability for Clinical Phase II is 15%, and the annual failure probability for Clinical Phase III and FDA Regulatory Review is 8%. The resulting failure probability for the compound at the beginning of Clinical Phase II is around 50%.[9] Annual sales are $20 million. The cost volatility is 50% and sales volatility is 35%. Net operating cash flows are 75% of sales.

For the licensing agreement, development cost and milestone payment schedules are: (1) $40 million development cost for Clinical Phase II, (2) $60 million development

[9] This failure probability may be a little higher compared with Schwartz 2003, which states that the failure probability provided by the survey includes the "abandonment" decision in addition to the catastrophic events. However, it should not affect the analysis result per se.

cost for Clinical Phase III and FDA Regulatory Review together, (3) $16 million annual research investment for Clinical Phase II, and $15 million annually each for Clinical Phase III and FDA Regulatory Review, (4) $5 million milestone payment after Clinical Phase II, and (5) $12 million milestone following FDA Regulatory Review.

We begin with the symmetric duopolistic case by introducing another comparable licensing group (i.e., the same development cost, milestone payments, royalty rates and catastrophic events). Based on NPV calculated: (1) at a risk-adjusted rate equal to riskless rate plus annual failure probability minus the risk-adjusted sales growth rate, and (2) with an initial licensing fee of $0.5 million dollars, we estimate a PSR of 1.33 (57%/43%) in the monopoly situation and, for the symmetric duopoly situation, it yields: (1) the licensee negative profits holding all else constant or decreasing licensing payments by half, (2) the licensee a PSR of 0.887 (47%/35%) when sales are estimated to be 1.75 times the initial monopoly sales holding all else constant, and (3) the exact profit and PSR as the monopolist when sales are double holding all else constant. Under NPV calculation, the duopoly situation makes the licensing agreement unattractive to the licensee without re-negotiating because the revenues from commercialization decrease by half. Without re-negotiating terms, the unattractive situation improves only if estimated sales can be increased (see Table 1).

Table 1

Item	Monopoly			Duopoly - No Change		
Parties	Static NPV	Upfront Payment	PS	Static NPV	Upfront Payment	PS
Licensee	33.2591	0.5	56.74%	-17.4863	0.5	0.00%
Licensor	24.4717		43.26%	16.0649		100.00%

Item	Duopoly - Half of Milestone and Royalty Payments			Duopoly - Initial Sales 1.75 times of Monopoly			Duopoly - Initial Sales 2 times of Monopoly		
Parties	Static NPV	Upfront Payment	PS	Static NPV	Upfront Payment	PS	Static NPV	Payment	PS
Licensee	-9.56549	0.25	0.00%	20.5728	0.5	46.74%	33.2591	0.5	56.74%
Licensor	8.03247		100.00%	22.37		53.26%	24.4717		43.26%

For project value with abandonment using the base case parameters (sales volatility 35%, cost volatility 50%) we estimate a PSR of 1.50 (60% / 40%) for the monopolistic case, and 0.59 (37% / 63%) for the symmetric duopolistic case[10] ceteris paribus. Because of the presence of a duopolist, the PSR decreases to less than one, and as expected, the profit decreases, too. Holding all else constant by increasing initial sales by 1.7 times, we estimate a PSR of 1.17 (54%/46%). It seems that the profit split ratio increases to a greater degree than in a monopoly case as uncertainty increases. However, total project value decreases, even with 1.7 times the initial sales, because of the presence of a duopolist (see Tables 2 and 3). We also find:

(1) Under both scenarios, the PSR increases as sales and/or cost volatility increases.

(2) Under both scenarios, licensee project value with abandonment option increases with greater amounts of either source of uncertainty. Licensor project value does not display the same monotone relation to changes in either sales or cost uncertainty.

(3) The smaller positive investment opportunity value based on NPV implies higher abandonment flexibility value to the licensee. NPV also implies that both licensee's and licensor's project value is relatively more sensitive to changes in either sales or cost volatility.

(4) Under both scenarios, sales volatility has a greater impact on licensee's and licensor's project value compared with cost uncertainty.

(5) The sum of option values is not zero. This is a byproduct of the characteristics of the relation between licensee and licensor, i.e., their positions are not symmetrical.

[10] For the number reported here, we assume the same technology shocks. The detailed results for different technology shocks will be reported in a future version of this paper.

Preliminary results from the symmetric duopolistic case reinforces our conjecture that the estimated project value is affected by competition. Based on our example parameters, it appears that the option to abandon is worth more in the symmetric duopoly case than in the monopolistic situation. This is not surprising given the smaller NPV accruing to the individual licensee under the symmetric duopolistic case. Research is under way concerning asymmetric duopoly situations. We will present asymmetric duopoly results in a future version of this paper.

IV. Conclusion

Preliminary results reinforce our conjecture that uncertainties embedded in competition for the project have an impact on the PSR. The true PSR will deviate from estimates obtained through the typical NPV analysis under uncertainty, and also from the contingent claims analysis under monopoly. Based on our example parameters, it appears that the option to abandon is worth more in the symmetric duopoly case than in the monopolistic situation. This is not surprising given the smaller NPV accruing to the individual licensee under the symmetric duopolistic case. The outcome from the Cournot-Nash game impacts our results, and we look forward to presenting results from more elaborate simulations in future research.

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Figure 1.
Simulated Total Cost to Completion – One of the Groups

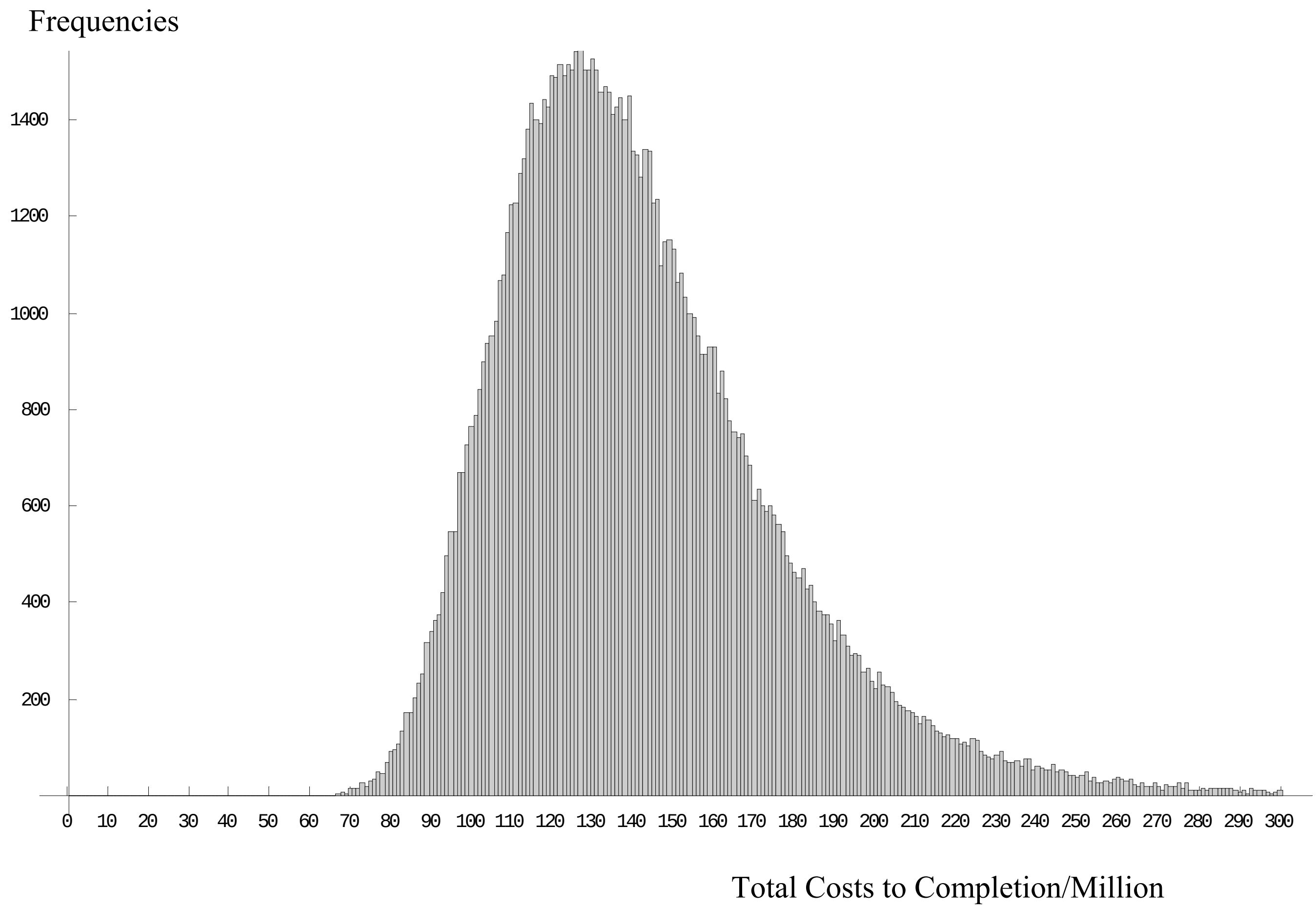


Figure 2.
Simulated Completion Times (Quarterly Time Step) – One of the Groups

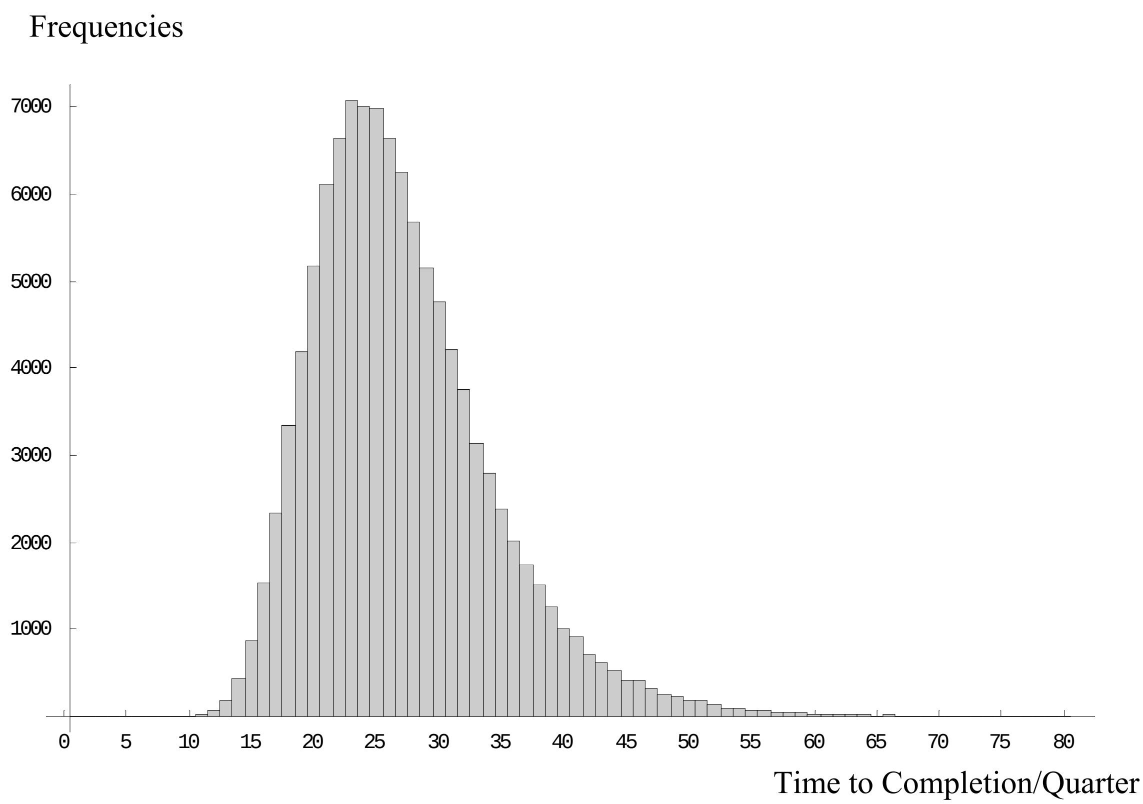
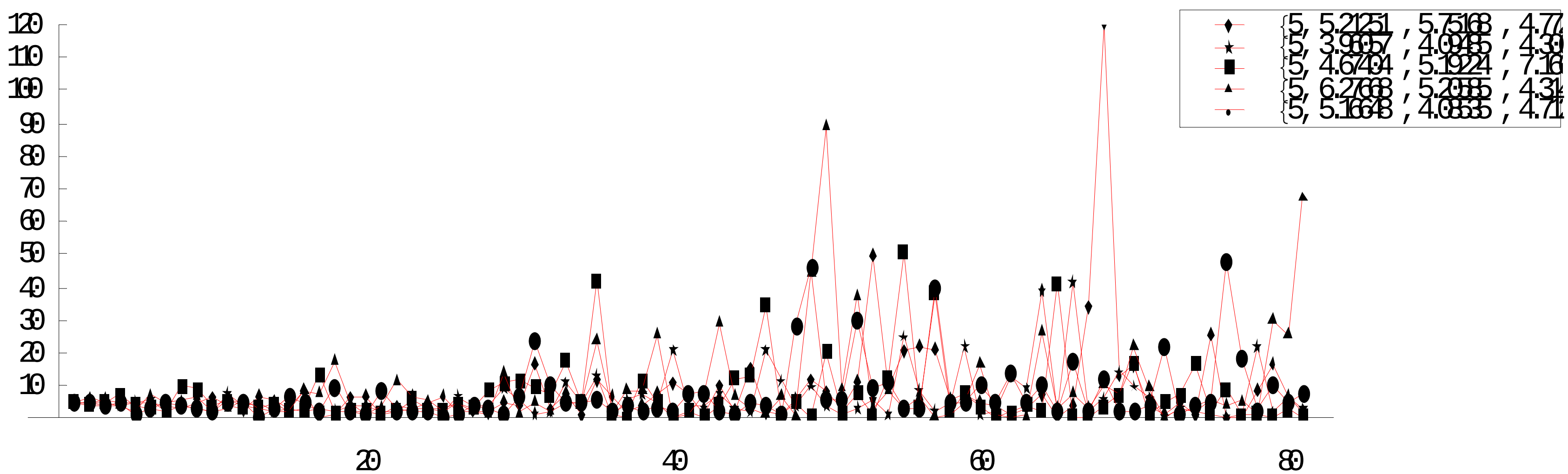


Figure 3.

Simulated Sales Revenue (Quarterly Time Step) – 5 Random Paths

Quarterly Sales Revenue/Million



Time/Quarter

Figure 4.

Simulated Mean Sales Revenue
(Quarterly Time Step; 110,000 Paths)

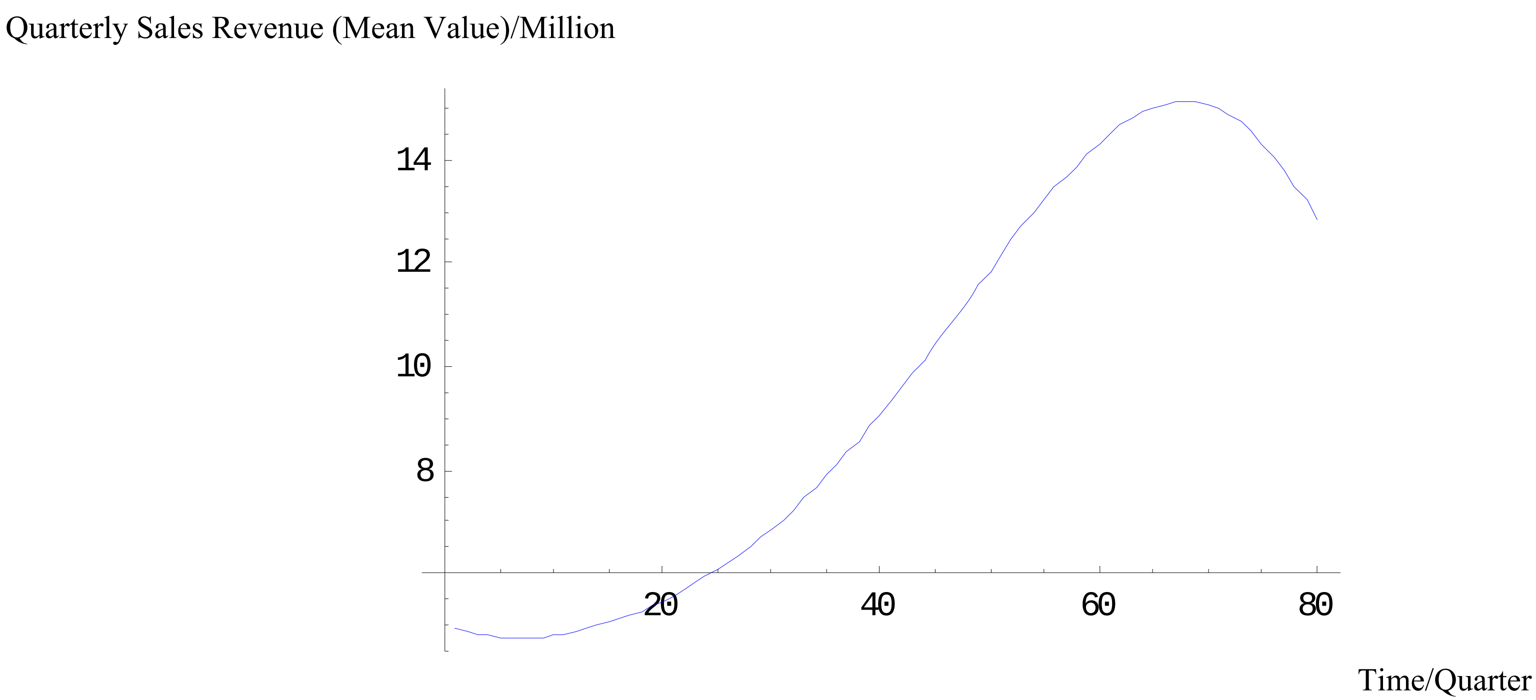


Table 2. Licensee and Licensor Project Values and Profit Allocations for Monopoly and Symmetric Duopolistic Case (1.7X Sales)

Project values and profit allocations are generated from the real options model described in Section 2. Licensor and licensee have a 20-year agreement for a project that has passed the Phase I clinical trial. The licensee pays a 10.5 percent royalty when marketed. Expected development times are 2.5 years, 3 years and 1 year for Phases II, III, and FDA regulatory review, respectively. The annual failure probabilities are 15% for Phase II, and 8% each for Phase III and FDA regulatory review. Annual sales are $20 million. Net operating cash flows are 75% of sales. Cost schedule includes: (1) $40 million for Clinical Phase II, (2) $60 million for Clinical Phase III and FDA Regulatory Review, (3) $16 million annual research investment for Clinical Phase II, and $15 million annually each for Clinical Phase III and FDA Regulatory Review, (4) $5 million milestone payment after Clinical Phase II, and (5) $12 million milestone following FDA Regulatory Review.

Cost Volatility = 0.5	Monopoly							
Cash Flow Uncertainty	Parties	Value with Abandonment Option	Value without Abandonment Option	Option Value	Upfront Payment	PS with Abandonment Option	PS without Abandonment Option	Sum-Opt Value
0.25	Licensee	33.9717	33.5219	0.4498	0.5	58.13%	57.21%	
	Licensor	23.61	24.1942	-0.5842		41.87%	42.79%	-0.1344
0.3	Licensee	34.5397	33.6991	0.8406	0.5	58.85%	57.32%	
	Licensor	23.3008	24.2218	-0.921		41.15%	42.68%	-0.0804
0.35	**Licensee**	**35.2958**	**33.8916**	**1.4042**	**0.5**	**59.72%**	**57.43%**	
	Licensor	**22.9645**	**24.2537**	**-1.2892**		**40.28%**	**42.57%**	0.115
0.4	Licensee	36.1058	34.0889	2.0169	0.5	60.64%	57.54%	
	Licensor	22.6115	24.2865	-1.675		39.36%	42.46%	0.3419
0.45	Licensee	37.1123	34.276	2.8363	0.5	61.65%	57.64%	
	Licensor	22.2717	24.3173	-2.0456		38.35%	42.36%	0.7907

Cost Volatility = 0.5	Symmetric Duopoly; Sales = 1.7X							
Cash Flow Uncertainty	Parties	Value with Abandonment Option	Value without Abandonment Option	Option Value	Upfront Payment	PS with Abandonment Option	PS without Abandonment Option	Sum-Opt Value
0.25	Licensee	22.3593	21.1368	1.2225	0.5	51.01%	47.75%	
	Licensor	20.4978	22.0787	-1.5809		48.99%	52.25%	-0.3584
0.3	Licensee	23.2674	21.314	1.9534	0.5	52.58%	47.93%	
	Licensor	20.0329	22.108	-2.0751		47.42%	52.07%	-0.1217
0.35	**Licensee**	**24.4025**	**21.5065**	**2.896**	**0.5**	**54.30%**	**48.13%**	
	Licensor	**19.6193**	**22.1399**	**-2.5206**		**45.70%**	**51.87%**	0.3754
0.4	Licensee	25.7486	21.7048	4.0438	0.5	56.10%	48.33%	
	Licensor	19.2591	22.1728	-2.9137		43.90%	51.67%	1.1301
0.45	Licensee	27.231	21.8909	5.3401	0.5	57.93%	48.50%	
	Licensor	18.913	22.2096	-3.2966		42.07%	51.50%	2.0435

Table 3. Licensee and Licensor Project Values and Profit Allocations for Monopoly and Symmetric Duopolistic Case (1.7X Sales)

Project values and profit allocations are generated from the real options model described in Section 2. Licensor and licensee have a 20-year agreement for a project that has passed the Phase I clinical trial. The licensee pays a 10.5 percent royalty when marketed. Expected development times are 2.5 years, 3 years and 1 year for Phases II, III, and FDA regulatory review, respectively. The annual failure probabilities are 15% for Phase II, and 8% each for Phase III and FDA regulatory review. Annual sales are $20 million. Net operating cash flows are 75% of sales. Cost schedule includes: (1) $40 million for Clinical Phase II, (2) $60 million for Clinical Phase III and FDA Regulatory Review, (3) $16 million annual research investment for Clinical Phase II, and $15 million annually each for Clinical Phase III and FDA Regulatory Review, (4) $5 million milestone payment after Clinical Phase II, and (5) $12 million milestone following FDA Regulatory Review.

Cash Flow Volatility = 0.35		Monopoly							
Cost Uncertainty	Parties	Value with Abandonment Option	bandonme	Value without Abandonment Option	Option Value	Upfront Payment	PS with Abandonment Option	PS without Abandonment Option	Sum-Opt Value
0.4	Licensee	33.9679	11.91%	32.7693	1.1986	0.5	59.02%	56.81%	
	Licensor	22.7379		24.0368	-1.2989		40.98%	43.19%	-0.1003
0.45	Licensee	34.5765	12.08%	33.2862	1.2903	0.5	59.34%	57.10%	
	Licensor	22.8464		24.1376	-1.2912		40.66%	42.90%	-0.0009
0.5	**Licensee**	**35.2958**	**12.38%**	**33.8916**	**1.4042**	**0.5**	**59.72%**	**57.43%**	
	Licensor	**22.9645**		**24.2537**	**-1.2892**		**40.28%**	**42.57%**	0.115
0.55	Licensee	36.1139	12.75%	34.5905	1.5234	0.5	60.14%	57.80%	
	Licensor	23.1016		24.3872	-1.2856		39.86%	42.20%	0.2378
0.6	Licensee	36.9937	13.50%	35.3613	1.6324	0.5	60.57%	58.20%	
	Licensor	23.2552		24.5333	-1.2781		39.43%	41.80%	0.3543
Cash Flow Volatility = 0.35		Symmetric Duopoly; Sales = 1.7X							
Cost Uncertainty	Parties	Value with Abandonment Option	bandonme	Value without Abandonment Option	Option Value	Upfront Payment	PS with Abandonment Option	PS without Abandonment Option	Sum-Opt Value
0.4	Licensee	23.1137	19.13%	20.3719	2.7418	0.5	53.25%	47.00%	
	Licensor	19.3528		21.9111	-2.5583		46.75%	53.00%	0.1835
0.45	Licensee	23.6988	19.26%	20.8943	2.8045	0.5	53.73%	47.53%	
	Licensor	19.4776		22.0175	-2.5399		46.27%	52.47%	0.2646
0.5	**Licensee**	**24.4025**	**19.47%**	**21.5065**	**2.896**	**0.5**	**54.30%**	**48.13%**	
	Licensor	**19.6193**		**22.1399**	**-2.5206**		**45.70%**	**51.87%**	0.3754
0.55	Licensee	25.271	19.60%	22.2131	3.0579	0.5	54.94%	48.80%	
	Licensor	19.8198		22.2806	-2.4608		45.06%	51.20%	0.5971
0.6	Licensee	26.149	19.90%	22.9934	3.1556	0.5	55.52%	49.53%	
	Licensor	20.0462		22.4167	-2.3705		44.48%	50.47%	0.7851